

## CHAPTER 10

### A GENERAL GEOMETRIC MODEL OF THE PREVIOUS DIALS

This chapter covers the following topics...

A general geometric model of the dials discussed so far  
By now many normal dial building methods have been discussed  
Is there a common architecture

This chapter encapsulates some important thoughts.

First, one simple model based on a gnomon explains all the hour angle based dials discussed so far. And an ancient concept from around 1525 bears out this thought.

Key points to remember are:-

Any dial of any type can be designed with an auxiliary equatorial dial.

And as a shortcut to that, many of the complex dials such as vertical south decliners can be designed using an auxiliary horizontal dial.

In other words, the horizontal dial is a short cut to designing vertical dials, even declined ones as will be seen later, just as was the equatorial dial a shortcut for the horizontal and polar dial, as well as the east or west facing dial.

This chapter starts with some examples of the sun's rays, moves to the generic model, and concludes with more examples of the sun's rays for polar, horizontal, and vertical dials. That repetition is intended to enhance the general model of the sundial. CAD is used here to show the interaction of a range of rays throughout the year for any given hour. Additional examples of CAD based drawings showing the interaction of shadows of an hour spread across the year can be found in chapter 18.

#### **Some further research:-**

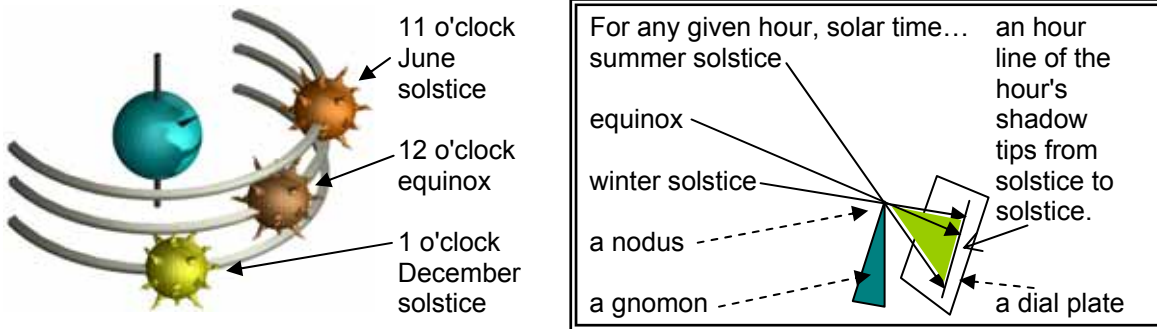
R. Sagot and D. Savoie of the sundial section of the French astronomical society developed a method, documented in Chapter 58 of Jean Meeus's work called "Astronomical Algorithms". An avid sundial enthusiast might choose to study their general methodology.

Frank Cousins in his "A Simplified Approach by Means of the Equatorial Dial" has towards the end a lucid explanation of the spherical triangle, and from that he develops the various formulae used in sun dial construction.

This chapter expands on these topics.

## HOW THE SUN'S RAYS CAUSE RANGES OF SHADOWS

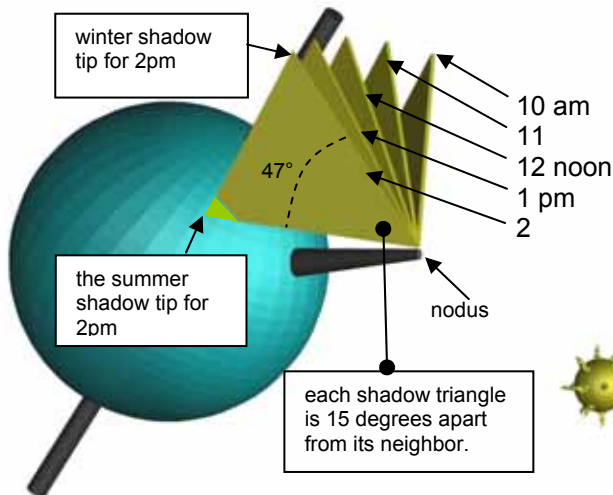
For a sundial, the sun rotates around the earth, and as there are 360 degrees in a circle and 24 hours in a day, that means the sun rotates around the earth's polar axis 15 degrees per hour. Those 15 degrees define solar time, and as we have seen, it varies from clock time due to many factors, one being that the earth's annual orbit around the sun is elliptical, not circular.



The sun moves north from December to June, south June to December as shown above. If we made a shadow casting device (called a gnomon) with a tip or other mark on it (called a nodus) that caused a clearly defined point or tip of a shadow, and at the same solar (not clock) time each day marked the shadow's tip, over a year a line would be drawn by that shadow tip, an hour line, see above right pictorial. The hour line results from the base of a triangle, solstice to solstice, of shadow lines for that solar hour.

If clock time were used as opposed to solar time, then that line would look like a figure of eight, and demonstrate the equation of time. Some sun dials have those figures of eight on their hour lines, and it is called an analemma, and that figure of eight is not symmetrical.

In the picture below the hour shadow triangles are in one hour increments for simplicity, they are triangular because, as above, they show the range of the hour's shadow tip throughout the year.

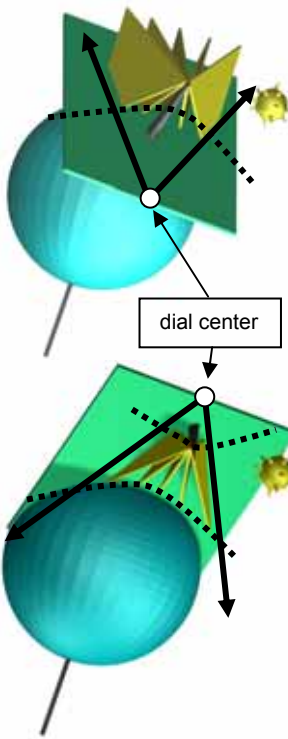


One hour increments are common on many sun dials, and each hour from winter to summer solstice builds a set of shadow ranges. Each is triangular. Each is 47 degrees at the apex, this being twice the sun's journey of 23.5 degrees (approximately). And the apex, as far as shadows go is a point (called a nodus) on a shadow casting device (called a gnomon).

To the left is planet earth shown rather large, and to its right is the sun, shown rather small. And some triangular shadow rays are shown from 10 am to 2 pm each being 15 degrees from its neighbor.

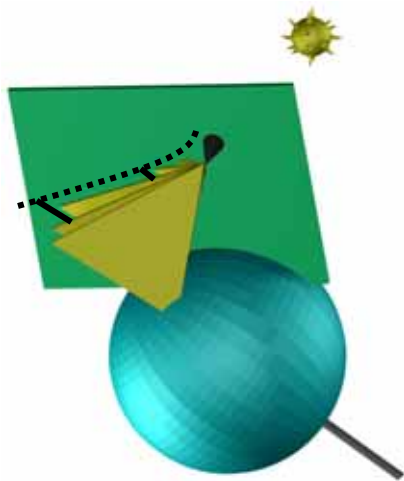
Each of the rays shows the winter and summer solstice limits for a given solar (not clock) time. Those limits form an hour line when they hit a surface. And each shadow ray triangle thus shows where each hour line will be.

The surface holding those hour lines (called a dial plate) can be at any angle to those rays, horizontal and vertical are the two most common. And vertical dials may be southerly or they may face east or west. The pictures below show a horizontal, then a vertical south facing dial, then a west facing vertical dial.

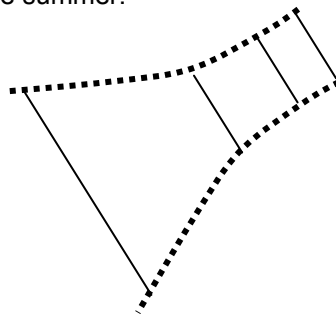
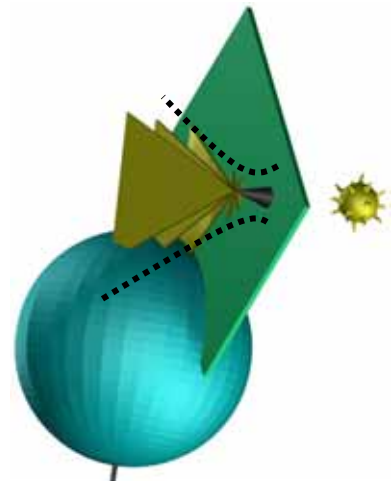


To the left is a dial plate for a horizontal dial. The solid lines are for 10am and 2pm, as on the previous page. The dashed line is the limit of the summer shadow tips, it marks the summer solstice, around June 21. The hour lines do not center on the base of this columnar gnomon, they have a "dial center" elsewhere. And if a line were drawn from the dial center to the nodus, the tip in this case of the gnomon, it would be called the style, and the angle it would form with the dial plate would be the dial's latitude.

To the left is a dial plate for a vertical south facing dial. The solid lines are for 10am and 2pm, as before. The dashed lines are the limit of the solstice shadow tips. The top "U" shaped curve is the winter solstice, around December 21, and the lower "∩" marks the summer solstice curve, around June 21. The hour lines do not center on the base of this columnar gnomon, they have a "dial center" elsewhere. And if a line were drawn from the dial center to the nodus, the tip in this case of the gnomon, it would be called the style, and the angle it would form with the dial plate would be the dial's co-latitude. Of course the angle it would form with the horizontal would be the latitude..



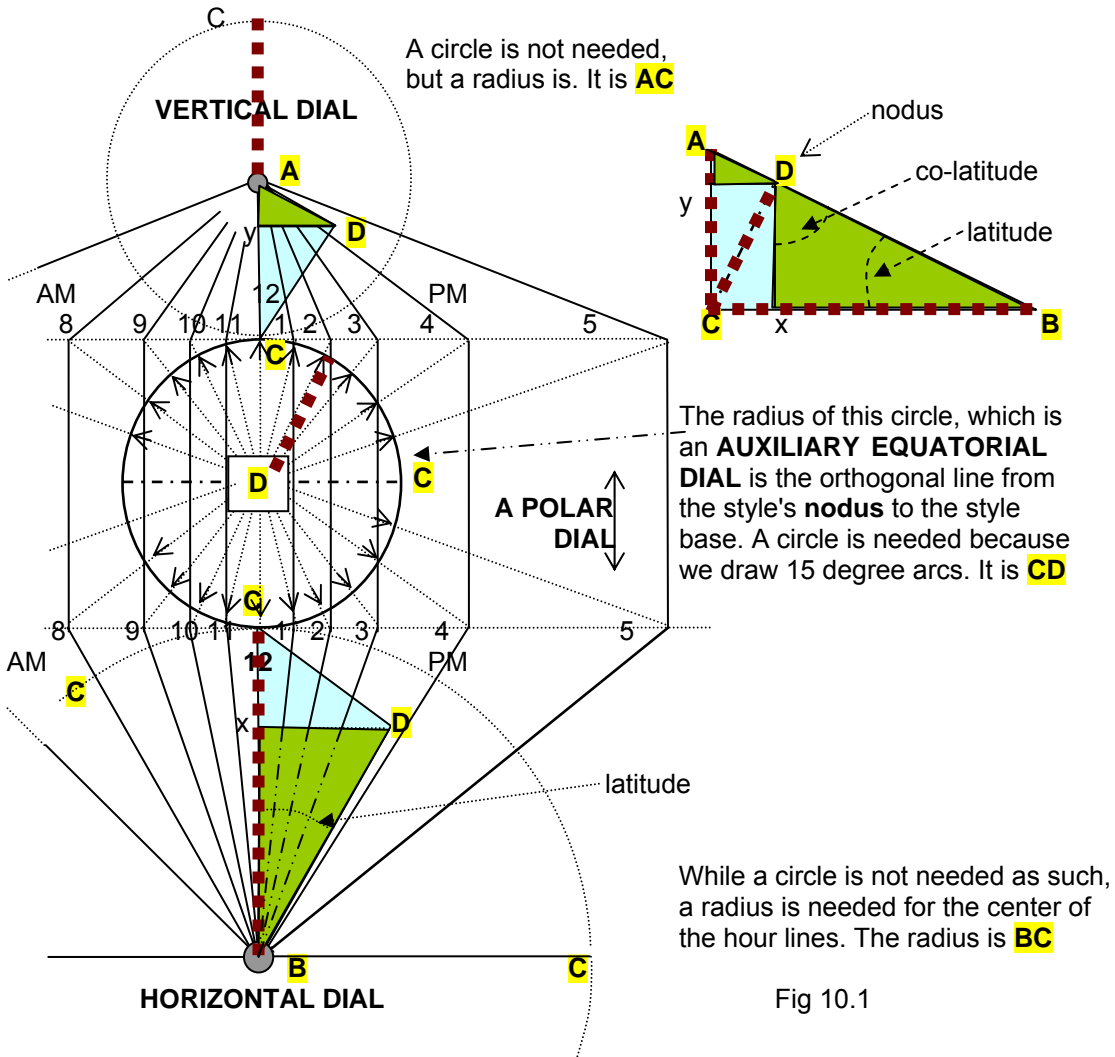
To the left and right are dial plates for the vertical dial which is facing true west. The pure east or west facing dial center is at infinity! Solid lines are hour lines, dotted lines are annual limits. The upper curve is the winter solstice curve, the lower curve is for the summer.



Any dial plate shape may be modeled in this manner, such as the armillary, equatorial, or any irregular shape.

## A GENERAL GEOMETRIC MODEL OF THE PREVIOUS DIALS

We have now seen the geometric methods for designing the armillary and equatorial dial, the polar and meridian dials (true east or true west), as well as the vertical (true south) and the horizontal dial. We can blend all those concepts into one easy to remember model. This is intended to show a natural symmetry in geometric dial design.



The above general geometrical model shows a symmetry in the design of the polar, vertical, and horizontal dials in the context of their gnomon's style. For a polar or a meridian dial, CD is the linear height of the style's nodus above the dial plate. In this model, CD does different things depending on dial type.

For a horizontal dial, this model is based on the ratio of the DC to CB. For a vertical dial the ratio is based on DC to CA. In all cases, DC is the equinoctial ray going to the dial plate at C from nodus D.

The model in figure 10.1 was developed by Albrecht Durer in 1525 and is pictured below as figure 10.2.

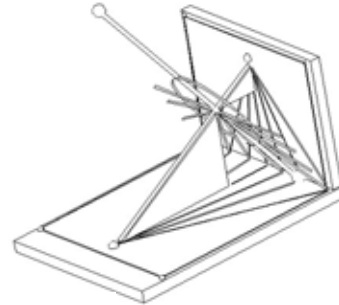
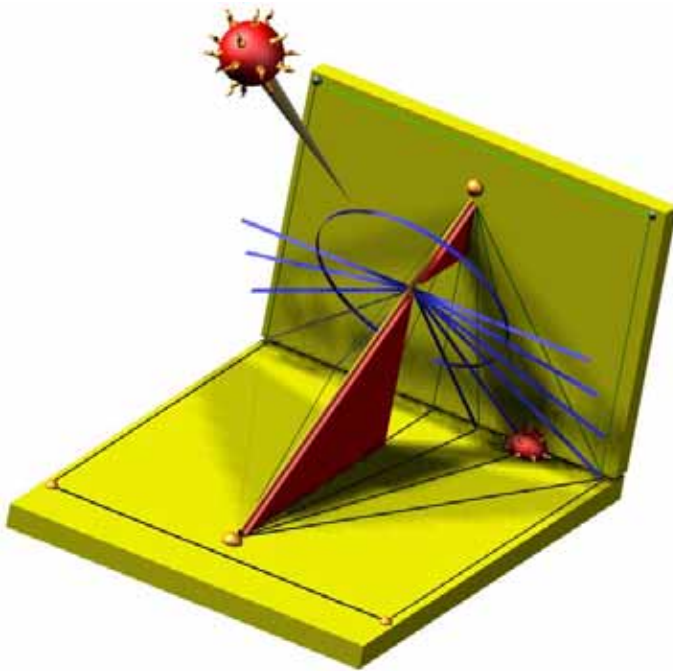


Fig 10.2

Any dial not susceptible to the methods described here can be designed because that dial has a horizontal or vertical dial that matches it somewhere else on the planet. The French dialists Sagot and Savoie developed such a model.

Dials may be designed for portability, or for a specified location.

Latitude adjustments can be simplified by tilting a dial.

Longitude adjustments can be made by either considering the longitude difference when the hour lines are designed, or the difference can be included into a tailored equation of time.

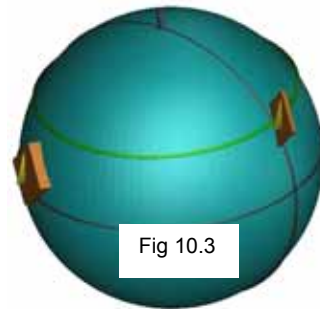


Fig 10.3

Portable dials should not have the longitude correction built in since it would limit their portability. If a dial has no longitude correction built into it, then it is simpler to design, and it can be relocated. The correction for the EOT is simply modified to include the longitude correction.

e.g.

EOT	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
	+9	+14	+9	=0	-3	=0	+6	+4	-5	-14	-15	-5



The longitude correction of say  $-8$  minutes that would have been built in to the dial, now becomes one correction table when you deduct the fixed 8 minutes from each EOT entry.

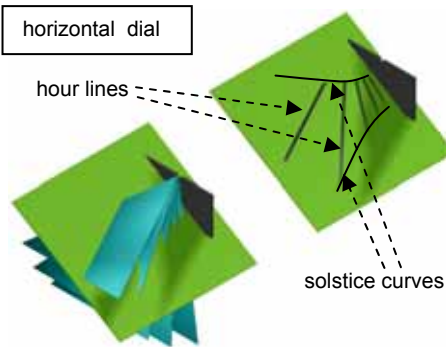
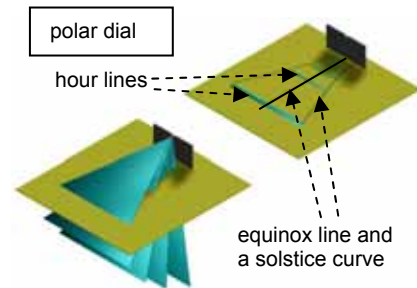
e.g.

EOT	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
	+1	+6	+1	-8	-11	-8	-2	-4	-13	-22	-23	-13

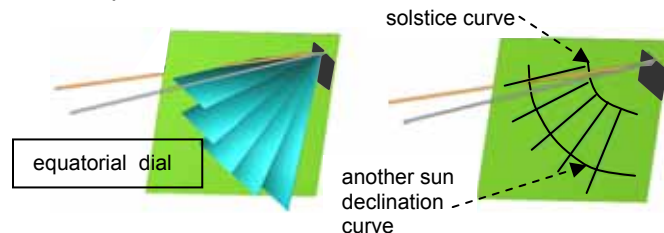


## CAD (COMPUTER AIDED DESIGN) CAN PROVIDE AN INSIGHT INTO DIAL DESIGN

Moving from the world of the sixteenth century to the present time, 3d CAD can create insights into how a dial may be designed and how it works. The chapter on spreadsheets and CAD has an example of a horizontal dial design using just CAD alone. As the Durer model indicates, a dial may be designed by considering rays coming from the nodus and those rays have a center (equinox rays), and the outer edges are the solstice rays. To the right is a dial plate, a vertical gnomon and a nodus for a polar dial. On the left half of the picture, the rays diverge from that nodus and create the solstice-equinox-solstice limits for a given hour. Then the 3d intersect tool of CAD is used on the right half of the picture, with the dial plate added back in, and it shows the hour lines as well as the solstice limits. Then using the CAD Bezier line drawing, the actual solstice and equinox lines are added and converted to solids so that they can show clearly. In essence, this is a computer based trigon, the very essence of Durer's concepts.



In the next example to the left, a horizontal dial is built and viewed at an angle, and here the gnomon has a style with a nodus about half way along the style. The nodus is used to indicate calendar information (sun declination curves), and the complete style makes the hour line more visible and easily readable. The same process was used as for the polar dial above.



In the third example, an equatorial dial is depicted with a square dial plate, and a gnomon not normally used for equatorial dials, a cone or rod is more functional. The part to the left shows the usual rays and the equinox ray which never intercepts the dial plate, and another ray for a random sun declination. The same process allows us to generate the arcs or circles for a given sun declination along with the hour angles which in this case are 15 degrees.

In each case, a triangular ray of light was built as a 3d solid. Then several were duplicated albeit at 15 degrees apart, and for convenience they were then 3d-added so the complete set of light rays could be handled at one. Then a gnomon and nodus was built, and finally a dial plate. The dial plate was copied and kept just below the actual dial plate, that copy being needed for the final figure because the 3d-intersect process will destroy the original dial plate, see next paragraph.

The CAD programs have a 3d intersect tool which allows just the intersections of solid objects to remain, so a 3d intersect between the rays and the dial plate will show where the range of shadow or of light will fall, solstice to equinox to solstice, for each hour indicated by that triangular ray. In so doing however, the rays and the dial plate are deleted as only their common intersection remains, hence why a spare dial plate was copied, and it is moved back into place where the original dial plate was, and then additional dial furniture may be drawn. Such dial furniture may be the curves showing the hyperbolic solstice lines. CAD programs supporting 3d can be purchased very reasonably, this book used TurboCAD Deluxe from IMSI.

In summary... There is a symmetry in all dial design, and flexibility in how to manage corrections.

## CHAPTER 18

### Computer aided design, spreadsheets, and vrml.

This chapter covers the following topics...

#### THE USE OF CAD, OR COMPUTER AIDED DESIGN, AND VRML ON THE WEB

- 2d drawing
- 3d object modeling

#### THE USE OF SPREADSHEET PROGRAMS

- Converting formulae used in math to  
Formulae usable in a spreadsheet
- Using graph functions for altitude curves and hour lines

Spreadsheets are software programs that allow data to be stored in individual cells, or in columns, or in rows, and for manipulation of that data to take place.

This seems simple enough, however there are a few traps along the way.

This chapter expands on these topics.

## THE USE OF CAD, OR COMPUTER AIDED DESIGN

CAD comes in a couple of flavors, one is 2d drafting, the other is 3d modeling. There are some freeware products available, and the program products for which a fee is charged, vary from \$100 to many hundreds of dollars. Free CAD programs may not provide the angular resolution required. The author elected to use TurboCAD by IMSI. This section glosses over a few techniques that may vary from package to package, and offers ideas based on the author's learning curve.

### DRAFTING:

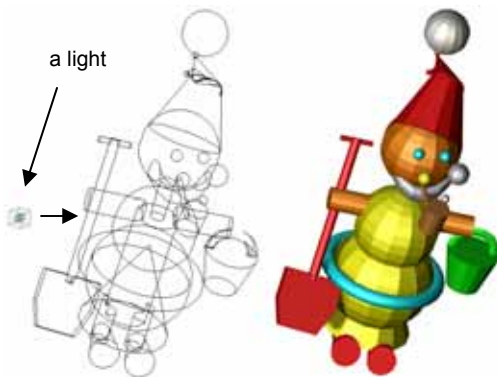
In 2d mode, drafting is simple with many options for lines and orthogonal lines. The first learning curve was selecting the starting point for an angle [OPTIONS, ANGLE, BASE ANGLE] for drawing or for measurement. Second, setting grid snaps could cause problems, as it does with any such program. A third problem was that the default tool was LINE DRAW and not SELECT which caused and still causes much consternation. Replication, linear or angular, requires caution when multiple copies are made. Otherwise 2d drafting is simple.

### MODELING:

In 2d mode, modeling a 3d object may cause confusion. For example drawing a cube may result in a message saying that you can't have a length of zero. However if one shifts from the normal view to say a south-west view (Isometric SW) then "no worries". When modeling an object, the color etc must be set before drawing the item. It can be set afterwards however the object must be selected first and then the attributes changed.

Positioning an object in 3d such as a column takes patience. The concept of a WORKPLANE and how to set it is critical, and affects all sorts of things such as subsequent objects, as well as lights. Failing to use the work-plane will result in frustration if you attempt to position objects since a movement on one dimension may cause an unexpected shift in another.

Objects are modified in wire-frame mode, not in the other display modes. Saving files regularly is critical. This book used a screen capture program to capture final rendered figures as JPG files.



The picture on the left shows the gnome in line as well as in rendered mode. TurboCAD provides excellent model rotation on any axis, and the 3d stereogram pictures were made by rotating the picture a few degrees to simulate the left and right eye's perspectives of the viewed object.

A major benefit to CAD is in drawing angles and distances. The Bezier line drawing is useful for curves such as used in azimuth, altitude, and calendar or declination lines for dial plates.

In 2d drawing mode, tables from a spreadsheet or pictures such as JPG can be pasted into the drawing which keeps the data to hand facilitating line and curve drawing. However the pasted item can dramatically increase the final file size. Also, such objects do not appear in 3d mode.

Text can be made to appear in 3d mode. Enter the text, then select it, then FORMAT, EXPLODE it twice, right click for properties, then enter a thickness in 3d.

Rotating a 3d object around the X axis is done by grabbing the Y axis of the model, rotating a 3d object around the Y axis is done by grabbing the Z axis of the model, and rotating a 3d object around the Z axis is done by grabbing the X axis of the model, The work-plane axis can be displayed if desired by OPTIONS, PREFERENCE, SHOW WORLD CS, it shows three little arrows in the bottom left of the screen showing where the X, Y, and Z axes are, which is critical when rotating an object, especially given the possibility of confusion when rotating an object.

Direction, point, and spot lighting are treated as objects, thus when a light is created, it goes on the work-plane, and its direction is adjusted using the same method as any other 3d object. Using four tiled windows with top, left, right, and southwest views facilitates object manipulation.

Different TurboCAD releases may not read their .TCW files. Use .DXF formats for compatibility, however colors may not be adjustable once saved in that format.

Should you use the free CAD program mentioned in this book for dial design, ensure you set SNAP to none, other wise line angles will not be suitable for dial plates.

### **MANAGING A CAD FILE AS AN INTERNET VIEWABLE VRML OBJECT**

A CAD file may be saved for viewing on the web in very simple steps. There are three steps depending on how your computer system is established. For people who only wish to view 3d, not generate it, steps 2 and 3 would apply. For people with a VRML browser plug-in already installed, step 3 only would apply. For designers, all three steps would apply.

1. Building a VRML (WRL) file for other to see
2. Installing the VRML browser plug-in if you do not have one
3. Viewing any 3d VRML file on the internet (WRL file suffix).

### **BUILDING A 3D WEB PICTURE FOR ANYONE ON THE WEB FROM TURBOCAD**

In TurboCAD save as WRL: The SETUP option in the SAVE panel was set to version 1.0, do not save as WRZ which while smaller, may have browser problems on the internet.

**STEP 1: SETTING UP TO VIEW 3D PICTIRES ON THE WEB (For designers)**  
Find out what plug-ins will work for you, there is a web site to advise you of your options.

VRML browser detect:  
<http://cic.nist.gov/vrml/vbdetect.html>

**STEP 2: INITIAL BROWSER PLUGIN INSTALLATION (First time viewers)**  
Download manually a browser plug-ins such as Cortona

Cortona VRML browser  
<http://www.parallelgraphics.com/products/cortona/download/iexplore/>

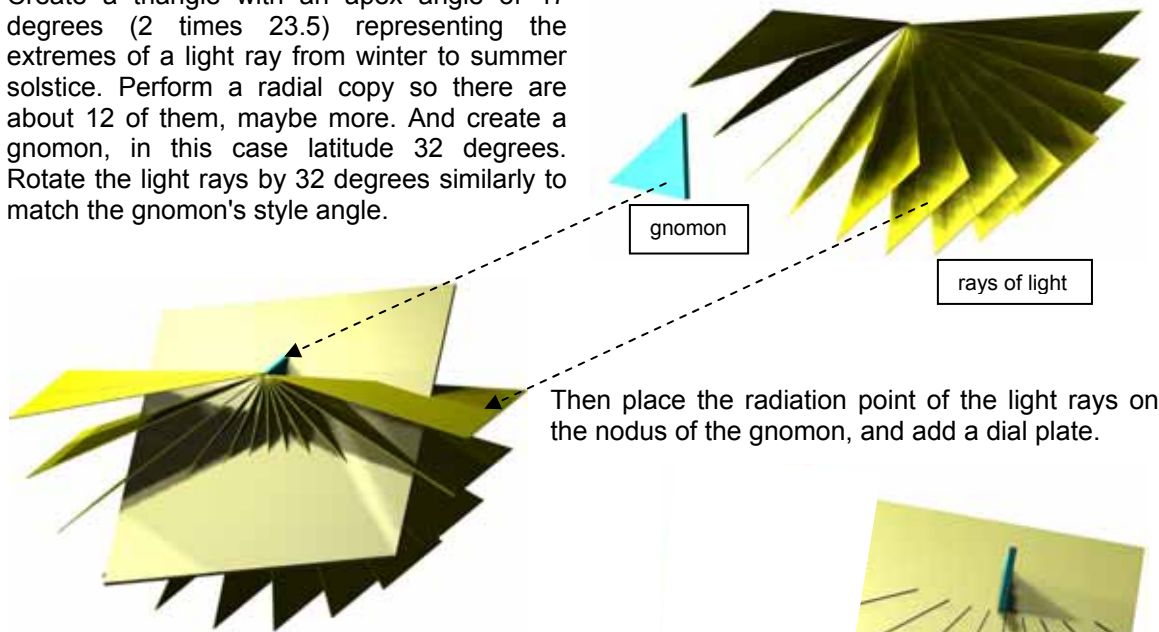
**STEP 3: VIEWING A 3D PICTURE ON THE WEB**  
Open a VRML file (WRL)

eg: there are several on the web site for this book.  
IF Internet Explorer gives a cautionary notice, accept it and let ACTIVE-X do its thing.  
Netscape and Internet Explorer both run Cortona well, and Cortona also works on Mozilla.  
the picture may be tiny, so zoom it by clicking FIT  
then PLAN to zoom back a bit  
then you may then manipulate the view with STUDY!

**OR USE A STANDALONE VIEWER (NOT USING A BROWSER) SUCH AS:**  
[http://www.sim.no/products/SIM\\_VRMLview/](http://www.sim.no/products/SIM_VRMLview/)

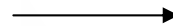
## USING CAD AS THE ACTUAL MODELING TOOL ITSELF

Create a triangle with an apex angle of 47 degrees (2 times 23.5) representing the extremes of a light ray from winter to summer solstice. Perform a radial copy so there are about 12 of them, maybe more. And create a gnomon, in this case latitude 32 degrees. Rotate the light rays by 32 degrees similarly to match the gnomon's style angle.

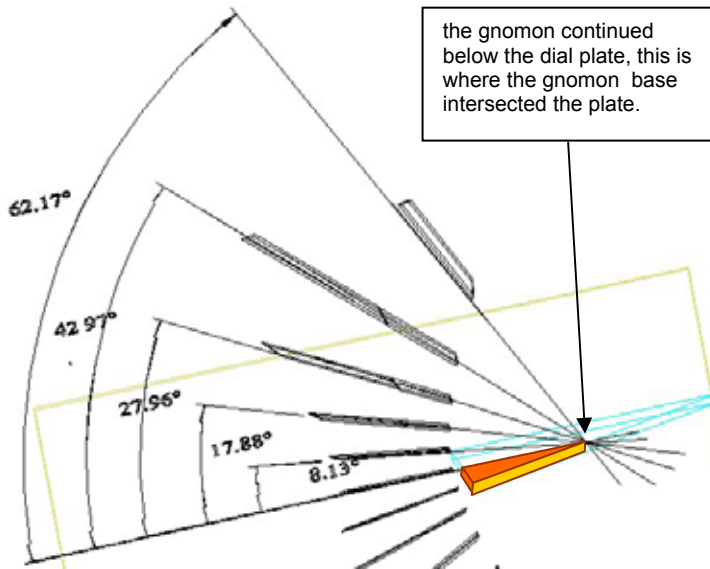


Then place the radiation point of the light rays on the nodus of the gnomon, and add a dial plate.

Then use the 3d intersect tool to cause just the intersection of the radiating rays and dial plate to remain behind. Then another plate is re-added thus obtaining a sun dial.



Having completed this task, rotate the model and draw lines from the dial center to the places where the light rays intersect the dial plate and measure them with the dimension tool, thus providing the hour line angles. The extremes of the intersecting rays on the dial plate form hyperbolic curves, they are the calendar lines for the solstices.



the gnomon continued below the dial plate, this is where the gnomon base intersected the plate.

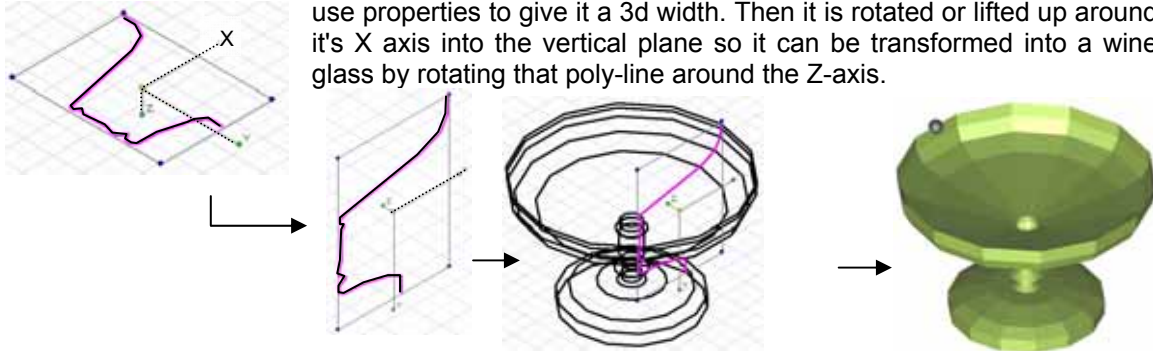
The angles from the noon line showed 8.1, 17.9, 27.9, 43, and 62.2, compared to the latitude 32 tables (appendix ~ A3.1a) offering 8.1, 17, 27.9, 42.6 and 63.2 which shows a close agreement.

The North American Sundial Society publishes a journal called the Compendium, and several articles have discussed this process in depth. The objective of this section is to show how a CAD program can not only be used for drafting dial plates, but can also be used as a direct tool to deduce hour lines and

calendar lines. And decliners and recliners can be thusly modeled. While this modeling concept was discussed in chapter 10 and other places, this reinforces the concept.

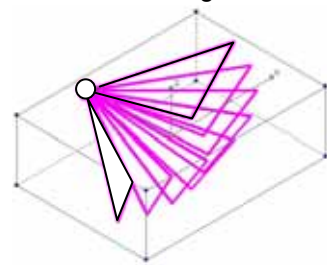
Another example of a more complex kind consists of a wine glass used as a sundial. In earlier centuries, a silver or gold wine goblet was sometimes used as a sundial.

FIRST: A 2-d poly-line was created. Then if needed to get x/y/z axes shown when it is selected, use properties to give it a 3d width. Then it is rotated or lifted up around its X axis into the vertical plane so it can be transformed into a wine glass by rotating that poly-line around the Z-axis.



Rotating that poly-line is done by INSERT, 3D OBJECT, REVOLVE, and the wine glass may then be rendered. A copy of this glass is made, because it will be destroyed when we do a 3d-intersect with the solar rays in a couple of steps.

SECOND: A single solar ray is created – it is a triangle about 47 degrees at its apex, and this time a radial copy is performed.



The solar rays are added together with the 3d-add. Thus there are now two objects. One is the glass and its copy, the other is the solar rays. A small sphere was added at the focal point of the rays as a nodus to help with placing the rays on the wine glass, and to solve a 3d-add problem that sometimes happens (see below). A similar small sphere or nodus was also added to the wine glass rim to facilitate placing the rays on the rims consistently.

THIRD: The rays and one of the wine goblets are then used in the 3d-intersect operation, leaving just the places where the rays intersected that wine glass, which is why we made copies of the wine glass. FOURTH: The result of the intersection is moved back to the wine glass copy.



NOTE: Sometimes during a 3d-intersect or a 3d-add operation a message will say that the resulting surface was degenerated, meaning there was nothing left after the intersect or it could not add the objects. This happened in the above example. That was one of two reasons why a ball was added to the solar rays. For this method to be of practical use, the wine cup or other object to be used as the curved dial plate has to be modeled accurately, along with reference points, an activity somewhat easier said than done.

