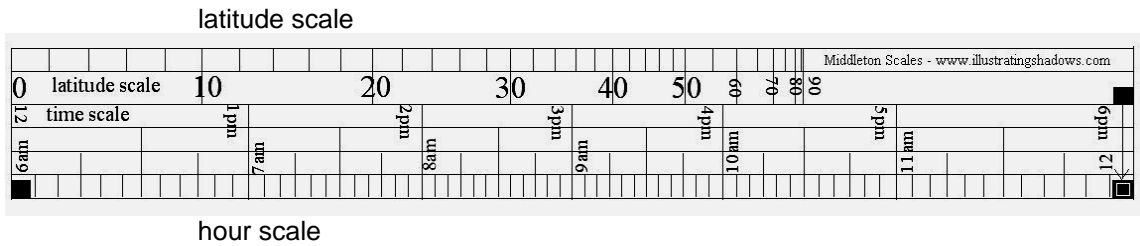


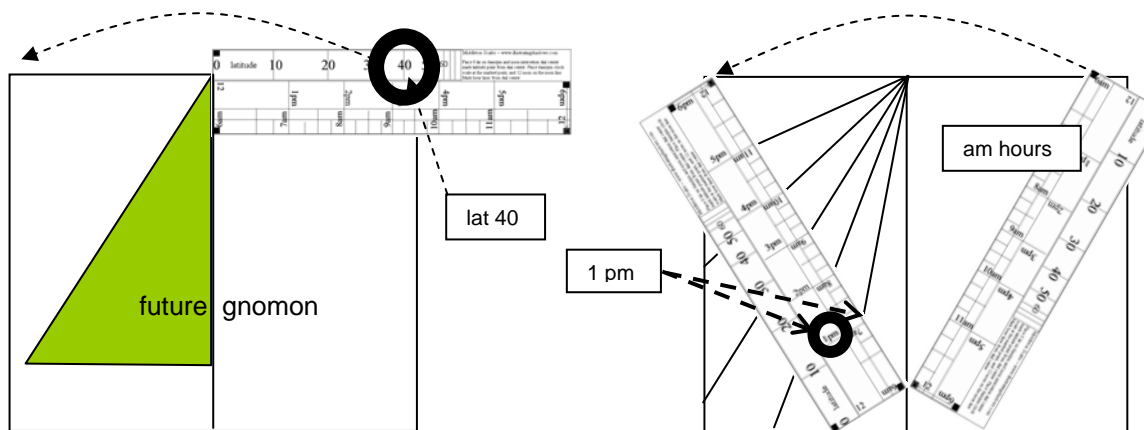
## OLDER METHODS ~ DIAL DESIGN USING THE MIDDLETON SCALES



Originally used in the mid to late 1600s, the scales were enhanced by E. C. Middleton in the late 1800s, and made more popular by Frank W Cousins in his book *Sundials - A Simplified Approach by Means of the Equatorial Dial*, also known as *Sundials – The Art and Science of Gnomics*. These scales may be used to design vertical, horizontal, polar, meridian, and declining dials.

### Horizontal dial design

A dial plate is drawn with the 6 am, 6 pm, and noon line, thus establishing a dial center. The latitude scale is placed with 0 degrees latitude at dial center, and the new dial's latitude marked, in this case latitude 40, and copied an equal distance on the other half of the dial.



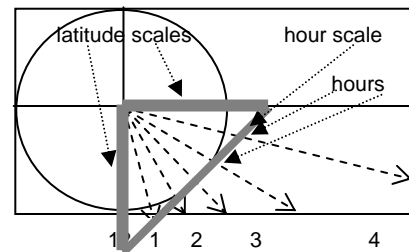
The hour scale is placed with the 6 am point on the 6 am line where the latitude was marked, and the noon point on the noon line, or the 6 pm point may be placed on the 6 pm line with the noon point on the noon line. Hour lines are drawn from dial center to the hour point on the hour scale.

### Vertical dial design

A vertical dial is drawn using the co-latitude. The hour line angles of a horizontal dial match the angles for a vertical dial at the co-latitude, and vice versa. Thus, proceed as above, however in place of latitude, use 90-latitude. Of course, the co-latitude is the angle between the style and the sub-style.

### Polar and Meridian (true east or true west) dial design

If the latitude scales were unfolded to be two sides of a triangle, the time scale being the third, then the hour marks would each be 15 degrees from where the two latitude scales meet. In this case a simple protractor may be easier!

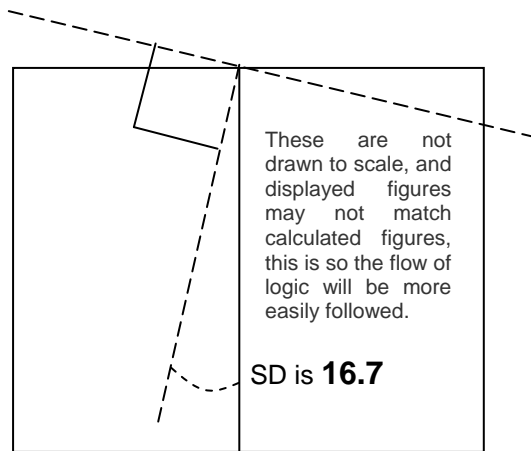


Vertical decliner design

A vertical decliner has the gnomon rotated by an amount called the style-distance based on the latitude and wall declination. Assume a latitude 30 dial with 10 degrees east wall declination.

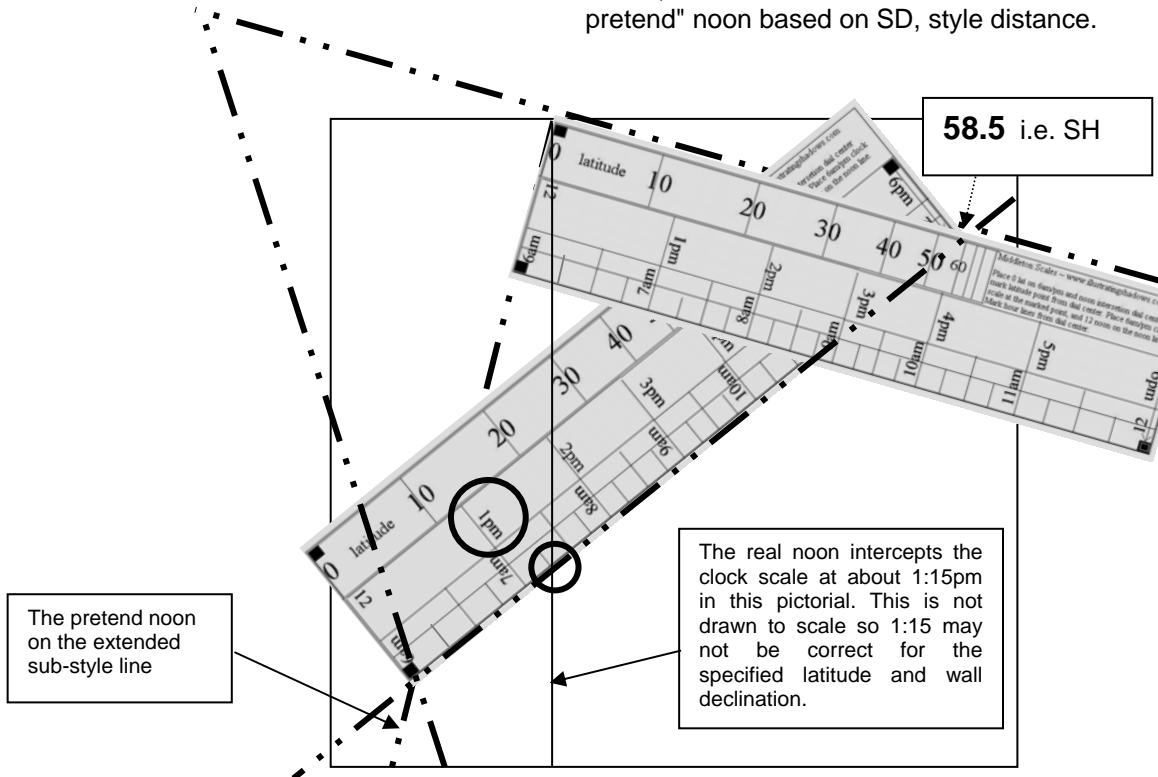
Style Distance:  $SD = \text{atan}(\sin(\text{dec}) / \tan(\text{lat}))$  **- 16.7**  
 Style Height: style and sub style angle is:  $SH = \text{asin}(\cos(\text{lat}) * \cos(\text{dec}))$  **+ 58.5**

Having calculated the style height (SH) and style distance (SD), draw a dial plate for the vertical decliner as shown below, the dashed line being angled from the vertical noon line by SD. At 90 degrees to that, draw another line as shown. [See Tables A5.1, A5.2 and formulae A8.22, A8.23.]

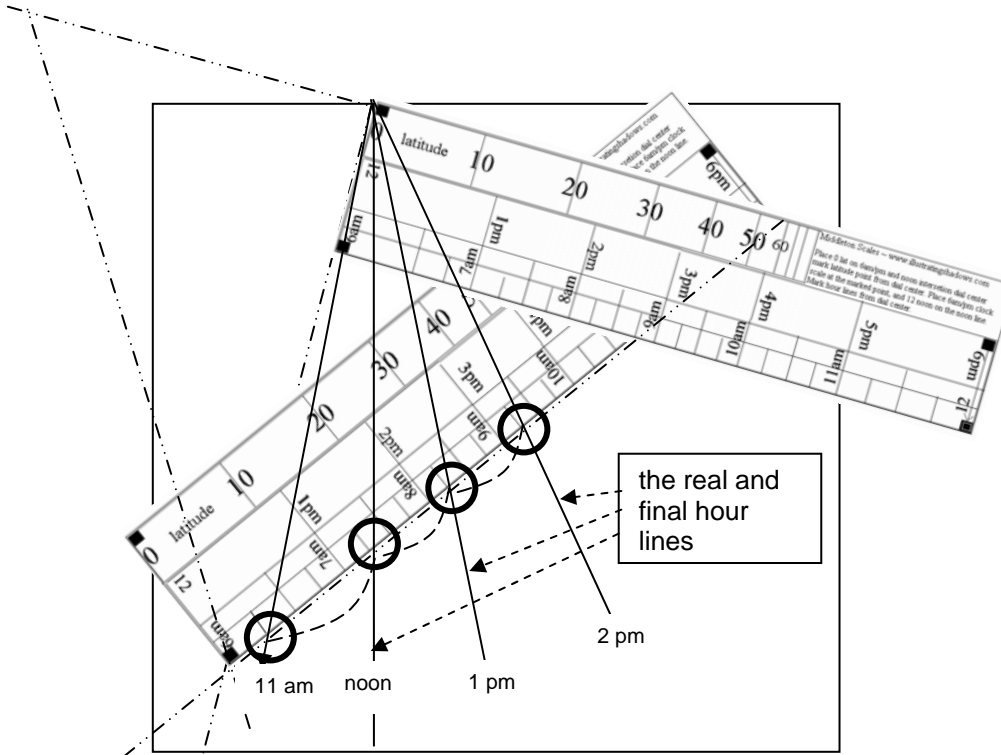


Using the two dashed lines, pretend they are noon and the 6 am and 6 pm lines (which they are not). But in so pretending, use the Middleton scales as for the vertical dial design, except use SH instead of the co-latitude (SH is the co-latitude were the wall declination zero). The Middleton scales create construction points.

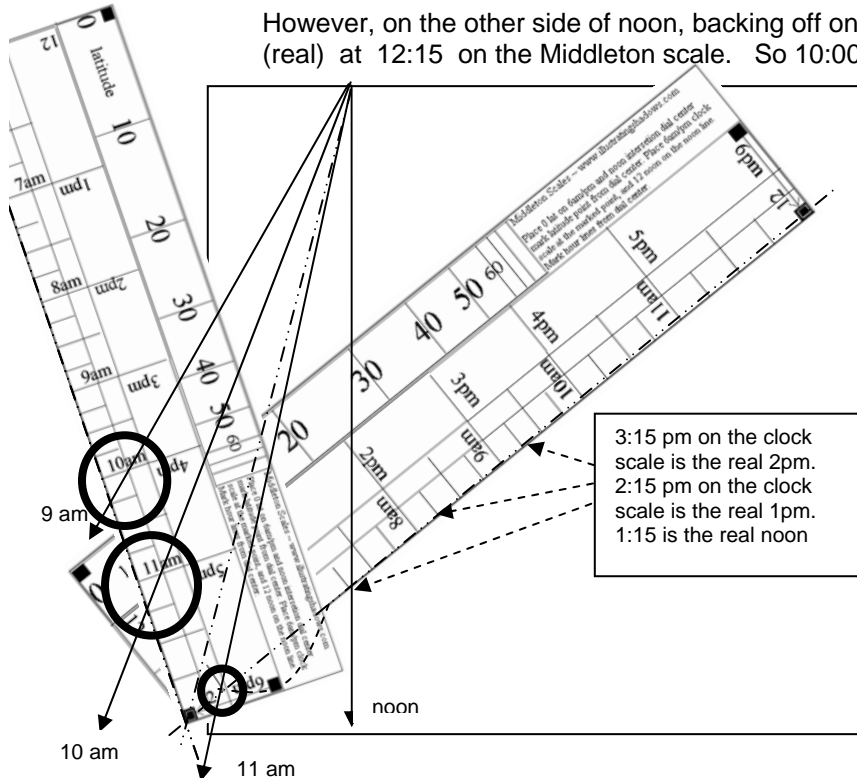
Then use the hours scale and draw the two edges as shown below. Note that the latitude used is the Style Height (SH). SH is the co-latitude if the wall declination was zero. Note where the real noon or vertical line intercepts the hours scale, and note the time difference from noon. These pictorials are not drawn to scale. Below, the real noon is at 1:15 from the "lets pretend" noon based on SD, style distance.



Again, assuming that the 1:15 time on the clock scale represents the real noon, then 2:15 on the clock scale will represent 1 pm, 3:15 on the clock scale represents 2 pm, and so on, at least on that side of the scale.



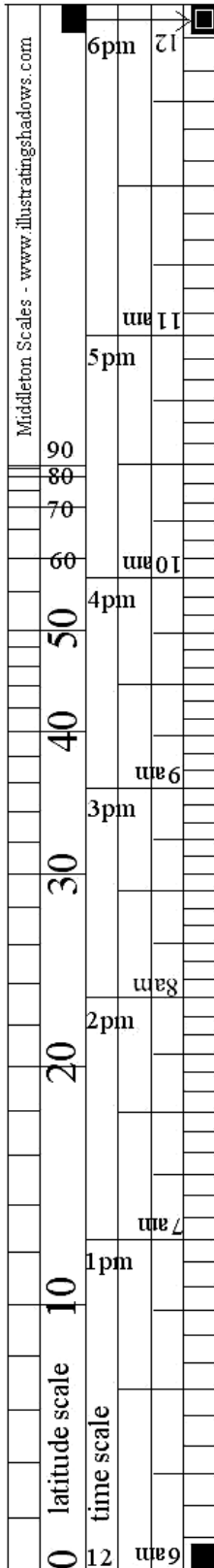
However, on the other side of noon, backing off one hour places 11 am (real) at 12:15 on the Middleton scale. So 10:00 am would be at the



11:15 on the Middleton scale, 9 am real would be at 10:15 on the Middleton scale, and so on.

This shows the symmetry of a declining dial around the extended sub-style.

### THE MIDDLETON SCALES



For a unit length on the hours scale of 10, the distances from III or IX are:-

Time h.hh	Dist	Time h.hh	Dist
6.00	12.00	5.00	9.00
6.25	12.25	4.38	9.25
6.50	12.50	3.84	9.50
6.75	12.75	3.34	9.75
7.00	13.00	2.89	10.00
7.25	13.25	2.47	10.25
7.50	13.50	2.07	10.50
7.75	13.75	1.70	10.75
8.00	14.00	1.34	11.00
8.25	14.25	0.99	11.25
8.50	14.50	0.66	11.50
8.75	14.75	0.33	11.75
9.00	15.00	0.00	12.00

$$\text{dist} = \text{scale [i.e. 10]} * \tan ( 15 * \text{hours} )$$

And the distances for latitude are:

Latitude	Distance	Latitude	Distance	Latitude	Distance
0	0.00	28	4.24	57.50	6.38
2	0.35	30	4.46	60.00	6.48
4	0.70	32	4.66	65.00	6.65
6	1.04	34	4.86	70.00	6.80
8	1.38	36	5.04	75.00	6.91
10	1.71	38	5.21	80.00	7.00
12	2.04	40	5.37	85.00	7.05
14	2.35	42	5.52	90.00	7.07
16	2.66	44	5.66		
18	2.95	46	5.79		
20	3.23	48	5.91		
22	3.50	50	6.02		
24	3.76	52.50	6.15		
26	4.01	55.00	6.27		

$$\text{dist} = \text{scale [i.e. 10]} * \sin(\text{lat}) / \text{sqrt}(1+\sin^2(\text{lat}))$$

A spreadsheet to calculate the scales is available at:-

[reference-middleton-scales.xls](#)

Note:  $\sin^2(\text{lat})$  means:-  $\sin(\text{lat}) * \sin(\text{lat})$

Note: Middleton scales can be used not only for horizontal, vertical, and vertical decliner, but also for polar and meridian dials.