## Illustrating Time's Shadow



Illustrating Time's Shadow completely replaces Illustrating Shadows and Illustrating More Shadows which were merged into one book, Illustrating Time's Shadow, after combining both original books, deleting duplicate material and less frequently used methods, while adding much new information.

This book addresses small indoor sundials of wood, glass, and PVC, as well as outside garden dials of glass, clay, tile, and common building materials. Less common dial features such as the inclined decliner and calendar or declination curves, are covered, as well as the astrolabe, other altitude dials and azimuth time keepers. This book uses empirical, geometric, trigonometric, CAD (computer aided design) both 2d and 3d, spreadsheet, procedural programming, tabular methods, and other techniques. Tables are provided.


# ILLUSTRATING TIME’S SHADOW 

Incorporating Illustrating Shadows and Illustrating More Shadows

a book by Simon Wheaton-Smith


a lino cut print

ISBN 978-0-9960026-0-8
Library of Congress Control Number: 2014904839
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## THE ILLUSTRATING SHADOWS COLLECTION

Illustrating Shadows provides several books or booklets:-

Simple Shadows
Cubic Shadows
Cutting Shadows

Build a horizontal dial for your location. Appropriate theory. Introducing a cube dial for your location. Appropriate theory. Paper cutouts for you to make sundials with.

Illustrating Times Shadow
the big book Illustrating Times Shadow ~ Some 400 pages covering almost every aspect of dialing. Includes a short appendix.

Appendices Illustrating Times Shadow ~ The Appendices ~ Some 180 pages of optional detailed appendix material.

Supplement Supplemental Shadows ~ Material in the form of a series of articles, covers more on the kinds of time, declination confusion, other proofs for the vertical decliner, Saxon, scratch, and mass dials, Islamic prayer times (asr), dial furniture, and so on!

Programming Shadows A book discussing many programming languages, their systems and how to get them, many being free, and techniques for graphical depictions. This covers the modern languages, going back into the mists of time. Legacy languages include ALGOL, FORTRAN, the IBM 1401 Autocoder and SPS, the IBM 360 assembler, and Illustrating Shadows provides simulators for them, including the source code. Then C, PASCAL, BASIC, JAVA, Python, and the Lazarus system, as well as Octave, Euler, and Scilab. And of course DeltaCAD and its Basic variant, Python as in FreeCAD and Blender CAD systems, VBS and Java Script as in NanoCAD, programming TurboCAD (VBS and parametric script), and LISP as in the ProgeCAD system. And so on!

Illustrating Shadows provides a variety of software tools:-

| CAD | DeltaCAD ~ macros for almost all dialing needs in BASIC |
| :--- | :--- |
|  | NanoCAD ~ dial macros written in VBS and Java Script |
|  | FreeCAD $\sim$ dial macros written in Python |
|  | Powerdraw $\sim$ dial macros in a Pascal subset |
|  | ProgeCAD $\sim$ dial macros writen in LISP |
|  | TurboCAD $\sim$ dial macros written in VBS, and parametric part scripts also |
|  | Blender $\sim$ dial macros written in Python |
| Languages | Programs in the languages are discussed in Programming Shadows |
| Spreadsheets | illustratingShadows.xls simpleShadows.xls cubicShadows.xls |

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the appendices for this book are printed separately, and are free on www.illustratingshadows.com however, some key tables are in some chapters, and an abbreviated appendix is also contained herein.

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(c) 2004-2014 Simon Wheaton-Smith All rights reserved. However limited selections may be copied provided credit is given to this book, author, and web site, no fee charged beyond copying costs, and the author advised at: illustratingshadows@yahoo.com Please check: www.illustratingshadows.com/reference every now and then for updatesor clarifications. An all purpose Excel spreadsheet covering most dialing functions is available. DeltaCAD and other CAD system's macros are provided for almost all dialing needs.

There are many books on sundials that provide a wealth of information, some may be hard to follow, and sometimes the math lacks a clear explanation.

I had grown up with sundials and clocks. When I was a teenager I was given a portable sundial, and later I acquired an astrolabe. In the east end of the lawn of the house I grew up in during the 1950s there was an old sundial, pictured to the right.

The objective of this book is to show clearly several methods for making sundials. This book merges illustrating Shadows and its sequel Illustrating More Shadows, adds new material, covers the theory, geometry, trigonometry, tabular and empirical methods of sundial design,
 as well as covering media for small and large dials. Advanced dial types which as one author has said "are a problem of the diallist's own making and thus easily avoided" are covered, as well as matters of general interest. I loved mathematics in school and college where I studied for a degree in Physics, and thus trigonometry was not hard for me to understand, nor geometry. This book makes no assumptions, and the math is extremely simple.

In one of my professions I was a software designer from 1967 to 1997. I designed the SHADOW teleprocessing control program, co-designed Manage-IMS, and designed the ITT Courier DOOMSDAY quality control system. In my other profession I started flying in 1963 and had flown part time since the mid 1970s, later full time with the airlines, lastly America West on the B737, then the Federal Aviation Administration as an Operations Inspector from whence I retired.

Most sundials have a shadow casting edge whose angle is usually the latitude and usually aligned true north/south, and a dial plate which marks the hours. Corrections are needed for longitude as well as the "equation of time" (EOT). The words "usually", and "nearly always", and "most", and "as a rule" are used because some types of dials may use special techniques.

This book has ease of use as its core value. Thus text will normally refer to figures that are on the same page, even if that means duplicating some figures.

Unless a figure or pictorial is clearly intended to be to scale such as in the templates, it should not be assumed that scale is used in explanatory diagrams. In many clarifying pictorials, angles are exaggerated so that the diagram is easier to follow and less cluttered. The term "LAT" means Local Apparent Time, and "lat" is the abbreviation for latitude.

Illustrating Time's Shadow came from the $4^{\text {th }}$ edition of Illustrating Shadows and the $2^{\text {nd }}$ edition of Illustrating More Shadows combined, while deleting duplicate or less frequently use methods, but adding a substantial amount of new material.

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## A WALK AROUND THE GARDEN



One azimuth dial that is sometimes found in public recreation areas is the analemmatic dial. The one to the left is 16 feet east to west, and great for recreation centers for children during school breaks. The photo is taken from the west north west to the east south east.

Another method of determining the time from the sun is to use the sun's altitude, or how high up in the sky the sun is, just like the mariner's sextant. This is latitude dependent. The altitude for any given hour varies dramatically from winter to summer. To the right is an altitude dial known as a Shepherds Dial.

The third method is to use the angle that the sun makes around the Earth's rotational axis, the sun's hour angle. This is always assumed to be 15 degrees an hour and is latitude independent. What makes dials that use the hour angle latitude dependent is the display of data on the face of the dial, or dial plate, or surface which has, among other things, the marks telling the time, or the hour lines. Hour angle dials are by far the most common.

Where I live there are a fair number of sundials. In no particular order, they are summarized below. There are a number of ways of telling the time from the sun.

One such method is to use the sun's azimuth, which means how far east or west of the north south line the sun is. This is latitude dependent. The azimuth for any given hour varies dramatically from winter to summer. Below is an azimuth dial.


A very simple example of an hour angle dial showing the 15 degrees per hour is the armillary dial sometimes found in parks and plazas. Usually the gnomon or shadow casting device parallels the north south polar axis. In the example to the right, the gnomon is a rod whose tip (nodus) establishes both the time and the calendar information. The armillary dial is so called as the dial plate is like a bracelet for the arm, the dial plate parallels the polar axis. Usually other "bracelets" exist and when they do, they model the solar system to some extent.


A mirror image of the armillary and equatorial dials is the globe dial shown to the right and uses a movable shadow caster, or gnomon, which is turned until the shadow vanishes, and the time then read by the position of that movable gnomon.

At the risk of being highly repetitive, for hour angle dials (as opposed to altitude or azimuth dials), the hour lines on a dial plate are always based on $15^{\circ}$ per hour, but may not be $15^{\circ}$ themselves. The hour line angles for such dials are latitude and hour dependant; plate orientation dependant; and the longitude may also be a factor if standard time is used.

Whereas the armillary dial plate paralleled the polar axis, the equatorial dial, sometimes called an equinoctial dial again works on the same 15 degrees per hour philosophy, however the dial plate parallels the equator, hence its name.
(Sometimes armillary dials are called equatorial, and vice versa.)

Just like the armillary dial above, the equatorial dial to the left is also longitude corrected.


Perhaps the next most common dial is the horizontal dial, often seen in garden, and often sold in garden centers. Sadly, they are often designed for a generic latitude as opposed to the latitude of the dial's final resting place, so while they use those same 15 degrees per hour basis, they display the results incorrectly because of the latitude difference, in other words how far one is from the equator or pole.

Their owners turn the sundial in circles hoping the time will be correct, they bend the shadow casting device, or gnomon, to set the time, and neither of these are correct. Tilting the dial will help a lot, unless the dial is completely ill-designed. However sun dial readers still need to make two corrections.


One correction is for longitude, and is a fixed amount based on where the dial resides. That is because the sun dial tells the local time, or local apparent time, or L.A.T., unless designed otherwise. Some dials have longitude correction built in, many do not. Portable dials do not so that they can be used anywhere. Permanent dials often, but not always, have the difference between local sun time and the legal time zone factored in.

Another correction is to enable the pocket watch to match the endless motions of the solar system, a correction that varies by the day. This is a plus or minus 16 minutes correction often provided to the sundial user as a graph or table. This factor is called the Equation of Time, or EOT.

With those two corrections, a sun dial and pocket watch may live in harmony.


Sundials that use hour angles often have an edge that casts a shadow, that edge is called the style, and is set at an angle equal to the dial's latitude. Some dials add interest by having a rod set at any angle, and whose tip, called a nodus, points to the time. In olden days the hour lines were drawn from such a column's base, and they were not equal hours. However, if the center of those lines, or dial center, is displaced, then the hours displayed are even all year round. Such a dial with a virtual style, is shown to the left.

In fact much of the delight of gnomonics is the depiction of hour lines on surfaces not immediately obvious to the observer.

Instead of being horizontal, a sundial can be vertical, as on a wall. Sundials may be made from transparent as opposed to reflective mediums, for example stained glass. To the right is a dial designed for latitude 32 north intended to be placed on a south facing window's sill. In the southern hemisphere such a dial would have the hours reversed, morning for afternoon, and would face north as opposed to south.


Sundials can be designed to face true east or west and be vertical. One such dial is shown to the right and intended to rest of a west facing window's sill,
 while the dial to the left was similarly designed but intended to hang in that west facing window.

Both of these dials have two added features.

One feature enables the reader to determine the date. This works because for any given date, the sun is orbiting at a certain angle north or south of the equator, that angle is called the declination of the sun.


As a rule there are two dates for each declination the exception being the shortest and longest days, or the solstices. The equinoxes are days when day and night are equal, and they share the same declination, namely zero degrees of declination, as the sun is orbiting over the equator.

Another feature in both of the dials above is the ability to determine how long it will be until sunset. The lines that provide sunset information are called Italian lines. Italian hour lines are totally latitude dependent and to be pedantic show the number of hours since the last sunset. Common usage is to have them show time until the next sunset however.

Other special hours exist, such as Babylonian hours which tell the time since the last sunrise, however neither of the dials above show them.

Glass dials may be transparent as above, or they can be reflective. The glass must throw a good shadow, so test it before you use it.

Cube dials have multiple faces, often declining. To the right is a cube dial, offset 45 degrees from true north. The face on the left is for the afternoon hours, the face on the right is for the morning.

Vertical dials that face true north or south are the complement of dials that are horizontal and aligned on the true north south line. While horizontal dials are aligned on a true north south line, the same is not always true for vertical dials, they are often off a few degrees and the wall they are on is said to decline. Wall declination has nothing in common with the sun's declination mentioned earlier. Same word, different meaning and context.



In fact, true east or true west facing walls are rare, thus while a portable vertical dial such as a glass vertical dial is straight forward, ones on walls are a little more involved. Declining vertical dials with large declinations are sometimes called great decliners since they decline a lot.

The hour lines slope, and in the picture to the left, converge above the dial. Compare this to the glass vertical dial an the preceding page where the hour lines parallel each other because that dial is designed for true west orientation.

These dials when facing true east or west are often called meridian dials because their dial face or plate parallels the meridian. The meridian being the north south line connecting the poles running through a location.


It was mentioned earlier that one of the joys of dialling was to produce dials on surfaces that are just not aligned on a natural orientation. In other words dials that are both declined (not on a north, south, east, or west orientation) and inclined also. Such inclining decliners, or reclining
 decliners, are as one author wrote "a problem of the diallist's own making and thus easily avoided." None the less, they exist, roof dials being one such common case, and to the left is one inclined by 20 degrees from the horizontal and declining by 50 degrees from the south towards the west.

If masonry, glass, clay, and copper seem to be used in many of these dials, it is because the author was trained in those media over the years.

In fact many of the dials in this section are easily made from common clay brick, common concrete blocks, copper wiring, and glass available at stained glass shops.

There is no complex engraving, although the artwork on the glass does get into that kind of work.

The author has no skills in metal work and thus the wonderful metal dials do not appear in this book. The intent was to enable the reader to build these dials for their own location, easily, and in a short time frame.


It was mentioned earlier that pocket watches do not track the daily movements of the heavens, however they do a very good job of counting the seconds. So how do you set a pocket watch accurately. While in the digital age that may be less of a problem, a hundred years ago it was a genuine concern. The result was the noon dial which is not a straight line but a distorted figure of eight. It is the depiction on a dial plate of the equation of time, or difference between clock and solar system time.

To the left is the December through June half of such a dial, next to it but not shown is a dial for June through December. For a given date, when the tip of the shadow crosses the curved line it is legal standard noon time. Usually both dials are combined into one figure of eight dial. In fact some dials have figure of eights all over the place, instead of straight hour lines. Those dials when corrected for longitude then show standard legal time. Except for summer time when one hour has to be added, a folly of politicians, which nature chooses not to observe.

Summer time is less meaningful as the traveler journeys towards the equator, and yet the political engine dictates summer time for all latitudes of the country, a folly caused by politicians who reside in northerly latitudes and whose brains do not seem to thaw out, especially when considering the rules they make to govern one and all. Few states have the courage to say enough is enough, Arizona being one of the few. Most have fallen in line with the federal system. But enough of politics. I am referring to the USA in particular, however other countries like lemmings seem to have adopted this folly, intended since its inception to save energy, whale oil for lighting, and so on. None of which does it do.

## THE PARTS OF A SUNDIAL

Below are horizontal, and vertical but declining dial layouts. They have a dial plate upon which things are inscribed or placed, such as hour lines and declination (calendar) lines, called furniture. The shadow casting assembly is called a gnomon, its angled edge which casts the shadow is the style. The bottom of the gnomon attached to the dial plate is the sub-style. An optional notch or blob is called a nodus usually used to show calendar or other information.

noon line for non longitude corrected dials.
gnomon linear height (linear, not angular)

## A QUICK HANDS ON PROJECT ~~~ BUILD IT FIRST - FIX IT NEXT

The next two pages offer a simple "go ahead and build it" horizontal sundial, then see what needs to be done to make it accurate. The rest of the book is the other way around, theory to understand what is going on, then build it. These two pages help you build a horizontal dial and is a good prelude to the rest of the book. NOTE: Please use the separately printed appendices.

1. What is your latitude (how far north or south are you from the equator). Many maps show this, and so does a GPS. Appendix 2 (separate book) may help with latitude data.
```
my latitude is:
```

2. EITHER download the Microsoft Office Excel spreadsheet at:
www.illustratingshadows.com/illustratingShadows.xls (works in Open Office)
then on the worksheet named "h dial with all figures", enter the latitude. OR go to tables in the appendix 3, look at the latitude at the top of the columns, then find the angles for the hours you wish marked.

| 6 | 7 | 8 | 9 | 10 | 11 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 5 | 4 | 3 | 2 | noon <br> 1 | noon |

morning hours afternoon hours
3. Mark the lines on the template "A DRAFTING SHEET FOR HORIZONTAL DIALS" from the appendices. The end result should look roughly like the fan of lines to the right.

Noon is going to be on the north south line and the 6 am and 6 pm lines will be at 90 degrees to that, The other lines will fan out, and their angles depend on your latitude.
4. Transpose the hour lines to wood, PVC, copper, glass, concrete, clay, or any other medium. This is called the dial plate.

5. Build a shadow casting device, called a gnomon, it is a triangle whose angle from the dial plate is equal to the latitude. The angle is important, the length is not, unless you want to add other information related to the time of year or sunset, discussed in the main part of the book. NOTE: the gnomon should be thin. If thick as shown above, then the am and pm hours should be separated at noon to account for it, there would be two noon lines.
6. Affix the gnomon to the dial plate, the latitude angle end rests on top of where the hour lines converge (dial center), and it lies on the north south line, or the noon line.

7. Take the dial plate, its affixed gnomon on the noon line and place it in the sun.
8. Align the noon line with north/south, and the gnomon's end placed on dial center (where it meets the hour lines), should point to the equator, south in the northern hemisphere or north in the southern hemisphere. True north is meant, not compass magnetic north. To find true north, first find magnetic north by using a compass but keep away from metal. Then find your location's magnetic variation, or declination as sun dial people call it.

If it is an easterly declination you must back away from north to the west.
If it is a westerly declination, you must back away from north to the east.
Magnetic declinations are often found on maps, and a table and a map in the appendix has declination information.

| my magnetic declination: | E |
| :---: | :---: |
| W Appendix 2 may help. |  |

9. With the dial plate level, the angled part of the gnomon pointing north south, the dial will now read sun-time, or local-apparent-time or L.A.T. for short. It is still not clock accurate. There are two corrections to make. One is to correct for your distance from your time zone's reference longitude, the other is for the fact the sun is predictably slow or fast (compared to man made watches) as the year progresses, this correction is called the equation of time or EOT for short.
10. Find your longitude, it is on your GPS unit, or on a map of your area. Aviation and geological survey maps work, but road maps may not. Once you know your longitude, and your time zone, find your time zone's reference longitude.


If your longitude is greater than the reference longitude, ADD the difference times 4, these are the minutes to add to correct for your location.

If your longitude is less than the reference longitude, then SUBTRACT the difference times 4 to correct for your location.
plus [ ] or minus [ ] _ _ minutes to correct for location.
11. We now have a dial built for your latitude, aligned north/south, and corrected for your location's distance from the time zone's reference longitude. However it may still be off by plus or minus up to 16 minutes due to the fact the sun's orbit around the Earth varies during the year, and because the sun also moves north or south of the equator. There is a table of corrections for this equation of time or EOT by the day in the appendices.
12. That is it! You now have a working horizontal sun dial. To delve into it more, read on . . . .

## CHAPTER ONE

## THE UNIVERSE

The universe is the collection of all the galaxies or collections of stars. And surrounding stars are planets. And other things go on as well.

Planet Earth orbits a star called the sun, which is in a galaxy called the milky way, but distances out there are somewhat large.

To an observer on planet Earth, we can see several things.

The moon which orbits our planet.

The sun which we orbit but to a sundial the sun orbits us.

The planets which also orbit
 the sun, as do the comets which have very large elliptical orbits.

The stars which are far far away, and with the exception that the Earth wobbles, are essentially fixed in the sky, as if they were on a sphere of enormous dimensions. In fact it is called the celestial sphere.

The Earth spins and the polar axis of that spinning wobbles in a cycle of 25800 years roughly. So, those fixed stars actually appear to collectively wobble over that time-span.

- The Earth wobbles somewhat slowly in a 25,800 year cycle, causing the "precession of the equinoxes", and causes the constellations to move around.
- One measurement of time is sidereal time, based on a star's position.

This relative stability allows star charts to be meaningful, and allows the time to be measured by the star's angular position around the Earth's rotational axis, or polar axis. In the northern hemisphere the north star Polaris is very close to that polar axis, now. That will change over the 25800 year cycle which results in the "precession of the equinoxes", because seasons will correspondingly change. The stars in the northern hemisphere seem to rotate around the north star, Polaris. They revolve about 360 degrees in a day. And for any given time of night, they rotate 360 degrees in a year.

This all means that a 360 degree map of the stars can be drawn, and throughout the year some stars come into view, others leave. And similarly, in the evening, the stars for the season rotate 360 degrees in a day. Approximately. This means that should one know the month, then the time can be approximated, and this is the basis for a nocturnal dial.

The nocturnal dial is described in chapter 27 on "Night Time Dials".
To be pedantic, we must add something to the model of the universe to explain why we see different stars in the winter from those of the summer.

The reason is that the Earth orbits the sun once in a year, which is about 365 days. So when the Earth is on one side of the sun, the stars behind the sun are not visible, and so the visible stars come and go over a year's time.

The bright sun will block out different stars in the winter than the summer. In fact every day the relative position of the sun and Earth have moved a bit and some stars come into sight at a given time and some become no longer visible.

Adding to that, the Earth is tilted by about 23.5 degrees as it orbits the sun.

The Earth's orbital plane is called the ecliptic. The ecliptic does not affect the nocturnal dial, but the planets for the most part lie close to the Earth's orbital plane, the ecliptic.

The ecliptic is not essential to
 understand for most normal sundials. It is used in the "ecliptic" dial, and in the astrolabe.


In the study of sundials, sometimes the Earth will be considered to orbit the sun, as for example to explain the seasons, and the sun's changing declination. Sometimes the sun will be considered to orbit the Earth, as for example to explain the changing star patterns throughout the year, this is shown in the picture above. It is all relative, it is not a mistake.

Chapter 27 has a simple yet very accurate clock that uses, in the northern hemisphere, Polaris as an axis of rotation, and a specific star in one of three constellations, it is called a nocturnal dial. And the templates are in appendix 9.

## CHAPTER TWO

## THE SOLAR SYSTEM

The solar system is comprised of planets in elliptical orbits orbiting the sun, asteroids, and the occasional comet. For the most part, the planets orbit the sun in the same orbital plane as the Earth, they are off by a few degrees. And they go the same way round. The Earth's orbital plane is called the ecliptic.

The ecliptic is the plane made by the Earth's orbit around the sun, and if a dial plate was placed on it and moved with the Earth as it orbits the sun, then in one year the shadow of the gnomon would rotate about 360 degrees. About, because things are not that tidy. Because it would take one year for the gnomon's shadow to rotate around that dial plate, and since that is one calendar cycle, it is possible to build a calendar dial using the ecliptic, as opposed to using declination lines.

The Earth's orbit is 23.44 degrees, or 23.5 for simplicity, from the Earth's polar axis. The pictorial below to the left shows several dial plates around the Earth's annual orbit as viewed from above the north.


Above to the right, the northern hemisphere is viewed from above the north and looking down, and the southern hemisphere is viewed from below the south while looking up.


The shadow is of course not a triangle, it would be an infinitely long box in the direction away from the sun. The triangle is used to more clearly indicate "a shadow"


There are some interesting results that come from such dynamics. First, how the intersection of the ecliptic varies throughout the day and the year. Second, how to construct a simple device to indicate the ecliptic and thus the date. Third, an understanding of the ecliptic, useful for locating the moon and the planets based on the seasons. The ecliptic plane slices the Earth, however, the slice angle varies by the second. The slice in the morning differs from the evening, and they both differ from noon. It differs by the season.

In the context of a full moon, in the December January time frame or year end, the full moon is effectively above the extended equator, so the full moon rises from the northeast and sets in the northwest. Mid year in June at full moon the moon is effectively below the extended equator, so the full moon rises in the southeast and sets in the southwest. For full moons at the March and September equinoxes, moonrise and moon set happens from the east to the west, the moon is effectively on the extended equator. The full moon path differs in the winter, summer, and equinoctial times. For full moons, the lunar path is almost the same as the sun's path 6 months in the past or future.

The first quarter moon's apparent path varies from the full moon. For example look at the first quarter moon in the December January timeframe. The moon rises during the daytime from the east, sets in the west, as the moon is effectively on the extended equator.

The lunar orbit is actually offset by about $5^{\circ}$ from the ecliptic, which can add to the variation, thus the above pictorial is stylistic as well as not to scale. Some time can be well spent on the pictorial above to see how the apparent lunar orbit varies by the month as well as by the season for several selected hours.

The planets are mostly in the same plane as the Earth with respect to the sun, thus the planets apparent path in the sky will also vary seasonally.

From this daily and seasonal variation of the ecliptic intersection with the planet Earth, comes an explanation of lunar activity, as well as the method for designing a calendar dial using the ecliptic.

The remaining question is how to build an ecliptic dial.
On a polar axis alignment such as the style of an hour angle gnomon, a canted device is built that rotates 23.5 degrees around the polar axis. In the picture below, PVC tubing was employed. To the 23.5 degree canted rotating tube is affixed a plate, in this case from an unused computer CD.


When the date wheel is affixed, June 21 is at one end, and December 21 is at the other. Of course, the winter solstice is when the sun's declination is lowest in the sky, and the summer solstice is when the sun's declination is highest. In the northern hemisphere, December 21 would be at that low point, June 21 would be at the high. In the southern hemisphere, December 21 would be at the high point, June 21 would be at the low.

A normal rotating gnomon is used to indicate the time on a linear plate. And the canted section is rotated around the polar axis until the dial plate casts a flat shadow. A rotating cursor is moved until the ecliptic gnomon casts a shadow as shown above, and the other end points to the date.

There are usually two locations at any time when the ecliptic dial plate casts a null shadow, this matches the two dates for any given sun declination, the solstices being the exception. Some degree of awareness about the probable month is likely to be beneficial at this point.

Just as the equation of time exists for the minutes to adjust a time dial, similarly an equation of day exists for the date. This is a sine wave indicating about plus or minus a couple of days, this is subtracted in the first half of the year from the indicated date, added for the second half.

The ecliptic is not essential to understand for most normal sundials. However, it is used in the "ecliptic" dial (above), and in the planispheric astrolabe described in chapter 21 on "Altitude Dials".

## CHAPTER THREE

## THE PLANET EARTH

The Earth is a planet and from history we know it orbits the sun. Back in medieval times the sun was believed to orbit the Earth. Galileo and others nearly got burned at the stake when they said it was the other way round, however it is pretty much accepted now that the Earth orbits the sun. And by the way, the Earth is no longer flat either, it is a sphere in space.


We call the line running around the middle of the Earth the equator, it is perpendicular to, and mid way between the poles.

The planet Earth's polar spinning axis is tilted about $23.5^{\circ}$, or $23.44^{\circ}$ to be more precise, compared to its orbit around the sun. That causes the seasons to vary.

The equator is at 90 degrees to that polar spinning axis and mid way between the poles. Looking at the figure below, when the sun is below the equator (bottom left in the figure below), the southern hemisphere has summer while the northern has winter. And when the sun is above the equator (top right in the figure below), it is winter in the southern but summer in the northern hemisphere. The Earth orbits the sun in an ellipse and that together with the Earth's tilt, are two reasons why day length varies and why the sun's hours appear to vary somewhat, and hence why the "equation of time" was developed which corrects the sun for being slow or fast when compared to the watches and clocks which are set based on an Earth day averaged through the year. The pictorial below shows the Earth spinning on its axis, and the sun in four different positions.


While the Earth rotates around the sun, to a sundial it is as if the sun goes around the Earth's polar axis. So for simplicity, we will assume it is before the days Galileo. When the sun is over the equator, top left and bottom right in the picture above, it is the equinox, and the days are equal to the nights in duration, late March and September. By the way, every day on the equator is an equinox, their days are always the same length as their nights. When the sun is over the tropics, i.e. latitude plus or minus 23.44 degrees, it is the solstices, late June and December.

Now let us define a few things that are needed in order to understand location on planet Earth. The pole around which the Earth spins is tilted by about $23.5^{\circ}, 23.44^{\circ}$ to be more precise, and perpendicular to it is the plane called the equator, see figure immediately below.


If the Earth looks bigger than the sun, it is because the sun looks small to us here on planet Earth, and the Earth looks big to us.

Referring to the figure below, the Earth is sliced into sections parallel to the equator, those slices are measured by their angle to the center of the Earth. Those angles are called latitude and $\varnothing$ is its common symbol.

The latitude tells you how far north or south of the equator you are. It is measured in degrees, the equator's latitude is $0^{\circ}$ while the north pole is $90^{\circ}$, and one degree of latitude is about 60 nautical miles on the surface, that is how nautical miles came into being.

The equator is an obvious slice for latitude references, as it is $90^{\circ}$ to the polar axis, but there is no such obvious place for the leftright position, so England graciously agreed to define Greenwich near London as the reference point.


Referring to the figure below and left, longitude determines how far you are east or west of Greenwich England. However as you go further north or south of the equator, the linear distance between one longitude degree varies. Whereas a degree of latitude is 60 nautical miles anywhere, for longitude the distance gets smaller as you travel north or south of the equator.

Longitude has one fixed relationship however, one degree of longitude always accounts for 4 minutes of time in the mean motion of the sun.

Positions on the planet are identified by latitude (north or south of the equator), and by longitude (east or west of Greenwich England). Because the Earth rotates on its axis, some places see the sun later and some earlier. As there are 24 hours in a day, and there are 360 degrees of longitude, 15 degrees equates to one hour, so standard longitudes are established every 15 degrees, but for political reasons they may zigzag around the place. Those longitudes in essence define legal standard time. Since $15^{\circ}$ is one hour or 60 minutes, then one degree is 4 minutes.

Those 15 degrees are split in half for those legal time zones, thus Greenwich is $0^{\circ}$ of longitude, and $7.5^{\circ}$ west to $7.5^{\circ}$ east marks the legal time zone. Lets put this into the real world.

| $\begin{aligned} & \text { Silver City } \\ & \text { SVC } \end{aligned}$ | $32.75{ }^{\circ} \mathrm{N}$ | 108.2 ${ }^{\circ} \mathrm{W}$ | mag var $10.6^{\circ} \mathrm{E}$ |
| :---: | :---: | :---: | :---: |
|  |  | mst is at $105^{\circ}$ | SVC is 3.2 from mst |
|  |  | i.e. | 12 mins 48 secs from mst |
| Phoenix <br> PHX | $33.5{ }^{\circ} \mathrm{N}$ | $112.0^{\circ} \mathrm{W}$ | mag var $11.8^{\circ} \mathrm{E}$ |
|  |  | mst is at $105^{\circ}$ | PHX is $7.0^{\circ}$ from mst |
|  |  | i.e. | 28 minutes from mst |
| Scottsdale SDL | $33.6{ }^{\circ} \mathrm{N}$ | $111.9^{\circ} \mathrm{W}$ | mag var $11.8^{\circ} \mathrm{E}$ |
|  |  | mst is at $105^{\circ}$ | SDL is $6.9^{\circ}$ from mst |
|  |  | i.e. | 27 mins 36 seconds from mst |



Things of interest to note are that when it is solar noon sun time on the 105 meridian (local apparent time or L.A.T.), it will be appear to be earlier in Silver City and earlier still in Phoenix, as far as sun time goes. Indicated sun time is called local apparent time, or L.A.T.

When it is solar noon on the Silver City meridian, L.A.T. (local apparent time) it is later at the 105 meridian and earlier on the 112 meridian of Phoenix, sun time wise.

One degree of longitude is 4 minutes of L.A.T. difference, which derives from the fact there are $360^{\circ}$ going around the entire planet and there are 24 hours in the day or 1440 minutes.

## SUN'S DECLINATION

It has been shown that throughout the seasons, the sun moves north and south of the equator. The angle the sun's rays make compared to the center of the Earth, is called the sun's declination.


## SUN'S DECLINATION

Simplified formula: DEGREES $=(23.45 * \sin ($ radians $(0.9678(j-80))))$
where $\mathrm{J}=1$ to 365 , the day of the year, see below.

## SUNS DECLINATION

More complex formula
Day number, J J=1 on 1 January, J=365 on 31 December. February is taken to have 28 days.

| Jan | Feb | Mar | Apr | May | Jun |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 31 | 59 | 90 | 120 | 151 |
| Jly | Aug | Sep | Oct | Nov | Dec |
| 181 | 212 | 243 | 273 | 304 | 334 |

Day angle: $\mathrm{da}=2$ * pi * $(\mathrm{j}-1) / 365$ (in radians, is an intermediate figure)

Sun Declination

$$
\begin{aligned}
\text { decl }= & \text { degrees }\left(0.006918-0.399912^{*} \cos (d a)+0.070257^{*} \sin (d a)\right. \\
& -0.006758^{*} \cos \left(2^{\star} d a\right)+0.000907^{*} \sin \left(2^{\star} d a\right) \\
& -0.002697^{*} \cos \left(3^{\star} d a\right)+0.001480^{\star} \sin \left(3^{\star} d a\right)
\end{aligned}
$$

A declination table is included in the appendices as well as at the end of this chapter.

## LONGITUDE AND EQUATION OF TIME CORRECTIONS ARE NEEDED

A dial's indicated time is called Local Apparent Time (L.A.T.), and when corrected for the Equation Of Time or EOT (see chapter 5), the result is Local Mean Time. When longitude is factored in, the result is "standard mean time". The word "mean" is often omitted.
L.A.T. local apparent time, time indicated by a sundial (sometimes also called true time)

EOT equation of time for the day, if the sun is fast compared to a clock (or the virtual or mean sun), then there is a minus correction, and if slow, there is a plus. Some tables show fast as plus, slow as minus. They are not wrong, they are designed by people who while otherwise normal, use a different convention. Many astronomers fall into this category.

Converting local apparent time to standard time is used when reading a dial:-

add if west of standard meridian subtract if east of standard meridian


Phoenix AZ is at a longitude of $112^{\circ}$ and is west of the standard time meridian for Phoenix time which is at $105^{\circ}$. When reading a dial with no built-in longitude correction in Phoenix, the longitude correction must be added because it is later at the $105^{\circ}$ standard time longitude than in Phoenix at $112^{\circ}$. In other words, if west then add the longitude difference times 4 minutes per degree. If east you subtract.

Since Phoenix is $7^{\circ}$ west of the reference longitude, being 112 minus 105 , and remembering one degree equates to 4 minutes, that makes Phoenix show four times seven, or 28 minutes earlier than the legal time, so you add those 28 minutes. It is 28 minutes later on the $105^{\circ}$.

For purposes of repetitive learning, all of October, November, and May the sun is fast so the EOT is negative, and in all of January, February, March, July, and August the sun is slow so the EOT is positive. There are four days when the EOT is effectively zero, they are somewhere near April 15, June 15, September 1, and December 25. The extreme values of the EOT are around February 11th when the sun is slow and the EOT is +14 minutes 12 seconds, and early November when it is fast and the EOT is now - 16 minutes 22 seconds. Other peak values are near May 13th and 14th when the sun is fast, so the EOT is -3 minutes 39 seconds, and July 25th and 26 th when the sun is slow with an EOT of +6 minutes 30 seconds.

## THE EARTH AND SUN INTERACT - EQUINOXES AND SOLSTICES

On the equator, at 0 degrees of latitude, every day is an equinox, that is to say that every day of the year has equal hours for day and night, regardless of whether the sun is overhead, north, or south of the equator. In the three pictures below the dashed arrow shows the Earth's rotation. The pictures show the Earth rotating on its axis, they suggest the Earth tilts back and forth yearly. In fact the Earth doesn't tilt back and forth as shown during the year, it only appears to do that because the Earth retains its axis as it orbits around the sun, and that orbit around the sun is what causes the sun to appear to move above and below the equator.


Looking at figure 3.4 the picture depicts summer in the northern hemisphere when the sun is north of the equator. As the sun's rays are parallel, the equatorial ray in the picture gets to travel the great circle around the equator. Because each place on the equator gets that ray, days and nights are of equal length. With northern visits of the sun, the sun always shines on the arctic, so the arctic has days with no night, and the bottom ray misses out on the Antarctic whose nights are dark, with no sun.
Fig 3.4

Summer solstice in the north


Figure 3.6 shows March and September, when everywhere on Earth has days equal to nights. The equinox happens around March 21 and September 23, plus or minus a bit. The extremes of the sun moving north and south of the equator by 23.5 degrees are the solstices, December and June 21, shown in figures 3.4 and 3.5 respectively.


The three figures have a line 90 degrees to the sun's rays, left of it is dark night time, right is bright day time. Assuming there are no clouds. The equator is always bisected, it is the north and south hemispheres that have an uneven division of time when the sun is not directly above the equator. The equator is thus in a permanent equinox.

Notice the arrows in the summer pictures, they show how one pole endures a long night time, while the other basks in warmer temperatures.

[^0]
## SUN'S DECLINATION

DECLINATION OF THE SUN BY THE DAY

|  | Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -23.1 | -17.3 | -7.9 | 4.2 | 14.8 | 21.9 | 23.2 | 18.2 | 8.6 | -2.9 | -14.2 | -21.7 |
| 2 | -23.0 | -17.1 | -7.5 | 4.6 | 15.1 | 22.1 | 23.1 | 18.0 | 8.2 | -3.3 | -14.5 | -21.8 |
| 3 | -22.9 | -16.8 | -7.1 | 5.0 | 15.4 | 22.2 | 23.0 | 17.7 | 7.8 | -3.6 | -14.8 | -22.0 |
| 4 | -22.8 | -16.5 | -6.7 | 5.4 | 15.7 | 22.3 | 23.0 | 17.5 | 7.5 | -4.0 | -15.1 | -22.1 |
| 5 | -22.7 | -16.2 | -6.3 | 5.8 | 16.0 | 22.5 | 22.9 | 17.2 | 7.1 | -4.4 | -15.5 | -22.3 |
| 6 | -22.6 | -15.9 | -6.0 | 6.2 | 16.3 | 22.6 | 22.8 | 16.9 | 6.7 | -4.8 | -15.8 | -22.4 |
| 7 | -22.5 | -15.6 | -5.6 | 6.5 | 16.6 | 22.7 | 22.7 | 16.6 | 6.4 | -5.2 | -16.1 | -22.5 |
| 8 | -22.3 | -15.3 | -5.2 | 6.9 | 16.9 | 22.8 | 22.6 | 16.4 | 6.0 | -5.6 | -16.4 | -22.6 |
| 9 | -22.2 | -14.9 | -4.8 | 7.3 | 17.1 | 22.9 | 22.5 | 16.1 | 5.6 | -6.0 | -16.7 | -22.7 |
| 10 | -22.1 | -14.6 | -4.4 | 7.7 | 17.4 | 23.0 | 22.4 | 15.8 | 5.2 | -6.3 | -16.9 | -22.8 |
| 11 | -21.9 | -14.3 | -4.0 | 8.0 | 17.7 | 23.0 | 22.2 | 15.5 | 4.9 | -6.7 | -17.2 | -22.9 |
| 12 | -21.8 | -14.0 | -3.6 | 8.4 | 17.9 | 23.1 | 22.1 | 15.2 | 4.5 | -7.1 | -17.5 | -23.0 |
| 13 | -21.6 | -13.6 | -3.2 | 8.8 | 18.2 | 23.2 | 22.0 | 14.9 | 4.1 | -7.5 | -17.8 | -23.1 |
| 14 | -21.4 | -13.3 | -2.8 | 9.1 | 18.4 | 23.2 | 21.8 | 14.6 | 3.7 | -7.8 | -18.0 | -23.2 |
| 15 | -21.3 | -13.0 | -2.4 | 9.5 | 18.7 | 23.3 | 21.7 | 14.3 | 3.3 | -8.2 | -18.3 | -23.2 |
| 16 | -21.1 | -12.6 | -2.0 | 9.8 | 18.9 | 23.3 | 21.5 | 14.0 | 3.0 | -8.6 | -18.6 | -23.3 |
| 17 | -20.9 | -12.3 | -1.6 | 10.2 | 19.1 | 23.4 | 21.3 | 13.7 | 2.6 | -9.0 | -18.8 | -23.3 |
| 18 | -20.7 | -11.9 | -1.3 | 10.5 | 19.4 | 23.4 | 21.2 | 13.4 | 2.2 | -9.3 | -19.1 | -23.4 |
| 19 | -20.5 | -11.6 | -0.9 | 10.9 | 19.6 | 23.4 | 21.0 | 13.0 | 1.8 | -9.7 | -19.3 | -23.4 |
| 20 | -20.3 | -11.2 | -0.5 | 11.2 | 19.8 | 23.4 | 20.8 | 12.7 | 1.4 | -10.1 | -19.5 | -23.4 |
| 21 | -20.1 | -10.8 | -0.1 | 11.6 | 20.0 | 23.5 | 20.6 | 12.4 | 1.0 | -10.4 | -19.8 | -23.4 |
| 22 | -19.9 | -10.5 | 0.3 | 11.9 | 20.2 | 23.5 | 20.4 | 12.0 | 0.6 | -10.8 | -20.0 | -23.4 |
| 23 | -19.6 | -10.1 | 0.7 | 12.3 | 20.4 | 23.5 | 20.2 | 11.7 | 0.2 | -11.1 | -20.2 | -23.4 |
| 24 | -19.4 | -9.7 | 1.1 | 12.6 | 20.6 | 23.4 | 20.0 | 11.4 | -0.1 | -11.5 | -20.4 | -23.4 |
| 25 | -19.2 | -9.4 | 1.5 | 12.9 | 20.8 | 23.4 | 19.8 | 11.0 | -0.5 | -11.8 | -20.6 | -23.4 |
| 26 | -18.9 | -9.0 | 1.9 | 13.3 | 21.0 | 23.4 | 19.6 | 10.7 | -0.9 | -12.2 | -20.8 | -23.4 |
| 27 | -18.7 | -8.6 | 2.3 | 13.6 | 21.2 | 23.4 | 19.4 | 10.3 | -1.3 | -12.5 | -21.0 | -23.3 |
| 28 | -18.4 | -8.3 | 2.7 | 13.9 | 21.3 | 23.3 | 19.2 | 10.0 | -1.7 | -12.9 | -21.2 | -23.3 |
| 29 | -18.2 |  | 3.1 | 14.2 | 21.5 | 23.3 | 18.9 | 9.6 | -2.1 | -13.2 | -21.4 | -23.3 |
| 30 | -17.9 |  | 3.5 | 14.5 | 21.7 | 23.2 | 18.7 | 9.3 | -2.5 | -13.5 | -21.5 | -23.2 |
| 31 | -17.6 |  | 3.9 |  | 21.8 |  | 18.5 | 8.9 |  | -13.9 |  | -23.1 |

DEGREES $\left(0.006918-0.399912^{*} \operatorname{COS}\left(\left(\left(2^{\star} 3.1416^{*}(\mathrm{jd}-1)\right) / 365\right)\right)+0.070257 * \operatorname{SIN}\left(\left(\left(2^{\star} 3.1416^{*}\right.\right.\right.\right.$
$(j d-1)) / 365))-0.006758^{*} \operatorname{COS}\left(2^{*}\left(\left(2^{*} 3.1416^{*}(j d-1)\right) / 365\right)\right)+0.000907^{*} \operatorname{SIN}\left(2^{*}\left(\left(2^{*}\right.\right.\right.$
$\left.\left.\left.3.1416^{*}(\mathrm{jd}-1)\right) / 365\right)\right)-0.002697^{*} \operatorname{COS}\left(3^{*}\left(\left(2^{*} 3.1416^{*}(\mathrm{jd}-1)\right) / 365\right)\right)+0.00148^{*} \operatorname{SIN}\left(3^{*}\right.$
((2*3.1416*(jd-1)) / 365)))
DEGREES $=\left(23.45^{*} \sin (\right.$ radians $(0.9678(j d-80))) \quad$ alternative formula agrees within half a degree

Different declination charts may disagree, factors affecting them would be leap year approximations, and the formula employed. Many formulae are approximations.

## CHAPTER FOUR

## THE EVOLUTION OF THE DIAL

Since the earliest days of the human race, it was important to know when to plant, when to hunt, and a calendar was needed and developed. The sun's angle in the sky compared to the horizon, is its altitude, and early Egyptians employed such dials.


The sun woke up and climbed, ran out of energy in mid day, and becoming tired, descended into the arms of Morpheus.

The early Egyptians built a simple dial that could be turned toward the sun, and its angle at mid day cast a shadow of decreasing length as the climate became warmer. An altitude dial was in use 3600 years ago.

Later in this book in chapter 21 several altitude dials are shown. One is the Capuchin dial, so called as part of the dial plate looks like the hood worn by the monks of than name. Another is the shepherd's dial which is cylindrical, and could be carved on a walking stick. A third is a horizontal dial that needs no compass alignment which measures the sun's altitude. A fourth is the ogee dial. For all altitude dials, north south alignment is not used but the date must be known if the one is to tell the time.

Altitude dials can be vertical like a wall, like a column, or they can be horizontal like a flat surface. A vertical altitude dial like the capuchin or ogee dial doesn't need a gnomon of a specific length, all it needs is a shadow. But the shepherd's cylinder dial and the horizontal altitude dial measure the sun's altitude by the angle the sun makes with the tip of a rod (gnomon) thus requiring a precise length for this shadow casting object when the time of day is being measured. If that gnomon's length is not accurate, then the time displayed will similarly be in error.

Next developed a sundial that measured the azimuth of the sun, its angle compared with true north or south. Such early dials often divided daylight into pieces, yet those pieces were not always of equal duration, sometimes they were just arbitrary divisions. As time became more important than the calendar, the Arabs (who did much early development of the sundial), early Greeks, and Romans had business to conduct, and advances in dial design were made. Azimuth dials do not required a gnomon of a calculated height. The line of the shadow aligns with hour lines, or points to hour points, and if anything, the height of the gnomon only becomes relevant when measuring the calendar.

Of course, gnomon height, time, altitude, and azimuth are all somewhat related. And a definitive study of dial history may not show such a sequenced development because different areas of the world evolved differently. The objective is to provide a general blended overview of dial development.


We shall discuss dials using the sun's angle around the Earth's polar axis in many later chapters, hour angle being the third method of sun time telling, and the one most used now-a-days.

However these "hour angle dials" evolved from the early scientific work of the Arabs.
While the hour angle dial evolved from azimuth and altitude dials, the reality is that both altitude and azimuth are derived from the hour angle of the sun as well as the latitude of the location, with the sun's declination taken into account. The declination of the sun is the angle the sun's rays make compared to the center of the Earth.

With the exception of Sir Charles Wheatstone's polarized sun clock, sundials need clear cloudless sunlight.

For obscured sun and nighttime use, water clocks were used alongside sun dials but ran slower as the water froze, and hour glasses ran faster as their sand eroded the connecting orifice.

Even the moon was used for half the month however its hour lines do not match the sun's hour lines, and the moon is a somewhat less predictable satellite.

And a once common instrument, the astrolabe, was frequently designed to measure time by both the sun's altitude as well as by a star's altitude, both a day and a night clock. See chapter 21.

Additionally, a "nocturnal" or star clock was used, that used the rotation of the stars by the year and the hour. See chapter 27.

But by now sundials were fairly accurate and dials using the sun's hour angle with the polar axis were commonplace. As the horse and carriage produced rapid transportation people needed common time keeping as well as equal hours. With the steam locomotive railway there came a clear and absolute necessity for a legal standard time, and so it happened. This need was so that people would not arrive late and thus miss the railway schedule.

The reader may have noticed that the sun runs slow or fast depending on the day of the year. Sun dials use the solar position as the basis for date and for time. The Earth orbits the sun, however not by an exact number of days, and even those days vary in an annual cycle. Even city slicker watches are not synchronized as well as their owners might think. Is the synchronization with the Earth's daily rotation relative to the sun, the averaged annual rotation around the sun, or with the fixed stars? Some planets have days longer than their years! Venus has a solar orbit of 225 days, but its own rotation is 243 of our days. Four and a half billion years ago the Earth's day was 13.5 hours long based on geological deductions, and 900 million years ago it had lengthened to 18 hours and 10 minutes. Now the moon is receding from Earth by 2 inches a year according to laser measurements, so while our days may continue to get longer, it may not be noticeable to those among us who are mortal humans.

## MAJOR MILESTONES FOR SUNDIAL DESIGN

- unknown maker, $15^{\text {th }}$ C BCE, Thutmose III era, Egypt, oldest sundial marking daily passage of time with some form of hours. Except, earlier dials keep being found!
- Ibn al-Shatir, 1371, Damascus, the earliest example of a sundial with a gnomon style aligned with true north.
- unknown maker, 1446, Germany, earliest European dial known with a polar aligned style
- always remember that history changes as new finds are discovered!


## THE THREE WAYS TIME CAN BE DETERMINED USING THE SUN

In reviewing the sun's motion, the Earth rotates on its own axis and the Earth itself rotates around the sun, to the dialist we can consider that the sun rotates around the polar axis of the Earth.


The gnomon's shadow producing edge (called a style) for hour angle dials should point to true north or south and be parallel to the Earth's polar axis. From the picture above, it can now be seen that there is a simple geometrical relationship between the sun's motion around the Earth, and the angles produced between the gnomon's style and also the sun's shadow. That geometry is for the most part simple and from that geometry also comes the trigonometric method as an alternative method for building sundials.

The figure above shows on the right pictorial two gnomons, one east and one west, and why the shadow differs depending on location. This is the basis of longitude correction. One degree of longitude results in a 4 minute difference.

This all translates into simple geometry.
Most sun dials use the sun's "hour angle", the angle the sun makes as it appears to go around the Earth's polar axis.

Some dials use the sun's altitude, how high up it is, and not its hour angle, so they don't need alignment with true north. This would include the shepherd's dial (whose gnomon length is critical), and the ogee and capuchin dials (whose gnomon length is not critical). For various practical reasons these tend to be less accurate than hour angle dials.

Some dials use the sun's azimuth, how far east or west the sun is of the noon day shadow. They use vertical gnomons as a rule and thus the gnomon has no north south alignment, however the dial plate with hour markings must be aligned properly with the north south line (meridian). Azimuth dials don't need a gnomon of an accurate length. For various practical reasons these tend to be less accurate than hour angle dials.

And, simple geometry translates into simple trigonometry (sine, cosine, tangent).
And, simple trigonometry translates into formulae for programs, spreadsheets, and nomograms.

While the Earth orbits the sun, to a sundial it is the other way round, and there are three ways a dial can display information. In general, the first historical method was altitude ~ the angle the sun makes with the horizon, measured by the distance of a shadow from the base of a pole.


The second method was azimuth ~ the angle the sun makes compared with true south. The shadow of a pole shows azimuth by the angle it makes with true south.

The third more common method is the hour angle, the shadow the sun makes as it rolls around a sloped edge (style of a gnomon) which is parallel to the Earth's north to south polar axis which is perpendicular to the equator.

Altitude, azimuth, and hour angle are all inter related.

Ignoring equation of time (EOT) issues in chapter 5, hour lines drawn at the same time over a period of months are not straight when using azimuth, but are straight when using the hour angle of the sloped edge, or style, of a gnomon. Azimuth and altitude methods have a practical tendency to produce less accurate results than hour angle dials.

Altitude angles for January and June are shown, notice that the angles differ dramatically.

| Method 1 altitude | SOLAR DECLINATION FOR TWO DAYS AND HOURLY SOLAR ALTITUDE |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | THIS TABLE IS FOR LATITUDE: am altitude |  |  |  |  | 32 |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  | Date | decl | 600 | 700 | 800 |  | 900 | 1000 | 1100 | 1200 | Julian |
|  | 1/1 | -23.1 | -12.0 | -0.3 | 10.5 | 20.1 | 27.9 | 33.1 | 34.9 | 1 |
|  | 6/1 | 21.9 | 11.4 | 23.7 | 36.3 | 49.0 | 61.6 | 73.3 | 79.9 | 152 |

Azimuth angles for January and June are shown, notice that the angles differ dramatically.

```
Method 2
azimuth
```

| SOLAR DECLINATION FOR TWO DAYS AND HOURLY SOLAR AZIMUTH |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| THIS TABLE IS FOR LATTITUDE: |  |  |  |  | 32 |  |  |  | Julian |
| Date |  | am | azimuth |  |  |  |  |  |  |
|  | decl | 600 | 700 | 800 | 900 | 1000 | 1100 | 1200 |  |
| 1/1 | -23.1 | 70.2 | 62.7 | 54.1 | 43.9 | 31.4 | 16.5 | 0.0 | 1 |
| 6/1 | 21.9 | 108.9 | 102.0 | 95.1 | 87.3 | 76.8 | 56.7 | 0.0 | 152 |

Sundials using solar hour angles on a latitude sloped edge tend to have more accuracy, however, calculating the hour lines will use differing techniques depending on dial plate orientation.

```
Method 3
hour angle
```

| e | 6am | 7 | 8 | 9 | 10 | 11 | 12 | 1pm | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| angle | 90 | 75 | 60 | 45 | 30 | 15 | 0 | 15 | 30 | 45 |

The 15 degrees per hour around the polar axis is deceptively simple. The actual hour lines on the dial plate are separated by angles that vary based on both latitude as well as time. Accuracy involves some work! More on this later, just be aware that three methods exist to tell the time, and hour angle is the primary method employed for its accuracy, consistency, and other benefits. Also, azimuth and altitude are all derived from the hour angle, latitude, and solar declination.


All hour angle dials derive from the equatorial dial, a dial whose dial plate parallels the planet's equator. The "Durer" model is a geometric derivation of how hour angle dials are derived from the equatorial dial. This is discussed in full in chapter 14. From this comes all the geometric models for all the different hour angle dials, as opposed to the azimuth and altitude dials. From these geometric models come all the trigonometric formulae used. From these trigonometric formulae come the spreadsheets, nomograms, tables, and other programming systems for dial design.

NOTE: Altitude and azimuth should not be ignored as old fashioned ideas. They are essential elements along with the equation of time (chapter 5) in most derivations of the figure of 8 analemma, see chapter 25.

## CHAPTER FIVE

## THE EQUATION OF TIME

## THE ECCENTRICITY EFFECT

The Earth's solar orbit is almost a circle, but it is still elliptic. Hence it speeds up and slows down as it is closer to or further from the sun. That means the Earth must rotate a bit more, or a bit less, each day for a fixed location to once again point directly at the sun. This generates an annual variation in the form of one complete sine wave.


## THE OBLIQUITY EFFECT

The earth's tilt results in the sun moving not only "westwards" but except for at the equinoxes and solstices, a northward or southward motion as well. That north or south motion means that the sun travels a little more or a little less each day. This generates two sine waves a year of variation.


## THE EQUATION OF TIME (EOT)

The equation of time is the summing of those two sine waves, and other lesser waves as well, and enables the real sun to be adjusted to the fictitious virtual or mean sun, and hence correlate to pocket watches.

To repeat and review, as the Earth orbits the sun in an ellipse, it goes faster approaching the sun and slower when receding. The Earth has two segments; leaving the sun when it slows down, and turning back when it speeds up. The Earth takes more days on one half, less days on the other. This gives the sun an apparent variation of plus or minus about 7.64 minutes between clock and solar time in one full annual sine wave which has one upper and a lower half. This is called the eccentricity effect.

Also, the Earth is tilted by about 23.5 degrees to its orbit around the sun creating the two solstices and equinoxes. At the solstices the sun moves neither north nor south but reverses its north/south direction, and moves from increasing latitudes to decreasing ones. At the equinoxes the sun is on the equator and moves from decreasing latitudes to increasing ones. When the sun is moving south or north, some angular movement shifts from the north-south travel to the sun's westward travel, or vice versa, meaning the sun moves westward slower or faster which in turn makes the sun appear slower or faster. Between the equinox and solstice points, solar and clock time differ by about plus or minus 9.86 minutes in two full annual sine waves. This is called the obliquity effect.


These two variations added together give the equation of time curve, or EOT, which varies by plus or minus about 16 minutes. The equation of time corrects apparent sun time so it matches a virtual perfect sun, or mean sun, which flows at the same rate as a clock. The appendices have several formulae and tables for the equation of time.

## USING THE EQUATION OF TIME EOT

Here are some uses for the equation of time (EOT):-

- To correct a sundial reading
- To build a sundial using the shadow to mark hour points
- To locate true north or south

|  | Feb | Apr | Jun | Sep | Nov | Dec |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $13: 34$ | $3: 49$ | $-2: 09$ | $\mathbf{0}: 02$ | $-16: 21$ | $-10: 52$ |
| 3 | $13: 48$ | $3: 14$ | $-1: 50$ | $-0: 41$ | $-16: 22$ | $-10: 06$ |
| 11 | $14: 12$ | $1: 01$ | $-0: 20$ | $-3: 24$ | $-15: 54$ | $-6: 39$ |
| 13 | $14: 10$ | $0: 30$ | $\mathbf{0}: 04$ | $-4: 07$ | $-15: 38$ | $-5: 43$ |
| 15 | $14: 05$ | $\mathbf{0}: 00$ | $0: 29$ | $-4: 49$ | $-15: 19$ | $-4: 46$ |
| 25 | $13: 00$ | $-2: 03$ | $2: 39$ | $-8: 21$ | $-12: 55$ | $\mathbf{0}: \mathbf{0 8}$ |

There are four days when the EOT is effectively zero, they are roughly April 15, June 15, September 1, and December 25.

The extreme values of the EOT are around February 11th when the sun is slow and the EOT is +14 minutes 12 seconds, and early November when it is fast and the EOT is then -16 minutes 22 seconds. Other peak values are May 13th and 14th when the sun is fast, the EOT is -3 minutes 39 seconds, and July 25th and 26th when the sun is slow with an EOT of +6 minutes 30 seconds. Those dates are approximate because of leap years, approximations, and other things.

One depiction of the equation of time (EOT) is the figure of eight.

Don't forget that for sundials to be accurate, longitude must be considered in addition to the equation of time, the longitude correction is a fixed number for a given place. Chapter 25 discusses this figure of 8 for many sundial types.

Sometimes the figure of eight chart may be seen on the hour lines of a sundial, this is called an analemma and is intended to provide graphical equation of time correction. Its presence can however make a dial look rather confusing.

There are a number of formulae available for predicting the EOT. Some are simple, others are astronomically accurate.

(where d=1 to 365)

A TWO SINE WAVE FORMULA

$$
E=7.36^{*} \operatorname{Sin}\left(2^{*} 3.1416^{*}(d-4.21) / 365\right)+9.92^{*} \operatorname{Sin}\left(4^{*} 3.1416^{*}(d+9.9) / 365\right)
$$

other formulae are used for different spreadsheets and tables, see below
A THREE SINE WAVE FORMULA (where $\mathbf{d}=1$ to 365 )

$$
\begin{gathered}
E=-1^{*}\left(9.84^{\star} \operatorname{SIN}\left(\operatorname{RADIANS}\left(2^{\star}\left(360^{\star d} \mathrm{~d}-81\right) / 365\right)\right)\right)- \\
7.53^{\star} \operatorname{COS}\left(\operatorname{RADIANS}\left(360^{*}(\mathrm{~d}-81) / 365\right)\right)- \\
\left.1.5^{*} \operatorname{SIN}\left(\text { RADIANS }\left(360^{*}(d-81) / 365\right)\right)\right)-0.3
\end{gathered}
$$

## ANOTHER THREE WAVE FORMULA (where dis 1 to 365)

$$
\begin{array}{r}
E=7.5^{*} \operatorname{SIN}(\operatorname{RADIANS}(d-5))-10.2^{*} \operatorname{SIN}\left(\operatorname{RADIANS}\left(1.93^{*}(\mathrm{~d}-80)\right)\right)+ \\
0.5^{*} \operatorname{SIN}\left(\operatorname{RADIANS}\left(1.5^{*}(\mathrm{~d}-62)\right)\right)
\end{array}
$$

Every approximation is just that, and this book uses several methods for the EOT to demonstrate the real world of approximations, with their benefits as well as drawbacks. Even established published tables vary by almost a minute. Part of this is explained by the year within a leap year cycle, part by the decade the table was printed, and so on.

The most accurate formulae use the astronomical Julian day. This is somewhat involved described next.

## ASTRONOMICAL FORMULA FOR THE EQUATION OF TIME

The most accurate formulae use astronomical elements. The astronomical Julian day is first calculated, and then other elements build up to a very accurate EOT formula. "Astronomical Formulae for Calculators" by Jean Meeus is referred to below, fourth edition, ISBN 0-943396-026. Its page 24 derives the Julian Day, page 90 derives the equinoxes and solstices for a given year. Do not mix formulae among different books, they may use different baseline epochs, these formulae use Jan 1, 1900 as their epoch, however the formulae work back a couple of thousand years and well into the future. The Julian day discussed here is noon at Greenwich, England.

```
Julian Day =INT(365.25*(4716+(IF((IF(MM>2,1,0))=0,YYYY-1,YYYY)))
    +INT(30.6001*((IF((IF(MM>2,1,0))=0,MM+12,MM))+1))) +DD -1524.5
    +(2-INT((IF((IF(MM>2,1,0))=0,YYYY- 1,YYYY )/100)
    +INT(INT((IF((IF(MM>2,1,0))=0,YYYY-1,YYYY))/100)/4))
        where: yyyy = eg 2005, mm=01 to 12, and dd=01 to 31
March equinox: =1721139.2855+365.2421376*YYYY+0.0679190*ZZ*ZZ-0.0027879*ZZ*ZZ*ZZ
June solstice: =1721233.2486+365.2417284*YYYY-0.053018*ZZ*ZZ+0.009332*ZZ*ZZ*ZZ
September equinox: =1721325.6978+365.2425055*YYYY-0.126689*ZZ*ZZ+0.0019401*ZZ*ZZ*ZZ
December solstice: =1721414.392+365.2428898*YYYY-0.010965*ZZ*ZZ-0.0084885*ZZ*ZZ*ZZ
    where yyyy = eq 2005, and zz = yyyy/1000
```

Page 79 provides four ingredients, T, L, M, e. Page 81 provides another two, Obliq and " y ". Page 91 deriving the final EOT which is the astronomically accurate EOT in radians, which you convert to degrees, then to hours and minutes.

$\mathrm{mm} . \mathrm{mm}$ EOT is the EOT above in radians converted to degrees, divided by 15, and multiplied by - 60 (to get from astronomical EOT to sundial EOT).

The above are employed in the astronomical EOT spreadsheets and all that is needed is for the year to be entered once. The spreadsheet then provides the EOT values for that year, the year's Julian day for the solstices and equinoxes, the high and low peak values, as well as a five year review of the EOT for the 15th of the month, and finally a highly detailed daily EOT listing. Appendix 2 has EOT values for several years in several centuries.

An equation of time (EOT) table is included in the appendices as well as at the end of this chapter.

## NET CORRECTIONS TO A SUNDIAL

To correct a sundial reading to find legal clock time, add the EOT (equation of time) to the indicated or local apparent time (L.A.T.). If the EOT is +5 , then add 5 minutes to the dial's indication because the sun is slow. If the EOT were -3 , you would subtract 3 minutes from the reading because the sun is fast. Then consider longitude and summer time.


CAUTION: some almanacs show the equation of time with opposite signs to those used here. To a dialist, a minus means the sun is running fast and needs the minus to "slow it down". To an astronomer, a minus means the sun is "slow" or "minus" and thus needs a plus to correct it. Neither is right, neither is wrong, it is just that astronomers and dialists have different perspectives. NOTE: Many private sundial designers build a longitude correction into their dials, a few also include the "analemma" to incorporate the EOT.

Local Apparent Time (L.A.T.) is solar time shown by the real sun at a particular place, the time most simple sundials show. It needs two corrections before legal standard mean time is known.

One correction is the difference in longitude between the dial's location and the standard time meridian.

The other correction comes from measuring time using the real sun which results in days of varying length. Instead of the real sun, we use an imaginary or "mean" sun that moves at a constant speed equal to the average annual speed of the real sun. Thus we need to correct for the difference between the real sun and the mean sun. This way sundials can match clocks, this is the purpose of the Equation of Time (EOT). Local Mean Time (LMT) is solar time corrected for the Equation of Time but not yet for longitude, which is why it is called "local". The difference between the Local Mean Time and the Local Apparent time is the Equation of Time, i.e. the Equation of Time (EOT) is the difference between Local Apparent Time (apparent solar time) and Mean Solar Time at the same place. So: Mean Solar Time = Apparent Solar Time + EOT, and Standard legal time = Mean Solar Time + longitude correction. Mean Solar Time and Apparent Solar Time match four times a year, i.e. when the EOT is zero. The leap year and the other three years are often averaged into one EOT table.

| L.A.T. (local apparent time) or solar time as shown on the dial | + EOT eg: | + longitude correction - if $E$ of legal meridian + if west of it | +1 if it is summer | $\rightarrow$ LEGAL STANDARD MEAN |
| :---: | :---: | :---: | :---: | :---: |
|  | Nov -15 |  |  |  |
|  | Feb +12 |  |  |  |
| Ou |  | egal standard time |  | TIME |
| is what you see | mean time | gal standard time |  | is what |



## NOTHING WRONG WITH TAKING A YEAR TO BUILD A DIAL

How about building a dial over time. This particular dial will indicate accurately the legal standard time at noon, assuming no daylight savings time.

A major benefit to such a slow method is the involvement of the entire family over an extended period of time, and the direct blending of the seasons, the sun's shadow, and the weather as a family event. A sundial will live for many, many years, and the entire family can look back at the fun and excitement of those earlier times when the dial was built. Any dial can be built this way.

The process is simplicity itself. A place on a vertical wall was selected, but any surface could be used. The surface can be at any angle or alignment, however, a surface should be selected that will be around as the family matures. In fact the surface does not even need to be flat!

Then a gnomon was built. It can be a rod, it can be any rigid object that will last a long time. It does not need to be set accurately, it just exists in space, attached to the surface. The nodus, or the tip of the gnomon is all that is being used in this example, which is a noon time dial.

A piece of copper sheet was cut and fashioned and its tip being sharp, a brass washer was soldered to it. This not only makes it safer, but also makes the nodus shadow easier to see. The gnomon was not set at latitude and not aligned with true north or south. The straight edge of the style was not used, only the shadow tip, enhanced by the nodus washer. It was affixed to the wall with epoxy and secured with screws, and tile.

Of course, a gnomon could be set at latitude and aligned true north-south, but since this dial only displays legal standard noon time, that is academic. The hour line is a distorted figure of eight. Every few days, the first and middle of the month, or every ten days, the shadow is marked at the selected hour, noon in this example. The result is a dot for a specific legal standard time (noon here) on a specific date. As time progresses, the hour line, actually a figure of eight, is completed and annotated with calendar data. While six months can hold the entire range of solar movement, the equation of time has an annual cycle, hence why a full year is used.



Part of the figure of eight analemma is shown offset above to the right, the marked points on the left match the analemma curve well. To the left is a close-up of the rapidly changing EOT at the December solstice. This is one of two such sister dials. This covers June through December, its sister covers December through June. As the family matures, children and grandchildren can remember fondly the dial being built, learn about the passing seasons and the sun's movement across the sky. What better way to unite a family.

## EQUATION OF TIME

| Equation of Time (EOT) mm.ss | The mm.ss average of four years <br> of astronomically accurate EOTs <br> using 2010, 11, 12, and 13 |
| :--- | :--- |


|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| 1 | 3.15 | 13.31 | 12.26 | 3.59 | -2.51 | -2.14 | 3.47 | 6.22 | 0.09 | -10.10 | -16.27 | -11.08 |
| 2 | 3.44 | 13.39 | 12.14 | 3.41 | -2.58 | -2.05 | 3.58 | 6.18 | -0.10 | -10.30 | -16.28 | -10.45 |
| 3 | 4.11 | 13.46 | 12.01 | 3.24 | -3.05 | -1.55 | 4.09 | 6.13 | -0.29 | -10.49 | -16.29 | -10.22 |
| 4 | 4.39 | 13.52 | 11.49 | 3.06 | -3.11 | -1.45 | 4.20 | 6.08 | -0.49 | -11.08 | -16.29 | -9.58 |
| 5 | 5.06 | 13.58 | 11.35 | 2.49 | -3.16 | -1.35 | 4.31 | 6.03 | -1.08 | -11.26 | -16.27 | -9.34 |
| 6 | 5.33 | 14.03 | 11.22 | 2.32 | -3.21 | -1.24 | 4.41 | 5.56 | -1.28 | -11.4 | -16.26 | -9.09 |
| 7 | 5.59 | 14.07 | 11.07 | 2.15 | -3.25 | -1.13 | 4.51 | 5.50 | -1.49 | -12.02 | -16.23 | -8.43 |
| 8 | 6.25 | 14.10 | 10.53 | 1.58 | -3.29 | -1.02 | 5.01 | 5.42 | -2.09 | -12.19 | -16.19 | -8.17 |
| 9 | 6.50 | 14.13 | 10.38 | 1.41 | -3.32 | -0.50 | 5.10 | 5.34 | -2.30 | -12.36 | -16.15 | -7.51 |
| 10 | 7.15 | 14.14 | 10.23 | 1.25 | -3.35 | -0.39 | 5.19 | 5.26 | -2.51 | -12.53 | -16.10 | -7.24 |
| 11 | 7.40 | 14.15 | 10.07 | 1.09 | -3.37 | -0.27 | 5.27 | 5.16 | -3.12 | -13.09 | -16.04 | -6.57 |
| 12 | 8.03 | 14.15 | 9.51 | 0.54 | -3.38 | -0.14 | 5.35 | 5.07 | -3.33 | -13.24 | -15.57 | -6.29 |
| 13 | 8.26 | 14.14 | 9.35 | 0.38 | -3.39 | -0.02 | 5.42 | 4.56 | -3.54 | -13.39 | -15.49 | -6.01 |
| 14 | 8.49 | 14.13 | 9.19 | 0.23 | -3.40 | 0.11 | 5.49 | 4.46 | -4.15 | -13.54 | -15.40 | -5.32 |
| 15 | 9.11 | 14.11 | 9.02 | 0.08 | -3.39 | 0.23 | 5.56 | 4.34 | -4.36 | -14.08 | -15.31 | -5.04 |
| 16 | 9.32 | 14.08 | 8.45 | -0.06 | -3.39 | 0.36 | 6.02 | 4.22 | -4.58 | -14.21 | -15.20 | -4.35 |
| 17 | 9.52 | 14.04 | 8.28 | -0.20 | -3.37 | 0.49 | 6.07 | 4.10 | -5.19 | -14.34 | -15.09 | -4.06 |
| 18 | 10.12 | 13.60 | 8.10 | -0.34 | -3.35 | 1.02 | 6.12 | 3.57 | -5.41 | -14.46 | -14.57 | -3.37 |
| 19 | 10.31 | 13.55 | 7.53 | -0.47 | -3.33 | 1.15 | 6.17 | 3.44 | -6.02 | -14.58 | -14.44 | -3.07 |
| 20 | 10.50 | 13.49 | 7.35 | -0.60 | -3.30 | 1.28 | 6.21 | 3.30 | -6.23 | -15.09 | -14.30 | -2.38 |
| 21 | 11.08 | 13.42 | 7.17 | -1.12 | -3.26 | 1.41 | 6.24 | 3.15 | -6.45 | -15.19 | -14.15 | -2.08 |
| 22 | 11.25 | 13.35 | 6.60 | -1.24 | -3.22 | 1.54 | 6.27 | 3.00 | -7.06 | -15.29 | -14.00 | -1.38 |
| 23 | 11.41 | 13.27 | 6.42 | -1.36 | -3.17 | 2.07 | 6.29 | 2.45 | -7.27 | -15.38 | -13.44 | -1.08 |
| 24 | 11.56 | 13.19 | 6.24 | -1.47 | -3.12 | 2.20 | 6.30 | 2.29 | -7.48 | -15.47 | -13.27 | -0.39 |
| 25 | 12.11 | 13.10 | 6.05 | -1.58 | -3.07 | 2.33 | 6.31 | 2.13 | -8.09 | -15.54 | -13.09 | -0.09 |
| 26 | 12.25 | 13.01 | 5.47 | -2.08 | -3.01 | 2.46 | 6.32 | 1.57 | -8.30 | -16.01 | -12.51 | 0.21 |
| 27 | 12.38 | 12.50 | 5.29 | -2.17 | -2.54 | 2.58 | 6.32 | 1.40 | -8.50 | -16.07 | -12.32 | 0.50 |
| 28 | 12.50 | 12.40 | 5.11 | -2.27 | -2.47 | 3.11 | 6.31 | 1.22 | -9.11 | -16.13 | -12.12 | 1.20 |
| 29 | 13.01 |  | 4.53 | -2.35 | -2.39 | 3.23 | 6.29 | 1.04 | -9.31 | -16.17 | -11.51 | 1.49 |
| 30 | 13.12 |  | 4.35 | -2.43 | -2.31 | 3.35 | 6.27 | 0.46 | -9.51 | -16.21 | -11.30 | 2.18 |
| 31 | 13.22 |  | 4.17 |  | -2.23 |  | 6.25 | 0.28 |  | -16.24 |  | 2.47 |
|  |  |  |  |  |  |  |  |  |  |  |  | 3.15 |

NOTE: The appendices have tables for the four years of the leap year cycle and in different centuries as well.

NOTE: This table comes from the "EOTandLONG" worksheet as a by-product, which is in the main spreadsheet: illustratingShadows.xls with no longitude consideration.

## NORTH AND SOUTH ON THE PLANET EARTH

A true north/south line or meridian is needed not just for dial plate alignment, but more importantly for calculations when a vertical dial is not aligned with the true cardinal points.

There are several methods of finding out where true north lies, and each has opportunities to screw it up. Some things to consider are that true north is usually not the same as magnetic north which varies over the years. Magnetic compasses are disturbed by local minerals, rebar in concrete block walls and proximity to automobiles; however the process is instant. The sun can give us the direction for true north, is not disturbed by local magnetic fields but the method takes either time or arithmetic, and no clouds. Surveys may have mistakes in them, be old and out of date. Stars require you to be able to identify them, and there are lots of them up there. Several other methods are discussed in this chapter.

## MAGNETIC COMPASS METHOD ~ highly susceptible to nearby magnetic deviation

Acquire a compass and set it level, sight the surface upon which the dial and gnomon will be affixed, then note the magnetic bearing. Avoid being close to metal such as a car, a metal garage door, or a block wall which may have rebar. Even fired clay bricks can acquire a significant magnetic field!

Using this surveyor's compass, the sight by definition was 0 degrees, and the magnetic needle at this location showed 6 degrees right (east) to magnetic north. So the wall must be 6 degrees west of magnetic north, or 354 degrees magnetic. Then find the magnetic declination or variation, in Phoenix AZ it is about 11 degrees east of true north, and then make an adjustment.

An easterly magnetic declination means true bearings are bigger than magnetic ones, and westerly declinations means true bearings are smaller than the magnetic bearings. Thus for an 11 degree easterly variation, 354 degrees magnetic becomes 365 true, or subtracting 360 degrees, the wall is aligned at 5 degrees true.

For easterly variations/declinations:TRUE $=$ MAGNETIC + variation/declination

For westerly variations/declinations:-
TRUE = MAGNETIC - variation/declination


The wall declination is used for surfaces not aligned to true north/south. The declination is named for the direction we face with our back to the wall, thus making the wall above "north 85 degrees west". Appendix 2 has magnetic declination/variation conversion aids, and magnetic variation is printed on many maps, is published in almanacs, and available on the web. Web sites are found by searching on "magnetic north variation".

Understand your compass. As shown above, they all produce the same results but for good reasons some indicate ascending clockwise, others ascending counter clockwise.

## TRUE NORTH - THE ASTRO COMPASS WAY ~ method one ~ all done with time

The astro-compass can be used as an accurate solar compass. It fits into a base, not supplied, which can be made from a 2.25 inch long PVC sprinkler pipe couplings whose inner diameter is 1.9 inches, and whose outer diameter is 2.25 inches. Two slits will need to be cut into it. For accurate time measurement, a clock showing hours, minutes, and seconds updated by an atomic clock such as the one in Colorado is needed. However, understand its use of summer time and that such clocks may drift substantially if the update fails. In this case we switch formula elements around, we need the L.A.T. (local apparent time) to calculate the sun's hour angle:


$$
\text { L.A.T. = standard time }- \text { [corrections] = L.A.T. }
$$

The astro compass is placed on a surface and leveled with the leveling knobs. Leveling only works when the compass is in its base. Then set the lowest adjustment (azimuth) so the lubber line marker is on N for north or S for south. Set the second adjustment up to your latitude. Set the third adjustment up (hour angle) until the sight is pointing roughly at the sun. Set the top adjustment (declination) to the sun's declination to enhance the shadow. Then calculate the corrections valid for the day. HINT: Don't use summer time.

|  | EXAMPLE | ACTUAL |
| :---: | :---: | :---: |
| LONGITUDE CORRECTION:  <br> your location:  <br> reference location:  <br> difference: 108.2 <br> times $4^{\circ}$  105.0 <br>  3.2 |  |  |
| EOT CORRECTION: appendix 2 | $\begin{aligned} & \hline \text { Sept } 26 \\ & -\quad 8 \mathrm{~m} \quad 41 \mathrm{~s} \end{aligned}$ |  |
| NET CORRECTION | $+4 \mathrm{~m} 7 \mathrm{~s}$ |  |
| en we apply this to clock time | - 4 m 7 s |  |

Next: select a time about 5 minutes hence and calculate the L.A.T. for that clock time: For September 26, longitude 108.2, the net correction is +4 minutes, 7 seconds which is subtracted from the clock indicated time because we desire L.A.T. Simplify work by using standard time.

Then: calculate the hour angle from noon for the L.A.T., this is $15^{\circ}$ per hour, and $1^{\circ}$ for four minutes (table A2.3).


Set the astrocompass hour angle dial, third adjustment up, in this example to 38.5 degrees from noon, or 321.5 which is 360-38.5.

Last: derive true north: At exactly the selected clock time, 10:30 in this example, slowly rotate the entire device until the sun's shadow is indicated in the declination adjustment's shadow box. The device is set, and the bottom or azimuth ring indicates true north or south. Appendix 2 has the above boxed work areas replicated several times as an easy to use work sheet.

HINT: simplify your life and use standard time even in the summer, it saves confusion.

## ASTRO COMPASS TRUE NORTH DETERMINATION ~ method two ~ done with degrees

This uses the same theory as backing off or advancing the clock time for a sighting, but instead of adjusting the time of the sighting, the hour angle is adjusted by advancing it or retarding it based on a degree equivalent (as opposed to a time equivalent) of the sum of the longitude and equation of time (EOT) corrections. Also, standard time is used, rather than considering daylight saving time in mid-calculation.

First, find the equation of time for the day, the EOT is in appendix 2. For March 27, this is +5.37 ( $\mathrm{mm} . \mathrm{ss}$ ), meaning that this is added to a sundial reading. Convert this to degrees, using 4 minutes per degree. This is 5 m 37 s , or 5.6 minutes and that becomes 1.4 degrees, positive when divided by four.

Next, derive the longitude correction for the dial's location, which is the location's longitude minus the legal time zone's meridian longitude. For a dial at location W108.2 and a legal time zone meridian of W105, the number is 3.2 degrees. A positive number.

Next, add the EOT in degrees and longitude correction in degrees giving 3.2 plus 1.4, in other words 4.6 degrees positive.

Next, look at your accurate watch and pick the nearest time

| EOT | mm.ss |  |  |
| :---: | :---: | :---: | :---: |
|  | Mar | Apr | May |
| 1 | 12.31 | 4.06 | -2.48 |
| 2 | 12.19 | 3.49 | -2.56 |
| 3 | 12.07 | 3.31 | -3.02 |
| 25 | 6.13 | -1.54 | -3.10 |
| 26 | 5.55 | -2.04 | -3.04 |
| 27 | 5.37 | -2.14 | -2.57 |
| 28 | 5.19 | -2.23 | -2.50 |
| 29 | 5.00 | -2.32 | -2.43 |
| 30 | 4.42 | -2.40 | -2.35 |
| 31 | 4.24 |  | -2.27 | whose minutes is a multiple of 4 . In this case it was 12:43 daylight savings, thus this is 11:43 standard time. The 11:48 daylight saving time simplifies calculations. This was 12 minutes before noon, or at 4 minutes per degree, this was $357^{\circ}$ ( $3^{\circ}$ before noon).

Finally, the sun's hour angle (in this case $357^{\circ}$ or $3^{\circ}$ before noon) is adjusted by the total EOT and longitude correction. Since the time was before noon, the 3 degrees is backed off by (has subtracted from it) the 4.6 degrees. This results in 7.6 degrees as the correct solar hour angle for $12: 48$. Note: $7.6^{\circ}$ is less than $3^{\circ}$ when you consider them to be $352.4^{\circ}$ and $357^{\circ}$ respectively. The astro compass having been set, it is used as described on the previous page.

Appendix 2 and the master spreadsheet illustratingShadows.xls make this process very simple.


## SOLAR METHOD ~ WHEN THE SUN IS EXACTLY AT SOLAR NOON ~ very effective

The longitude, equation of time, and summer time corrections for converting local apparent time (L.A.T.) into legal standard time can be used to identify true north.

For accurate time measurement, a clock showing hours, minutes, and seconds updated by an atomic clock such as the one in Colorado may be purchased for less than $\$ 20$ USD, and its alarm can remind one when to take readings. However, understand its use of summer time, and recall that there are sometimes political moves to adjust the dates of summer time. Also, some such clocks do not always synchronize very well and thus drift, so check them with an internet synchronized clock. Even cell phone clocks are not as accurate as one would wish them to be.

| L.A.T. (local apparent time) | + i EOT eg: | + longitude correction | + 1 if it is | $\rightarrow$ LEGAL |
| :---: | :---: | :---: | :---: | :---: |
| or solar time as shown on | ¡ Nov-15 | - if E of legal meridian | summer | STANDARD |
| the dial | Feb +12 | + if west of it |  | MEAN |
|  |  |  |  | TIME |
| is what you see | gets local mean time | gets legal standard time and daylight saving | e or standar g time if rele | mean time vant |

Solar noon indicates true north because the sun is at its highest point. Thus a shadow produced at solar noon will point to true north. Solar noon happens when the local apparent time (L.A.T.) is 12:00:00 and the legal standard time that matches it is found as follows:-


Silver City NM is longitude 108.2 west, and is thus 3.2 degrees west of the legal time meridian for its mountain standard time zone (mst) which is 105 degrees west. At 4 minutes per degree, the 3.2 degrees becomes in clock terms 12 m 48 seconds west of the legal time meridian. To find the legal time for when the sun is at high noon for Silver City, we add 12m 48s (we are west of the meridian), thus getting 12:12:48 on the clock. That takes care of the longitude correction. However we must also correct for a slow or fast sun using the equation of time, or EOT. And summer time if appropriate.

On November 12 which was when this experiment was performed, the equation of time shows the correction to be " $-15: 46$ " which is minus because the sun is running fast by 15 minutes and 46 seconds. We subtract the 15:46 correction so our time synchronizes with the sun.

| legal noon on the 105 degree meridian: |  | 12:00:00 |  |
| :--- | :--- | :--- | :--- |
| longitude correction: | + | $00: 12: 48$ | we are west so we add |
| result | $=$ | $12: 12: 48$ |  |
|  |  |  |  |
| equation of time correction | $=$ | $00: 15: 46$ | use EOT table's sign |
| result |  |  |  |
|  |  |  |  |
| Summer time correction | $=$ | $00: 00: 00$ | not in November |
| result | $11: 57: 02$ |  |  |

To simplify this method, a "transit" table of noon plus the appropriate equation of time (EOT) can help, and the spreadsheets provided with this book can add in the longitude correction which further simplifies this method. Appendix 2 has a noon transit table for each day of the year so you only need to add in the longitude correction, it is also included in this chapter.

## TRUE NORTH BY COMPARING ACTUAL TO CALCULATED AZIMUTH ~ very effective



A flat board has a nail inserted perpendicularly into it at point "c" and its length from board surface to tip is "ch".

A vertical line "cx" is drawn and a perpendicular "cy" is drawn, and a plumb line "cv" attached to the nail so that the plate when attached to a wall can be set so that "cy" is horizontal.

The standard time is noted, and the tip of the shadow " $p$ " is drawn.

Distance $x p$ is measured, this happens to equal distance yc. And since angle hcy is a right angle, and since the sun's azimuth from the wall's perpendicular (ch) is angle chy, then the azimuth from the wall's perpendicular at the time must be:

$$
\text { azimuth at the noted time }=\text { atan (yc / ch) }
$$

When a wall's perpendicular is true south, i.e. ch points true south then life is simple, however few walls are so aligned.

If we have the standard time, we can convert it to local apparent time (L.A.T.) using the formula:


$$
\begin{aligned}
& \text { standard time }=\text { L.A.T. }+ \text { EOT + long.corr + summer.time thus } \\
& \text { L.A.T. }
\end{aligned}
$$

NOTE: When empirically marking a dial, the hour line's time has the EOT added to it. For example to mark the 11:00 am line, with an EOT of +10 , the line would be marked at 11:10, because when the shadow is on the 11:00 hour line, it would be 11:10, there is no sign reversal. In this case however, we are reversing the signs of the EOT, longitude correction, and summer time since we are now going the other way. HINT: forget summer time, use standard time.

Then, given the L.A.T. and the date, we can calculate the sun's azimuth from true south (formula A8.4 in the appendices). First, the L.A.T. must be converted to a local hour angle (LHA)(table A2.2 or 1 degrees per 4 minutes), the latitude must be known, and finally the sun's declination (formula A8.2a or A8.2b or appendix 2)

```
suns azimuth = ATAN{ SIN(lha)/
[as calculated]
    ( (SIN(lat)*COS(lha))-(COS(lat)*TAN(decl)) ) }
```

Once we have the sun's azimuth from true south calculated, and the sun's azimuth from the wall's perpendicular measured, the difference in azimuth's will be the wall's declination from true south.

There are a number of steps, and the spreadsheet illustratingShadows.xls has a worksheet dedicated to facilitating this reverse azimuth process. The steps are: (1) measure an actual azimuth at (2) a legal standard time, then (3) calculate the L.A.T. or local apparent time and (4) calculate the azimuth for that L.A.T. and date, and (5) the difference in azimuths is the wall declination. Multiple readings increase the final accuracy. The spreadsheet makes this so simple.


Some readings and basic data are collected and saved in "illustratingShadows.xls", this sheet uses latitude, longitude, and Julian day and derives the day's corrections.


The rod perpendicular to the wall or measuring board was 3 cm . The longitude correction was 12 minutes and 48 seconds, the date was April 4 giving an EOT of +3 minutes 7 seconds. However, the spreadsheet does all the work. Measurements were made at legal winter time MST (not daylight savings which was in effect) of 0907, 0950. 1025, 1047, and 1106, with respective vertical and horizontal values from the base of the perpendicular rod of (9.2, 7.9), (6.2, 8.0) cm, and so on. This provided azimuths of $71.94,64.18,53.13$ and etc degrees, and altitudes which are not essential.
measured azimuth [proof in appendix 7] = atan ( horizontal / rod length ) measured altitude [proof in appendix 7] = atan ( $\sin ($ azimuth $)$ * vertical / horizontal )

Having gathered raw data, all that is left is to calculate the azimuth for the local apparent time, the difference between the measured azimuth and the calculated azimuth is the wall's declination.

Local apparent time or L.A.T.: $\quad=\quad$ standard $-(+$ EOT + long.corr + summer.time $)$
The spread sheet does all the hard work, and even plots the " $x, y$ " points of the shadow which it graphically portrays as a double check. That is the only reason for the " $y$ " vertical measurements. The graph visually suggests that the first reading may be out of tolerances, so that could be ignored, and the last also seems erroneous. The average of the middle figures would probably be accurate, and that is 4.17 degrees. The above was checked with the astro compass which offered 4 degrees, and with three orthogonal compass readings which also showed 4 degrees.



The compass method uses simple math, a compass and is quick, but the compass can be swayed by nearby metal, such as wall reinforcements, and it is easy to make a mistake in the math unless you draw pictures (see appendix 2 for charts, tables, and some useful aids).

This solar method is slower, and requires a level surface. The figure to the left is a photo of a dial plate with a vertical pin, except circles are drawn around the pin. The surface is leveled with bits of wood between the upturned bucket and the dial plate. The compass is not used except as a rough and ready quick start to get the dial plate roughly pointing in the right general direction.

You begin before lunch, and note when the tip of the sun's shadow crosses an arc. And you come back in the afternoon and see when it crosses it again.


The sun crosses the circle at $x$ and $y$. You draw a line from the center to $x$ and to $y$, then bisect the angle. The bisected arc's center points to true north. The equation of time has nothing to do with this as it doesn't matter if the sun is slow today or fast. The sun orbits around the Earth's polar axis and that is what is actually being measured, not the time. In reality you do several circles and average the results, and several circles reduces the chances of a cloud shadowing things and ruining your afternoon.

## STELLAR METHOD ~ THE NORTH STAR ~ POLARIS

The skies at night may be used. The southern hemisphere has the southern cross and another marker, the northern hemisphere has the north star or Polaris which can be found by locating some constellations. Locate the pole star and from you to it is the north south line. This method is less accurate the further north you are since the altitude of the star increases. And the north star is not exactly where the celestial pole is, but off by a little bit.


Anyone using the stars to locate true north might search the web for "star maps", and some web sites are listed in appendix 10. One such web site might be:-
http://www.skymaps.com/downloads.html
The method of drafting the meridian is to use two suspended plumb lines one or both of which are rotated until both verticals coincide with the north star.

## IMPORTANT CONSIDERATIONS WHEN DETERMINING WALL DECLINATION

ASTRO COMPASS USAGE: The astrocompass method consists of placing the astro compass on a board whose edge is against the wall in question. For a wall $\mathrm{S} 4^{\circ} \mathrm{W}$, the astro compass will be rotated counter clockwise (left) when correctly set up as it points to the polar axis, true north.


SURVEYOR COMPASS USAGE: The compass points to magnetic north, and the true north is found by considering the magnetic declination (or variation as navigators and pilots call it).


With the compass aligned perpendicular to the wall, the same house would show the compass needle deflected clockwise (to the right) reading 354 magnetic with $10^{\circ}$ easterly variation. This translates to a $364^{\circ}$ true bearing, or $004^{\circ}$. This might seem to be contrary to the astro compass, however it is not. True north is still deflected to the left.


It is easy to become disoriented in the heat of field measurements. Work out the rough alignment first with a compass, then estimate some astro compass or wall-declination-by-azimuth figures next, then perform the actual final measurements. The spreadsheet: illustratingShadows.xls will take the guess work and frustration out of the scenario.

When using a magnetic compass, consider taking three readings, one east, one west, and one south of the wall, several readings from different places will reduce errors due to rebar or minerals.

## NORTH AND SOUTH HEMISPHERE DIFFERENCES

Most books that discuss sundials, this one included, select a hemisphere and work from its perspective. The benefit is that a fixed system is used, as opposed to being more abstract. These are the differences and similarities when looking at a different hemisphere

A shorter shadow in the northern hemisphere means a longer one in the southern hemisphere, and vice versa.


For similar dials, the $U$ shaped declination line in the northern hemisphere is a $\cap$ declination line in the southern hemisphere, and vice versa.

Comparing the north and south hemispheres, winter and summer are reversed, summer in the north means winter in the south, short shadows in the north mean long shadows in the south. While the sun moves from east to west in all cases, looking at the equator, the sun moves from left to right in the northern hemisphere, and right to left in the southern hemisphere. The celestial pole is north in the northern hemisphere, south in the south hemisphere. When a gnomon points to the equator, that means north in the southern hemisphere, and south in the northern hemisphere. Some things that stay the same, all formulae are the same, hour line angles are the same, the equation of time is the same. However, the hours on a dial plate themselves are reversed. For example, below are two horizontal and vertical dials, one for each hemisphere.

Horizontal dial in England


Vertical dial in England


Horizontal dial in Australia


Vertical dial in Australia


See also chapter 25 for notes on the north south hemisphere differences, especially when considering the analemma.

## NOON TRANSIT



The table above shows the time of noon transit corrected for the equation of time. The time shown must be corrected for the difference between the dial's longitude and the legal meridian.

If the dial is WEST of the meridian, noon happens later, so ADD the minutes difference which is 4 times the longitude difference.

If the dial is EAST of the meridian, noon happens earlier, so SUBTRACT the minutes difference which is 4 times the longitude difference.

## CHAPTER SEVEN

## Geometry and Trigonometry, a refresher

If one excluded empirical dial design, and focused on planned dial design, then geometry would be the first line of attack. The following is for flat surfaces, not curved surfaces such as a globe. However, for most purposes we can assume that this works on the surface of the planet.

## GEOMETRY

Geometry for simple dials, is a simple process. For example, the polar dial derived from the $15^{\circ}$ per hour radials is not involved.

A circle is draw, this represents an equatorial dial, and the $15^{\circ}$ radials extended to a horizontal line, this represents the dial plate, in particular, the equinoctial line.


As an aside, the $60^{\circ}$ radial shows a distance of 5.2 inches from the bottom of the circle with a radius of 3 inches. The distance along that horizontal line is the trigonometric tangent of the angle, times the radius.

| angle | 60.00 |
| :--- | ---: |
| radians | 1.05 |
| tan | 1.73 |
| times 3 | 5.20 |

Geometry gets slightly more complicated when three dimensions are involved. Something needs to be folded to make the geometric figure two dimensional. The above polar dial can be used, especially when it comes to lines (equinox) or curves (declination or calendar data). The hour lines are simple enough and were drafted using a flat two dimensional surface. But for the sun's declination to be depicted, a third dimension is added because the rays can hit the gnomon at angles ranging from -23.44 (winter), to 0
 (the equinoxes), and +23.44 at the summer solstice.

There are two lines which are 23.44 degrees from the vertical and show the range of the nodus shadow from the winter to summer solstice, with the equinoxes sharing the straight center line.

How can the three dimensional picture above be converted into a two dimensional flat drawing, such that all the relationships are retained.

The triangle representing the shadow range can be rotated flat.


After the gnomon has been rotated flat, then the rays of light are projected from the nodus now on the dial plate. This shows what happens for the solstice range for noon.

For different hours, that triangle of rays is from the nodus to the hour point on the equinox line.


In this case, two hours from noon (30 degrees from the vertical), results in a nodus to dial plate distance that is longer than the one for the preceding noon example.

The end result is that where the + and -23.44 degree lines intercept the associated hour line, will form a plot that is hyperbolic.

Convert the above three dimensional pictures into a flat drawing where relationships are retained, the polar dial uses a simple geometric construction, or a simple trigonometric formula.


As an aside, the trigonometric formulae also derive from that picture:-

```
tan (hrd ) = dh / rh = dh / th, thus dh = th * tan (declination ) and:-
cos (ath)= ta / th thus th = ta / cos (ath)
so given: }\quad\textrm{dh}=\textrm{th}*\mathrm{ * tan (declination )
then:- dh = ta * tan (declination) / cos (time )
```

The distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style linear height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines, then the points joined to form a hyperbolic curve for the solstices, or a straight line for the equinox.

## TRIGONOMETRY

Geometry cannot be used as is in programs or spreadsheets. So, the geometric relationships must be converted into things that programs and spreadsheets can use, and that something is trigonometry.

Accurate geometric figures can be used when developing trigonometric formulae, or, they can be stylized.


The same diagram as above can be stylized, as shown below.


Pure diagram for a vertical decliner retains relationships but has more steps.


Stylized diagram for a vertical decliner has fewer steps, relationships are not retained, but with a little imagination, it is simpler to see how trigonometric formulae are

Trigonometry, as used in this book, uses what are called circular functions.

Circular functions are sin, cos, and tan. They relate the sides of a right angle triangle that is contained in a circle.
the vertical in this triangle constrained in a


The tangent of that angle, abbreviated "tan": circle is the "sine" of the angle times the hypotenuse, sine is abbreviated to "sin".
the horizontal in this triangle constrained in a circle is the "cosine" of the angle times the hypotenuse, cosine is abbreviated to "cos".
the cosine is the complement of the sine.
The other way of looking at things is that the:-
$\sin (a)=$ opposite $/$ hypotenuse
$\cos (a)=$ adjacent $/$ hypotenuse
$\tan (\mathrm{a})=$ opposite $/$ adjacent

The above functions enable lengths of sides of a triangle to be calculated if the angle "a" is known and the length of one of the sides. The sentence "the old aunt, sat on her, coat and hat" helps remind people of those relationships.

Certain standard relationships exist:- $\quad \tan =\sin / \cos$
And for any triangle, whether or not it is a right angled triangle,

the law of sines says:-

$$
A / \sin (a)=B / \sin (b)=C / \sin (c)
$$

While normal humans use the "degree" which is $1 / 360$ of a circle, the "radian" is more appropriate for mathematicians, there are 2 * 3.1416 radians in a circle.

The sin, cos, and tan can be calculated from a series

```
sin = x - (x** 3)/3! + (x**5)/5! - (x**7)/7! + . . .
cos = 1 - (x**2)/2! + (x**4)/4! - (x**6)/6! + . . .
tan = x + (x**3)/3 + 2* (x**5)/15 + . . .
```

The opposite of sin, cos, and tan are called arcsin, arccos, and arctan, often abbreviated to asin, acos, and atan, or $\sin ^{-1}, \cos ^{-1}$, and $\tan ^{-1}$, or even asn, acs, or atn.

The arcsin and arccos can be developed simply from the following:-

```
asin = atan(x/sqrt(1-x*x))
acos = (3.1416/2) - asin (x)
```

however, atan is more involved. There are two series often found, one only works for a value of up to 1 , which is $45^{\circ}$, which is not very helpful. The other is:-

```
atan = (x/(1+x*x)) * ( 1 + (2/3)* (x*x/(1+x*x)) +
    ((2*4)/(3*5))* (x*x/(1+x*x))* (x*x/(1+x*x)) ...
see the Appendices for notes on the series.
see Supplemental Shadows for notes on the series including
    iterative methods.
```

Thus, if a sundial can be reduced to geometry using the projections discussed earlier, then they can be converted to trigonometry. Most such conversions are straight forward. Some are complex.

Different people use different parts of a geometric figure to develop a trigonometric formula.
For example, the standard formula for the hour line angles of an inclined decliner differs from the formula derived in this series of books.

The results are the same for all ranges of numbers. In fact, one formula can be converted to the other.

In the context of sundials, all hour angle dials derive from the equatorial dial, a dial whose dial plate parallels the planet's equator.

The "Durer" model was a geometric derivation of how dials are derived from the equatorial dial.

From this stem all the geometric models for all the different hour angle dials, as opposed to the azimuth and altitude dials.

From those geometric models stem all the trigonometric formulae used.

From those trigonometric models stem the spreadsheets and other programming systems for dial design.

This entire series of books, booklets, and supplements, was developed with no spherical trigonometry. In spherical trigonometry, biangles can have angles (other than $0^{\circ}$ ), and angles in a triangle do not add up to 180 either.

Other functions exist that compare to the circular
 functions of $\sin , \cos$, and tan. There are hyperbolic functions called sinh, cosh, and tanh. They use a hyperbolic curve rather than a circular arc. They are useful in electronics, some sundial authors use them for exotic dial plate furniture, however, they are not used in this series of books.

## CHAPTER EIGHT

## General Interest

There is a well known observatory in India at a place called Jaipur. The Jaipur Observatory was established by the local ruler in the 1700s, and has been modeled by Illustrating Shadows using virtual reality markup language (VRML).

With a browser plug-in from one of several sources, for example Parallel Graphics provides the Cortona plug-in, you can bring up this observatory and walk around it in full 3d. For interest, Illustrating Shadows also provides some other worlds. Some have an animation of the moon revolving around the earth which in turn orbits the sun.

Appendix 10 discusses virtual world viewing software, along with issues related to different Windows variants such as XP, and Vista both 32 and 64 bit.


A virtual world in VRML, built using the Parallel Graphics software scene builder ISB, animated with their ISA animator, of the Jaipur Observatory in India, and other places. Animated models are in those worlds which are on this book's web site.

Further, some of the dials designed for pictures in this series of books were themselves saved in VRML form, and using the Cortona web browser plug-in, the viewer can move those CAD generated sundial models and study them by rotating them and seeing how they are built.

## SUN DIAL STAMPS

Many countries have issued postage stamps with sundials on them, one of my favorites depicts an observatory of many sundials in Jaipur, India. Another depicts a dial in the Dominican Republic.


Some advertisements from the 1950s, for beer, liquor, and soft drink products used a sundial to send a message of gentle tranquility. The author has several in his collection.

## SUN DIAL CHINA

Here are some examples of china ware depicting sundials.

## SUN DIAL MEMORABILIA

Here are two examples of old biscuit tins in which biscuits were purchased. Cookies to an American.


## SUN DIAL POSTCARDS

In the author's collection of several hundred postcards are some of sundials in interesting places. One is of Jaipur, an interesting solar observatory established in the 1700s.

## SUN DIAL COLLECTORS CARDS

In the early 1900's companies issued collector's cards. In particular a series from a cigarette company, depicted various sun dials is in his collection. This one is of a "mass" dial.

## SUN DIAL BOOKS

Even books may have a sundial as their theme. Such books that the author possesses tell of some young chums and their associated adventures involving sun dials. Not all books with titles like "The Sundial" are about sundials however. The ones to the right do relate to sundials and are rather fun.


## ANTIQUE BOOKS OF EPHEMERIS

Some pages from an ephemeris in the author's collection, this is for 1773. Longitudes and latitudes are shown and a table of declinations from which sunrise and sunset may be deduced.


## MAGAZINE ARTICLES

Some magazines from the mid 1900's also discussed sundials. One in the author's collection has an interesting technique for building the hour lines for any desired latitude

Other sources of interest:
The North American Sundial Society has available to members a CD with all sorts of interesting collections. For example, there is a whole section on stamps related to sun dials. Another has many of the early patents issued. The society also has a register of sun dials. Membership in the society is recommended. Similarly, the British Sundial Society has many useful resources.

## DIAL MOTTOES



There are thousands of mottoes used on dials that help make up the dial furniture. Many are well known, however, some less well known ones are reproduced below. Many have a theme of doom and gloom, reminding all who stop to read of graveyard epitaphs. Sometimes written in Greek, often in Latin, these add a subtle charm for the passer by. Latin, in the far off distant mists of time was written with no gaps between the words. Equally interesting is the Roman numbering system, one of the few with no weighting, and it had no zero. Julius Caesar introduced the idea of using a dot to identify the first letter of a word, thus making reading more practical. This evolved into a dot between the words, and later a space was used. From the author's collection are some books of sundial mottoes etc between 1903 and 1919.

Let there be light.
Each one wounds, the last one kills ~ OMNES VULNERANT ULTIMA NECAT
As time and hours pass away, so doth the life of man decay
Time has been, is, and IS TO be. ~ TEMPVS RERVM ET ERIT
The sun returns but not so time
At my back I always hear Times winged chariot drawing near
The greatest maxim I can give, is make the most of how you live
I know mine hour, dost thou know thine
The time thou killest shall in time kill thee
Serene I stand amidst the flowers to tell the passing of the hours
The last is hidden so that we have to watch them all

~ ULTIMA LATET UT OBSERVENTUR OMNES
Use them don't count them ~ UTERE NON NUMERA
Beware of one of them ~ EX IIS UNAM CAVE
One of these will be your last ~ UNA EX HIS ERIT TIBI ULTIMA
The hour is slow but the years run quickly
~ LENTE HORA CELERITER ANNI
Life's but a shadow, Man's but dust, This diall says, Dy all we must
No man can call again the past time [pun] Whilst Phoebus on me shines, view my shades, view my lines


Common sources for mottoes come from scriptural works such as the psalms, and from classical works such as Chaucer and Shakespeare.

Harking back to a time when sundials were popular and dialing was a daily functional activity, stereoscopes were used for a casual past time. For interest only, some 3D pictures are provided which may be viewed by placing a cardboard between the two photos and letting the eyes relax.


A horizontal and vertical dial with an equatorial, after Albrecht Durer.


A gnomon on an east facing declining vertical dial in 3D.


A large Shepherd's dial

## CHAPTER NINE

## The basic equatorial and armillary hour angle dials

This chapter introduces sun dials that use simple hour angle principles, the hour angle of the sun as it rotates around the polar axis of the Earth. The sun rotates around the polar axis 15 degrees per hour. Thus, should a gnomon or in this case, a style be parallel to the polar axis and thus set to point to the pole, and, if the dial plate paralleled the equator, then hour lines would be 15 degrees apart. This is true for the armillary dial as well as the equatorial dial.


The equatorial dial has a polar pointing gnomon in the form of a rod, however the dial plate is flat and parallels the equator.

Winter hours appear on the side facing the Earth, and summer hours appear on the side facing the sky. Hours of the equinox are not readily available. The seasons are indicated by the length of the shadow cast from a nodus, and the sun describes circles on the dial plate. Hour lines are drawn as straight lines from the center, and calendar lines are circles surrounding the center as well.

The same principle applies to the armillary dial except the dial plate is not a flat surface paralleling the equator, rather it is a curved dial plate, whose radius is constant from the gnomon or style which in turn parallels the north south polar axis.


Hour lines are also drawn 15 degrees apart, or they
 can be measured linearly. Calendar lines are straight lines, indicated by a nodus on the gnomon or style, and parallel each other. The equatorial dial, or the armillary dial, can be the basis for designing almost any other type of sun dial that uses the 15 degree per hour solar rotation around the Earth's polar axis.

While the polar dial is not quite as simple as the equatorial and armillary dial, it is easily derived from them. The dial plate is angled for the latitude, but is flat. And like the armillary and equatorial dials, the gnomon is aligned to true north/south and sloping at the latitude angle. The hour lines are made by projecting the 15 degree lines to the flat dial
 plate, thus the hour line separation is not constant. A polar dial is an armillary dial whose dial plate has been flattened. The polar dial is the next step in the evolution in understanding dial design, and thus is introduced here, but discussed in detail in the next chapter.

## THE EQUATORIAL DIAL ~ HOUR LINES THEMSELVES ARE $15^{\circ}$ APART

The armillary and equatorial dials are very similar. In the armillary dial the hours were inscribed on a band, and the gnomon or style created a shadow on that band. The hours were 15 degrees apart, and summer or winter, reading the dial was easy.

The equatorial dial, named as the dial plate parallels the equator, is designed rather like an armillary dial. Whereas the armillary dial can find you using distances instead of degrees to lay out the time scale, with an equatorial dial it is simpler to use just the 15 degree angles.

This is because the armillary used a curved band to hold the hour lines, but an equatorial dial uses a flat, often circular, plate. That plate parallels the equator, hence the name "equatorial" dial.

The gnomon is pointing to true north/south angled at the location's latitude. this side of the dial plate which is uppermost


Longitude adjustment is made by rotating the dial, as was done for the armillary. Equation of time is usually managed with a table. However both the armillary as well as the equatorial dial can have a device to rotate the dial based on the date. This method of handling the equation of time works with armillary and equatorial dials because they are dials whose hour lines are separated by a constant number of degrees. The armillary or equatorial dial can be used as the basis for all other hour angle dials in existence, they can all be derived from it. However, for those other dials, adjusting for longitude is more involved.

NOTE: An equatorial dial can be built on a square or oblong and fitted with a protractor, and then fitted onto the gnomon of almost any other type of sundial, and the equatorial dial's hour lines projected using a cotton thread from the equatorials dial's center to the dial plate on the new dial. A small laser pointing device may be fitted so it can rotate on the equatorial dial, and this might be easier than using a cotton thread. When used in this fashion, the auxiliary equatorial dial may be called a trigon.

NOTE: If the gnomon's rod is of appreciable thickness then the hour lines must be adjusted to consider that thickness.

EQUATORIAL DIAL (dial plate parallels the equator, gnomon parallels the polar axis)


Geometrically, the hours are multiples of 15 degrees. The calendar lines are circles however, and together with the tip of the style or of a nodus, they form a cone. The solstice is of course 23.5 degrees from the tip of the style. And the circles get larger as the declination of the sun decreases. The equinox has a declination of 0 degrees, and that calendar circle would be at infinity, in other words it cannot be marked on the dial plate. Latitude corrections are applied by tilting the entire dial to the latitude. Equations of time are applied mentally. The radius from the nodus or tip of the

style is simple geometry. For example if November 22 were chosen, then from formulae or tables in the appendix the sun's declination is found to be -20.0 degrees. A right angled triangle is drawn with the height of the style or nodus as one length which with the hypotenuse was angled at $90-20.0$ or 70 degrees. Simple measurement provides the calendar lines circle.

Trigonometrically, the issue is the radius of the calendar lines. Their radius is the style height divided by the tan of the declination. While no special software or spreadsheet is needed, the DeltaCAD macro on the website and CD for this book does provide the calendar radial distances as well as the sunrise and sunset line, if desired.

To the left is an equatorial dial made from a circular concrete paver with the lines engraved with an engraver. The style is a fair sized bolt, and that size must be considered when marking or reading the hour lines. In this case the hour lines are not offset, rather the observer averages the shadow to locate its center. This dial has longitude correction, and its case study is presented in a few pages.

Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25.

An interesting phenomena happens on the equinox as far as equatorial dial shadows go. There is no shadow on the dial plate.


The only visible shadow at the equinox.

Note: Calendar or declination lines and curves are discussed in chapter 22, with other lines discussed in chapter 23.

The top left picture is the north or upper or summer plate of an equatorial dial, taken from the north west, on September 22, or thereabouts.

On the equinox the sun's path is edge on to the dial plate, so the gnomon casts no shadow on either the upper summer plate, nor on the lower winter plate either.

Looking carefully at the lower picture, you can see a gnomon shadow on the dial plate support structure, but no shadow on the dial.

Thus, if an equatorial dial has a small armillary or other device to detect shadows secured to the circumference, not large enough to interfere with the normal dial plate shadows, then the dual purpose equatorialarmillary dial will display the time year round.

Below is a template drawn by the DeltaCAD macros on this book's CD and web site for an equatorial dial with calendar lines, longitudinal correction and a sun-rise and set line. The sunrise and set line is simply the horizontal projection from the nodus tip to the dial plate. For the summer that will be a line above the summer gnomon's base, for the winter it will be a line below the winter gnomon's base, as discussed in chapter 22.


## THE ARMILLARY DIAL ~ HOUR LINES THEMSELVES ARE $15^{\circ}$ APART

In olden times, many centuries ago, the first sundials used vertical poles, or obelisks. They drew hour lines from the bottom of the pole and the end result was that the hours were variable length day after day. As time went on, people wanted constant hours and they also found that the sun appears to rotate around the polar axis of the Earth.

a vertical pole with hour lines radiating from the base does not indicate hours of the same duration day to day.

a pole sloped to parallel the Earth's polar axis, perpendicular to the equator, with lines drawn from its base will indicate hours whose duration is the same from day to day.

The idea of using a pole that was parallel to the Earth's rotational axis, not perpendicular to the Earth's surface, i.e., slanted to match the latitude, resulted in a shadow line which with a bit of calculation provided lines for hours (hour lines) whose duration would be constant hour to hour, day to day, and all year round. So the slanted pole gnomon (the shadow casting edge is called a style) is used for time telling when the sun's angle around the Earth's poles is used.

The line of the shadow indicates the time, its tip or other "nodus" such as a notch or blob, indicates the date. The date is ambiguous except for the solstices (June and December). For example the sun is over the equator in both September and March.


The armillary dial is a hemisphere or part of a circle of constant radius, and each hour is 15 degrees of arc. It derives its name from the Latin for "arm bracelet".

At solar noon, the 12 o'clock line is always on the same place on the circular dial. If the style or gnomon has a nodus then its point up or down the dial plate will also tell the date.

NOTE: If the gnomon's rod is of appreciable thickness then the hour lines must be adjusted to consider that thickness.

Note: Calendar or declination lines and curves are discussed in chapter 22, with other lines discussed in chapter 23.

The hours indicated are solar local apparent time (L.A.T.) hours. They need some corrections, one for longitude, one for the equation of time, and some for summer time if desired.

The latitude can be adjusted by adjusting the slope of the style and dial plate. This should always be the latitude of the place where the sundial is located. It goes without saying that the style should also point true north/south. True north is not magnetic north except on a specific meandering line where such is true. Finding true north was addressed earlier.

The longitude correction is very simple in the case of armillary and equatorial dials. The dial has 12 o'clock directly below the style when the sundial is on the longitude associated with the legal meridian or time. But if you are west of it, the dial plate must be rotated to show a later time than it would otherwise do, and if you are east of it, it should show an earlier time. Why? Because if it is 12 noon on the legal longitude, say 105 degrees, the sun will be almost vertical above the 12 mark, but at the very same instant at for example the 108 degree longitude, the sun will not have reached the vertical yet. And so on for any other hour.


The above picture shows why the sun reads an earlier time at 108 degrees of longitude than it does on the 105 degree line. For an armillary or equatorial dial, the adjustment for longitude is simple ~ rotate it.


The picture to the left is a modern permanent dial. The latitude is adjustable but needed a device to measure the angle because the scale was not marked. Also, this dial has no longitude adjustment, so a fixed amount must be added or subtracted depending on its location. Similarly the equation of time must be considered.

If you think this dial isn't working because there is no shadow of the style on the dial plate, fear not, the sun was not shining when the photograph was taken. However it does raise the point that shiny surfaces such as this can make it hard to read a shadow.

If the style, the rod thing looking like an arrow, had a notch or other mark on it such as a blob (nodus) then declination lines could be drawn on the circular band or dial plate so that it would provide calendar information as well as the time, although this particular armillary band would not be wide enough. In this chapter, examples are shown of armillary dials made from PVC as well as with other materials.

The equation to be used for any sundial that was designed for a specific latitude is:


And how is the date indicated? On an armillary or equatorial dial the style would need a notch or blob on it, called a nodus, and its shadow would move up or down the dial plate based on the sun's angle with the equator, called the sun's declination. Of course tables exist that estimate this declination, or one may acquire an almanac to look up such things. The appendices provides just such a table, and the section on formulae also provides such a formula.

Building an armillary dial is straight forward. There are three common ways of drawing dials, and the armillary and the equatorial dial are no exception.

Geometrically, the radius of the dial plate is found, then an isosceles triangle drawn with 15 degrees at the apex and the two long sides are equal to that radius. The base of the triangle, between the two resulting 82.5 degree angles, is the horizontal straight line distance between the arc segment.

Trigonometrically, the arc straight line distance " $x$ " is about 0.261 times the radius, which is derived from the law of sine's $[x / \sin (15)=r / \sin (82.5)$ ] or from twice the sine of half the angle of 15 degrees $[x / 2=r * \sin (7.5)]$.

\(\left.$$
\begin{array}{ll}\begin{array}{ll}r \\
x\end{array}
$$ \& =the radius of the dial plate <br>
= \& the straight distance that 15 degrees <br>

produces\end{array}\right]\)| 15 | $=$ the arc for one hour |
| :--- | :--- |
| $82.5=$ | the other two angles, as all angles |
|  | add up to 180 degrees |

Empirically, place a protractor on the style, the center of the protractor has the style running through it. Then use a cotton thread to measure the protractor's 15 degree lines to the dial plate.

The above method for the equatorial dial has no longitude correction. That correction is made, only for equatorial and armillary dials, by rotating the dial plate. Other dials use differing techniques for longitude correction, and there is nothing wrong in a tailored equation of time (EOT) chart or table that adds the longitude and EOT corrections together, thus making only one mental correction to apply, not the two separate ones.

## INTRODUCING THE POLAR DIAL ~ HOUR LINES ARE PARALLEL

The next sun dial in the complexity chain is the polar dial, named because the dial plate parallels the polar axis. Whereas the armillary dial and the equatorial dial had hour lines $15^{\circ}$ apart ...

... the polar dial hour lines increase in distance the further away from the style they are.


In the polar dial, the dial plate and the style are both angled to match the location's latitude, they are both aligned with the polar axis, hence the dial's name. The style is a certain distance above the dial plate. Because the dial plate is flat, the hour lines are no longer evenly spaced. They can simply be deduced from the equatorial dial, or by simple trigonometry. And whereas a calendar was easy to draw on an armillary or equatorial dial where the lines of the sun's declination were parallel to each other, it is a bit more involved here. The equinox line is straight but the other lines such as the solstices will be hyperbolic curves. Just as the equatorial dial had three ways of making it, so does the polar dial.

Geometrically, we can draw a triangle that matches the dial plate and the style, the apex of the triangle is 15 , or 30 , or some other multiple of 15 degrees. And this diagram provides the distance from the sub-style (the point the style is directly over on the dial plate) for the appropriate hour line. The protractor can be rotated flat onto the dial plate and the style's linear height used as a point for drawing angles if desired, this is called projecting the protractor and style's linear height.


Trigonometrically, using the above figure, $\tan (15)=x / y$, thus the distance from the sub style " $x$ " is given by: $x=y * \tan (15)$, or, hour line distance = style height * tan(hours*15). One hour would be 15 degrees, two hours would be 30 degrees and so on. The appendix reviews trigonometry and has a table of tangents.

Empirically, a protractor can be placed on the style, just as with the equatorial dial, and then using a cotton thread " c ", the hour lines drawn. You get a single point and that is all that is needed because the hour lines are parallel to the style and sub style.

The above method for the polar dial has no longitude correction. That correction is made by rotating the protractor or adding or subtracting from the 15 degree angles, etc before the hour lines are drawn. It cannot be made by sliding the plate left or right under the style, doing that would not take care of the ratios that exist between the hour lines and the base of the style, the sub-style. Expanded details of the polar dial's design are covered in the next chapter.

The hour lines are a simple 15 degrees. The calendar (declination lines) are arcs whose radius is:-
radius $=\frac{\text { gnomon linear height }}{\tan (\text { declination })}$

| DECLINATION | RADIUS |  |
| :--- | :---: | :--- |
| solstice $6 / 21$ | $23.5^{\circ}$ | $2.30 *$ gnomon |
| month from | $19.8^{\circ}$ | $2.78 *$ gnomon |
| two from | $11.2^{\circ}$ | $5.06 *$ gnomon |
| three months is an equinox $\sim$ infinity |  |  |
|  |  |  |
| Table A3.4 has all these figures as well as the |  |  |
| sunset/rise horizontal line data for most latitudes. |  |  |

$\checkmark$ one month from solstice ~ May/July or November/January 21 decl is about 19.8 two months from solstice ~ April/August or October/February decl is about 11.2

Also, sunrise and sunset are the intersection of the calendar arc and a horizontal line drawn as the intersection of a horizontal line from the nodus to the dial plate; above for the summer side, below for the winter side. This is latitude dependant. The distance to that horizontal line is $=$ $\tan$ (latitude) * gnomon linear height. For latitude 32 the distance is 0.62 . The CAD drawing below

was placed between two plexi glass sheets and shown in use, above right. Summer time was not used in the real dial as it was to be used in Phoenix AZ where there is no summer time switch.

Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25.

## AN ARMILLARY DIAL

A 4.5 inch PVC pipe cap-end was cut and used, the radius was thus 2.25 inches,, being thusly the gnomon to dial plate distance.

The hour lines are a simple 15 degrees, or 3.1416*diameter/24, i.e. $3.1416 * 4.5 / 24=0.58905$ inches apart. Useful when using CAD to draw a template. Appendix 1 may help here. The calendar (declination lines) are lines whose distance from the equinox line is:-
$=$ gnomon : plate distance * tan(sun's declination)

| KEY DATES AND DISTANCES |  | diameter <br> radius | $\begin{array}{r} 4.5 \\ 2.25 \end{array}$ | solstice to equinox distance inches |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1/20 | 5/20 | 7/20 | 11/20 | 0.82 |  |
| 2/20 | 4/20 | 8/20 | 10/20 | 0.45 |  |
| 3/20 | 9/20 |  |  | 0.00 | equinox |
| 20-Jun 20 | 20-Dec |  |  | 0.97 | solstice |
| 6am-6pm: di | distance | . 0686 | and /12:- | 0.58905 |  |

Using a CAD program, a dial plate was laid out 7.1 inches east to west ( 6 am to 6 pm ), and 2 inches north to south (twice the solstice to equinox distance). Hour lines are 7.0686/12 inches apart, namely 0.58905 or 0.59 inches. The sloping lines are Italian lines (see later) and show the time until sunset, however those lines make the dial latitude dependant.


The dial shows 2:10 pm winter time in October, just over 3 hours to sunset. Sunset lines, or Italian lines, are covered later, as are calendar or declination lines, they enhance the usefulness of the dial over the common pocket watch.

For a location with no summer time differential, the summer hours would not be one plus the winter hours. In the above example, summer time is considered when the sun is above the equator. If the metal used is magnetic, then it will deviate a magnetic compass, so alignment must be done before magnetic nuts and bolts are used.

Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25.

## CASE STUDY ~ AN EQUATORIAL DIAL WITH CALENDAR CIRCLES

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ} \mathrm{N}$ |
| :--- | ---: |
| location long: | $108.2^{\circ}$ |
| magnetic declination: | $10.6^{\circ}$ |
|  | E |

A concrete paver from a garden supply shop was acquired and a hole drilled in dead center with a masonry bit. Into that hole was inserted a threaded rod as the gnomon, its length was cut to be 5 cm on each side, and the paver itself was 4 cm thick, making a total length of 14 cm .

From that center were drawn 15 degree radials for the hours, and since this dial would be longitude corrected, another mark was drawn 3.2 degrees offset from noon, and when installed, that offset mark would be the vertical.


Then the sun's declination was established for the calendar lines.

|  | Dec solstice | -23.5 |
| :--- | ---: | :--- |
| Jan | Nov | -20 |
| Feb | Oct | -12 |
| Mar | Sep | 0 |
| Apr | Aug | +12 |
| May | Jly | +20 |
|  | Jun solstice | +23.5 |

Given the sun's declination angle and the height of the gnomon, the distance for the calendar circle was established, and the appendices have it tabulated or the formula can be used:-
calendar circle radius $=$
gnomon linear height $\tan$ (declination)

The design notes were all of one page, and the only geometric or trigonometric work was limited to:-

- how long the gnomon was to be in order to have reasonable calendar circles
- what is the radius of those circles

The radials and the calendar circles appear on the upper (summer) side as well as the lower (winter) side. They were first scribed with a Dremel engraver. The hour lines were scribed against a straight edge, the calendar lines were scribed using a $1 / 4$ piece of wood with some holes in it to act as a rotating rule. One hole fitted over the gnomon, the other two were set to the calendar circle radius and were large enough to accommodate the Dremel engraver but with no slack. The wooden radial rule was rotated, and all lines were engraved well. Then a Dremel rotary tool was used with the masonry circular disks to refine those lines.


While little dust may be thrown up by the engraver, there is still some. And the rotary cutter will clearly create clouds of the stuff. There is a health risk of silicosis, thus this operation should be well ventilated or outside, and a tight, not loose, fitting face mask should be worn, as well as eye protection. The clothes will become saturated with dust as well.

To the right is shown the brick support mechanism, it limits winter hours somewhat, but not excessively.


The picture to the left shows the solstice circle and one month after that. Two months after that was in fact the periphery of the paver. The third month after was the equinox and that radius is of infinite size.

The backing of the dial, the winter side, was supported by clay bricks cut with a masonry saw at co-latitude, so the paver paralleled the equator and the rod paralleled the polar axis.

Two holes were drilled in the paver and in the bricks into which rods were inserted to add stability. One hole was coincidently on an hour line, the other was not, this was because the dial plate was rotated to correct for the dial's longitude difference from the legal time meridian.

The final touches were to mortar the entire dial in place. This dial was placed on one of the columns of the analemmatic dial. This dial took one day from start to finish, and if $1 / 4$ inch copper tube was used in place of the threaded bolt, the entire dial would look rustic and a wonderful addition to a country garden.

To the right is the equatorial dial in place on the 6 am column of the analemmatic dial.

Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25.


## CASE STUDY ~ AN ARMILLARY DIAL WITH CALENDAR LINES

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ} \mathrm{N}$ |
| :--- | ---: |
| location long: | $108.2^{\circ}$ |
| magnetic declination: | $10.6^{\circ}$ |
| E |  |



Another line was drawn which was the longitudinal offset from the dial's location and the legal time meridian. This was offset by 3.2 degrees, being the dial's location (108.2) minus the legal time zone's meridian (105).

Being west of the legal meridian, the dial would show earlier times than the legal time, so the dial plate is rotated to show a later time, in this case by 3.2 degrees. Were the dial east of that legal time zone meridian then the dial would show a later time, and would be rotated to display an earlier time.

The hour lines were scribed and so was the longitude correction line, a Dremel engraver was used.

longitude correction line and its mate on the brick that would hold this armillary dial plate.

The armillary and equatorial dials are simplicity itself when designing the calendar lines. Everything is linear, they are straight lines and not hyperbolic curves.


There are 7 angles of interest, what you use is what suits you best.
+23.5 June solstice
+20 May and July
+12 August and April
0 September and March equinox

- 12 October and February
- 23.5 December solstice

The radius, or in this case approximate radius was found so the 7 angular lines are drawn. From this come distances. That is the geometric approach. The trigonometric approach would be to observe that

$$
\tan (23.5)=\frac{\text { linear distance from } 0 \text { to the June solstice }}{\text { radius of the semi-circle }}
$$

or $\quad$ linear distance from 0 to the June solstice $=\tan (23.5)$ * radius of semi-circle
The tangent of 23.5 is 0.4348 or 0.44
20 is $\quad 0.3640$ or $\quad 0.36$

Whereas the equatorial dial can never show the equinox dates since the sun's rays parallel the dial plate, the armillary can easily show all the calendar lines.

The dial plate selected here has the solstices at the extremes of the dial plate, the equinox line is in the center, and the May/July line next to the equinox line, the April/August line between that and the dial plate edge which displays the solstices.

The nodus can be seen in the shadow as just removed from the summer solstice.

Note: Calendar or declination lines and curves are discussed in chapter 22, with other lines discussed in chapter 23.


The gnomon is made from 1/4 inch copper tubing commonly found in hardware stores, soldered to establish the latitude, and a 10 or 12 gauge copper wire soldered into that. This was inserted into a hole drilled into the brick with a masonry bit. Grid powered electric drills are better for this sort of work than the portable battery drills because portable drills tend to go at slower speeds.

The gnomon support shaft was coated with epoxy and inserted in the hole after carefully verifying the level and angular sections.


Then the copper wire was displaced with two small bends to accurately align it, this corrects any errors when its support was stabilized with epoxy. The copper wire then received a blob of solder after accurately determining the correct place, being 90 degrees or perpendicular from the equinox line on the dial plate. The whole was copper plated with a mix of water and copper sulfate.

The dial plate was not truly circular, this means that those calendar lines would not in a pure world be straight lines, however for a garden dial they are close enough.

The dial plate was fixed with epoxy into a slice cut from the brick with a circular masonry cutting blade. After a number of days the dial was mortared into place and mortar used to fill the gaps where needed.

The final resting place for the dial was on one of three columns of a large garden analemmatic dial.

## CASE STUDY ~ TWO GNOMON-LESS ARMILLARY DIALS SIDE BY SIDE SOMETIMES KNOWN AS AN OPEN BOOK DIAL

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ} \mathrm{N}$ |
| :--- | ---: |
| location long: | $108.2^{\circ} \mathrm{W}$ |
| magnetic declination: | $10.6^{\circ} \mathrm{E}$ |

The dial plate from 7 am to 5 pm will be a maximum of 16 inches east to west and 8 inches top to bottom (approximately)

First, a few notes on how these dial work. An armillary dial can use the edges of the armillary as the gnomon, there would be one edge for the a.m. hours, the east edge, while the west edge would be the p.m. hours. The geometric figure below might suggest that calculating the angles would be difficult. However, simplicity itself reigns!

The 15 degree lines are drawn from one edge. Where they intersect the semi-circle is where the hour lines would be. A set of lines from the semi-circle's center drawn to those intercepts shows that the angles of the hour lines from the center are actually 30 degrees. This is "The
 Half Angle Theorem".

This is explained simply since each is an isosoles triangle. Since an angle at the base is 75 degrees, being 90 degrees minus the 15 , and since the other base angle must be the same as both are on the circumference, and both go to the semi-circle's center, then they total 150 degrees, leaving 30 when subtracted from 180. Thus, 30 degree radials from the dial center are drawn, which can easily be translated into linear distances once the radius of the semicircle is known.


A nodus can be added for calendar information, both edges would then need that nodus. However, those calendar lines might look somewhat confusing as they would diverge since the nodus to dial plate point of impact increases with the time from noon. The armillary must be set at latitude. The edges must be pure, thus also aligned to latitude, and be exactly where the 6 am or 6 pm lines, or the noon lines, are.

However, such a dial can be rotated, and can be longitude corrected.

This system can be analyzed in more detail, and a 3 inch radius or 6 inch diameter will be used as a basis for that study.

What is the width of the armillary dial plate? Using geometric projections whereby we rotate a view from the top sideways, so the plan view is rotated to the profile view. For a three inch radius semi-circle, or a six inch diameter semi-circle, the maximum spread for the 23.5 degree declination would be twice 2.58 inches, or 5.2 inches. Or in more general terms, the solar declination spread is a bit under the diameter.


However, this assumes a nodus is used. It must be remembered that the gnomon-less dial has two styles, and if declination lines or calendar lines will not be used, then all that is needed is to ensure that the style's shadow is always on the dial plate. So the southernmost tip of the style can be used for the winter shadow, and the northernmost tip for the summer shadow.


In this case, we can halve the length of the style, and thus dial plate. Thus a six inch diameter semi-circle need only have a width of 2.58 inches, or in more general terms, the width of the dial plate, summer to winter, is $2.58 / 6$ inches or 0.48 times the diameter. Or in very loose terms, the dial plate is about as wide as its radius.

Alternatively, the dial plate has a breadth from 6 am to noon, or noon to 6 pm of 3.1416 (pi) times the radius. Thus a 6 inch diameter semi-circle would have a dial plate whose breadth of the hour span was 9.428 inches. And 2.58 divided by 9.428 yields 0.27 which we shall verify next.

The above figures were based on empirical CAD (computer aided design). How well does that match a trigonometric approach? The tangent of 23.5 is 0.4348 , and that times 6 is 2.61 . Obviously 2.61 is the more accurate number to use, however the use of CAD adds to an understanding of what is going on. Using 2.61 as the true figure of the minimum style's length in order to keep the shadow on the dial plate, the ratio of dial plate ( 6 am to noon) to summer-winter range is: 2.61 to 9.428 , or 0.276 , which again is close to the empirical 0.27 .

## Solar ray examples and notes on hour line accuracy



To the left are shown 1 degree lines from the opposite style which is to the right, and 2 degree lines from the semi-circle center which is from the top. The sun's lines intersect perfectly, they are close to perpendicular with the dial plate, so accuracy is highest.

To the right are shown 1 degree lines from the style which is to the right, and 2 degree lines from the semicircle center which is from the top. The sun's lines intersect perfectly, they are about 45 degrees with the dial plate, so accuracy is still good.


To the left are shown 1 degree lines from the style which is to the top right, and 2 degree lines from the semi-circle center which is from the top left. The sun's lines intersect perfectly, they are close to flat with the dial plate, so accuracy is lowest.

The accuracy, or rather sensitivity, decreases around noon. For the perfectionist, a small polar dial can be added for an hour either side of noon.


On the left the complete picture of 1 degree solar rays and their intersection with the 2 degree radials from the semicircles center can be seen, and the three boxes correspond to the three expanded pictures above.

All these pictorials were done with TurboCAD.

The above pages lay the foundation for the dual gnomon-less armillary to be placed on a column, which will be described in the following pages.

The dial will be two gnomon-less armillary semi-circles, each rotated so one shows morning hours, the other displays afternoon information. The dial plate will be two semi circles, back to back, canted at 30 degrees, and will be longitude corrected.


With a gnomon-less armillary, the normal 15 degrees per hour is still 15 degrees from the style, but becomes 30 degrees per hour from the center, as shown to the left, and as discussed on the preceding pages.

In this case, two such armillary dials will be back to back, canted at 30 degrees.

The 30 degrees was selected as it was functional as well as pleasing to the eye.

As a result they can show time from 8 in the morning until 4 in the afternoon local apparent time or L.A.T., although longitude correction will modify this by almost 13 minutes at this dial location.



To the left, the radials have been rotated 3.2 degrees, being the dial location (108.2) minus the legal meridian longitude (105). This works because the complete dial, including the gnomon, is rotated around the style, i.e., around the polar axis, see Atkinson's Theorem in chapter 15.

The process used for the two armillary semi-circles matches that of the single armillary. Tracings are used and each tracing will have its own 15 degree radials, rotated for the longitude correction.


The two semicircles are tacked together with an epoxy and allowed to set overnight. Later the upper space between the curved dial plates as well as the lower space adjacent to the mounting paver will be filled with mortar.

When cured, the pair of dial plates is rotated and placed on a sheet of paper which has a horizontal line, the bottoms of the dial plates rest on this horizontal line.

Then the outline is traced. And perpendicular to that horizontal line are drawn lines up to the styles. From those styles are drawn the 15 degree hour angles, adjusted for the longitude correction.


The dial plate is returned to the outline drawing, see below, and the hour lines as well as half hour lines marked where the paper meets the edge of the armillary dial plate.


This methods works well on dial plates that are uneven, and these semi-circles cut from a flu liner are not of a constant radius.

A right triangle tool was used with the surface on the style and an edge aligned on the hour and half hour index mark. And long lines drawn for the hour, half length centered lines drawn for the half hour.

Those lines would then need engraving with an engraver.

Two small "T"s were made, one had the half and quarter hour detents, these were small nails in the $1 / 4$ inch wood. A second template was needed for the hours. The dial plate was engraved with a Dremel engraver.

Again, a face mask was used, as were glasses as a precaution.

The two dial plates were then given a small amount of epoxy to affix them to the concrete paver. When set on their
 supporting column, then mortar would be added.

Building the column this dial would rest on was standard procedure. Except that after the 2 inch by 1 foot square, base paver was set, the author was distracted by another work project and Alfonzo the contractor who was working on re-stuccoing the barn, decided that the base paver should align with the fence, and not with true north. Positive intentions, but not discovered when the author returned to complete the column.


So the column was built up, but part way up it was time for a cup of tea. After the cup of tea, work resumed however the author mistakenly didn't alternate one layer of bricks.

So when it was finished, the final paver used to rest the dial plate was no longer correctly aligned, and one set of bricks looked out of place. The resolution was to align the final dial plate by offsetting it from the paver it rested on.

To the left the realigned dial plate can be seen, and below the dial rests in a serene part of the garden. It was tested and found to be within a couple of minutes.


## CHAPTER TEN

## The polar dial in detail

## AN OVERVIEW OF THE THREE TECHNIQUES FOR A POLAR DIAL.

We have discussed the armillary and equatorial dials. This chapter revisits the polar dial. The polar dial to the right is engraved on a ceramic tile and shows about 9:00 am standard time as it is longitude corrected, but not mean time as there is no EOT correction. Designed by the author for Somerset, England.

Hour angle dials use gnomons whose style parallels the Earth's rotational axis. For polar dials, the dial center is actually an infinite distance away! Usually the
 style is an indicator of the time and the nodus an indicator of the date. In early times, the date was often a major function of some dials.

Whereas the armillary and equatorial dials have hour lines evenly spaced every 15 degrees, the polar dial has line spacing based on the trigonometric tangent of the hour difference from noon.

Dials are designed for a specific latitude or distance from the equator, and the hour lines should be adjusted for the longitude of the dial's location, or a fixed adjustment must be mentally made when reading the time. Like the armillary and equatorial dials, the polar dial works at any latitude as long as the dial plate and style are set at latitude. The sun runs slow or fast and a correction must be mentally made to account for it, the correction being the equation of time, or EOT. Most dials have no longitude correction built into the hour lines, but instead that time difference is folded into a tailored equation of time for that longitude.


There are several methods of designing sun dials. And there are many types of sundials as well. Dials may be designed for horizontal use, or vertical facing due the equator, east, or west, and the polar region. And dials can be designed for walls that don't quite face true south, east, west, or north, in other words dials that decline. The word "decline" has several different meanings, three to be precise. Magnetic (variation), sun angle from the equator (varies plus or minus 23.44 degrees over the year) which should not be confused with the sun's altitude, and, finally a wall's angular offset from true north or south is also called a declination. And dials can be designed that lean back or forward, dials that incline or recline. This chapter focuses on the polar dial where the dial plate and the gnomon's style parallel the Earth's polar axis.

There are three general methods of designing a dial once the type has been selected.
One method is to build the gnomon and do the rest by empirical methods, i.e. "practical geometry". The picture above is a "trigon" which is a custom built protractor. The word trigon comes from the same base as the word trigonometry. This trigon was used to get points on the equinox line for the hour lines for a vertical almost south facing dial discussed in a later chapter.

A truly empirical sundial would have the gnomon placed in position, and the hour lines built from hour points, longitude correction would thus be implied. This was discussed earlier. Simple math is done with an accurate clock and the equation of time, abbreviated to EOT.

To build a sundial using the sun's shadow to mark the hour points, an accurate standard time clock is used. This could be a cell phone with the time set by the provider, however often seconds are omitted. A watch set by the computer whose clock is corrected by the internet is a practical means of getting the accurate time. And cheap clocks which reset themselves daily based on a radio atomic clock may be had at department stores.

The formula for time is:-
dial indicated time + EOT
=> standard time
thus we can deduce:-

$$
\begin{aligned}
& \text { standard time - EOT } \\
& =>\text { dial indicated time }
\end{aligned}
$$

Notice that the longitude difference between the dial's longitude and the legal meridian is not computed. A byproduct of using this technique to calibrate a dial is that the longitude correction is implicit, this is because you are using the standard time, and the dial location's L.A.T., which is why the difference in longitude between the dial and the legal standard meridian was not computed.

To get a 1200 hour point therefore on November 10th when the EOT is $-16: 00$ we would:-

$$
\begin{array}{llll}
\text { clock time -EOT } & \text { has to be } & =>1200 & \text { thus } \\
\text { clock time }-(-16 & \text { has to be } & \Rightarrow 1200 & \text { thus } \\
\text { clock time }+16 & \text { has to be } & =>1200 & (-- \text { is a plus }) \\
\text { clock time } & \text { is thus } & =>1200-16 & \text { => } 1144
\end{array}
$$

So, with an EOT of - 16 minutes, at a standard clock time of 1144 pm a dial should show 1200 as the sun is fast by 16 minutes. In other words, when the shadow is on the "12" and the EOT is minus 16 minutes, the time is 1144 . So the hour mark is made in this case 16 minutes early on the clock time. If the EOT was positive, the opposite would be the case.

TO MARK A SUNDIAL SO IT WILL WORK ON ANY DAY USING THE EOT, PUT THE HOUR MARK WHERE THE SHADOW IS WHEN THE CLOCK READS THE STANDARD TIME ADJUSTED BY ADDING THE EOT

Some more examples:-

| hour point | + | EOT | $\boldsymbol{\rightarrow}$ | legal time to make the observation |
| :--- | :--- | :--- | :--- | :--- |
| $12: 00$ | + | -8 | $\boldsymbol{\rightarrow}$ | $11: 52 \quad($ eg December 8$)$ |
| $12: 00$ | + | +13 | $\boldsymbol{\rightarrow}$ | $12: 13 \quad($ eg February 15) |

It is very well worth the time to draw some hour lines, some shadows, and then play around with the numbers. Before doing this with a real dial under construction, draft a table of time that you will mark the hour points. It is easy to make a mistake otherwise.

The second method is to use geometric methods. The trigon above was used for empirical purposes, but could have been made more accurate and anyway, we can use its concept as the basis for geometric methods.

Geometric methods convert the three dimensional dial into a two dimensional figure and then use that figure to generate appropriate angles and lines of appropriate length. The polar dial for example took a real equatorial dial and rotated the nodus of the style or gnomon flat so that drawings could be made. A picture is worth a thousand words, so look below.

geometrically produced distance from the noon line to the hour line

The third method is to use trigonometric methods.
The trigonometric method uses the geometric model so that trigonometric functions can be used to create formulas that can then be used on the dial plate.


11am and 1pm = 15 degrees off center, tan (15) $=.2679$
10 am and $2 \mathrm{pm}=30$ degrees off center, $\tan (30)=.5774$ 9 am and $3 \mathrm{pm}=45$ degrees off center, tan $(45)=1.000$

$$
\begin{aligned}
\mathrm{nh}= & \text { dist to hour line } \\
& \text { from sub-style } \\
\mathrm{ns}= & \text { style linear height } \\
\mathrm{nh}= & \text { tan (time) * ns } \\
\mathrm{nh}= & 0.2679 \text { style height } \\
\mathrm{nh}= & 0.5774 \text { style height } \\
\mathrm{nh}= & \mathrm{sh}
\end{aligned}
$$

and of course the morning hours would be west of the style, with afternoon hours east.
All methods can use longitude correction. The geometric method simply rotates the protractor or adds or subtracts the adjustment to the hour angles before marking the point. The trigonometric method adds or subtracts the angle similarly before the tangent is calculated. The dial plate cannot be slid left or right however as hour line ratios would no longer be correct.

The equation of time is factored in mentally, otherwise the dial gets more complicated.
Empirical, geometric, and trigonometric methods can also be used for calendar information.

## THE EMPIRICAL METHOD FOR A POLAR DIAL USING A TRIGON

First build the dial plate and the gnomon with its style, there are no markings at this point. Then attach a protractor to the top of the style and decide what the longitude correction is to be, if any. For Phoenix AZ at 112 degrees using the 105 degree legal time meridian, the protractor must be turned 7 degrees. From personal experience, write down which way, or write down all the new angles to be used. Fail to do that and you will screw some of them up. Alternatively, no correction is made, and instead it is folded into the EOT table.

Then attach a cotton thread from the tip of the style or the center of the protractor and holding it over the appropriate hour angle on that protractor, see where the cotton line touches the dial plate. And mark that point. And repeat it for the other hour lines. Laser pointers may also be used.


For the angle, you use the time in hours multiplied by 15 and then add or subtract the number of degrees by which the dial's longitude differs from the legal time meridian assuming you desire built-in longitude correction. In the case of Phoenix AZ, the hours must be shifted to the west, or left in the above picture. That way noon will happen earlier than when the sun is actually overhead which is the correct adjustment for longitude when the dial is west of the legal meridian.

Personal experience shows that when making a dial it is easy to forget which way to correct a protractor, or if using trigonometry, it is easy to forget whether you are adding or subtracting the longitude correction.

When making a sundial empirically, the hour lines, and declination lines if they are used, should be penciled in, and then you go outside and wait for the sun with the sun dial in situ to fine tune the results.

Then use the equation of time and wait. If the equation of time is -15 (as in October) then that means that at 2 pm you would subtract 15 minutes. So, at 15 minutes to 2 the styles shadow should be on the 2 o'clock hour line. You can double check this by using reverse logic.

Again, from experience, rushing out every hour to do this calibration has the ability to cause confusion. So write it down, and set an hourly alarm on your PDA (personal digital assistant), and then you will not make as many mistakes.

As always, the polar dial under construction must be in place, and tilted to the latitude, both dial plate and style. And the style must point to true north and south.

Longitude correct can be omitted from the above, and instead it can be folded into an EOT table which then allows one mental arithmetic operation to account for both longitude as well as EOT. This is a wise approach especially if the dial may be relocated to another location at a later date.

## THE GEOMETRIC METHOD FOR A POLAR DIAL

First establish how far above the dial plate the style is (style linear height). Then on the 12 o'clock line draw a protractor of the same radius as the style's distance from the plate. Then proceed around the protractor at 15 degree increments and project those lines to the dial as shown.

No longitude correction employed in this example


Where the 15 degree line intercepts the dial plate, there is the 1 pm line, or if on the other side, the 11am line. The three o'clock pm (or 9 o'clock am) line is the 45 degree angle ( 3 times 15 degrees per hour, and its distance from noon is always equal to the style's distance from the dial plate which is a clue to the trigonometric method, the tangent of 45 degrees is 1.

Latitude is managed by the dial plate and the style being tilted so they make an angle with the planet's surface equal to the latitude.

Longitude is corrected so that indicated solar time, properly called local apparent time, or L.A.T., reflects the solar time at the standard time longitude or meridian. That is done by rotating the protractor by an amount equal to the difference between the sun dial's location and the reference meridian, or more often, a fixed number may be applied to the indicated time. The dial plate cannot simply be slid or shifted however, because the hour line ratios would no longer be correct.

For example in Phoenix AZ the longitude is 7 degrees west of the standard.

| Phoenix | $33.5^{\circ} \mathrm{N} 112.0^{\circ} \mathrm{W}$ | mag var |
| :--- | :--- | :--- |
| PHX | mst is at $105^{\circ} \mathrm{PHX}$ is $7^{\circ} 0^{\prime}$ from mst |  |
|  | i.e. 28 minutes from mst |  |

A Phoenix dial's protractor would be rotated 7 degrees if the longitude correction was to be built in. Unlike the equatorial and armillary dials, for a polar dial one cannot slide the dial plate a certain amount, because the line ratios would no longer be correct, instead the protractor employed is rotated. This is shown below, and notice that the 12 o'clock noon line is offset, in this case to the left or west.


So 12 o'clock will be indicated earlier than it would normally be, because the legal 105 degree meridian gets to 12 o'clock before it does at the 108 degree meridian.

As an aside, this exact same geometric method can be used for a wall facing exactly true west or exactly true east and is discussed in the next chapter.

The time is indicated, but how about the calendar? Except for the equinox, the calendar lines will be curved, in fact they will be hyperbolic. On an equatorial dial they are circles, on an armillary dial they are straight lines, but because the dial plate is flattened out when moving to a polar dial, geometry comes into play. These calendar or declination lines can be used to indicate the date, or even the number of hours in the day if so desired, and some have used them with the hour lines to show the hours until sunset, those are called Italian hours!

Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25 which enable legal time reading.


The geometric method for constructing calendar or declination lines is to use a similar idea with the protractor that was used for the hour lines.


The equinox line is drawn and then the style line is drawn upwards whose length equals the linear distance of the style from the dial plate. Then [1] a line is drawn from the selected hour line base at the equinox line for whom we wish a calendar point (in this case the three o'clock line) to the style top, and rotated down [2] to the equinox line, and from thence a line drawn to the selected hour line [3] at an angle that matches the declination of the sun for the date in question, this would be 23.5 degrees for the solstice. The winter solstice would be above and the summer solstice would be below. It is an equal distance below because the dial plate is aligned with the Earth's polar axis. For vertical and horizontal dials that is not the case. After this process is repeated for all the hours, the dots are joined and a hyperbola [4] is drawn for the date.

For the date to be shown on calendar or declination lines, the style needs a knob or a cut out or other definite point (nodus) so a shadow point can be seen, otherwise all you would see would be a straight hour line! Remember, this is the method for polar dials, other dials use other methods.

## TRIGONOMETRIC METHOD FOR A POLAR DIAL

HOUR LINES:


The equatorial line is along "ah", and the style is "t", where "at" equals the linear height of the style above the dial plate.

The angle "ath" is the time in hours from noon times 15 (there are 15 degrees in an hour).

$$
\begin{equation*}
\text { we see that:- } \tan (a t h)=a h / a t, \quad \text { thus } a h=a t * \tan (a t h) \tag{1}
\end{equation*}
$$

in other words the distance along the equatorial or equinox line to the hour line is equal to the style height times the tangent of the time.

And the angle can be adjusted for longitude by adding or subtracting the difference between the dial's location and the standard time meridian in degrees before calculating the tangent. Or it can be omitted and then deferred to an EOT table specific to the dial's longitude.

EQUINOX, OR ANY OTHER CALENDAR DECLINATION LINES (such as length of day):
Note: Calendar or declination lines and curves are discussed in chapter 23, with other lines discussed in chapter 24, and analemmas in chapter 25.

we see that:- $\quad \tan (\mathrm{hrd})=\mathrm{dh} / \mathrm{rh}=\mathrm{dh} /$ th, $\quad$ thus $\mathrm{dh}=\mathrm{th} * \tan$ (declination $)$
and:- $\quad \cos ($ ath $)=$ ta $/$ th thus $t h=t a / \cos ($ ath $)$
so given: $\quad \mathrm{dh}=\mathrm{th} * \tan$ (declination )
then:- $\quad \mathrm{dh}=\mathrm{ta} * \tan ($ declination $) / \cos ($ time $)$
in other words the distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines.

POLAR DIAL: Geometric:

Longitude correction is made by rotating the protractor, never sliding the plate


Each 15 degree line intercept to the dial plate marks an hour from solar noon. At 45 degrees the distance from the noon hour line is always equal to the style's distance from the dial plate.

Latitude is managed by the dial plate and the style being tilted so they make an angle with the planet's surface equal to the latitude.

Longitude is corrected so that indicated solar time, L.A.T., reflects the solar time at the standard time longitude or meridian. That is done by rotating the above protractor by an amount equal to the difference between the sun dial's location and the reference meridian. Alternatively, the longitude correction can be folded into an EOT table which is still only one mental arithmetic operation, and enables the dial to be portable.

Calendar or declination lines are constructed thusly:


POLAR DIAL: Trigonometric:
Hour line linear distance from

$$
\text { the noon line or sub style } \quad=\text { gnomon style linear height * tan (15 * hour) }
$$

Calendar or declination lines are drawn by connecting points from declination points on each hour line. Each such point is measured from the equinox line as:-

Linear distance along an hour
line to a declination point = linear style height * tan (declination) / cos (15 * hour)
Note: APPENDIX 3 has tables for the polar dial hour line and calendar line distances.
Note: A paper cutout polar dial is included in APPENDIX 9 which works for any latitude, however, it has Italian hours (sunset data) which were designed for and thus only valid for a specific latitude of 32.

## ACTUAL DESIGN NOTES

POLAR DIAL: Trigonometric:
A polar dial with calendar curves was designed for latitude 51 degrees north. The hour lines needed due to building shading were 10 am to 4 pm . The dial was to be 12 inches wide.


First, 10 am to noon consumes two morning hours, and at 15 degrees each, that is 30 degrees, and similarly from noon to 4 pm consumes 4 hours, or 60 degrees.

The total width was style height * $(\tan (30)+\tan (60))$, or 2.3 times style height.
Next, 12 inches must equal style linear height * 2.3 so style linear height is $12 / 2.3$ or 5.1 inches.
Geometrically, we can verify this, in this case CAD (computer aided design) was used.


Hour line linear distance from
the noon line or sub style $\quad=$ gnomon style linear height * tan (15 * hour)
Calendar or declination lines were drawn by connecting points from declination points on each hour line using the formula:-

Linear distance along an hour line to a declination point = linear style height * tan (declination) / cos (15 * hour)

The latitude was only relevant in setting the dial in place, it does not affect hour or calendar lines for this dial type. It would affect optional Italian hour lines however, discussed elsewhere.

Note: APPENDIX 3 has tables for the polar dial hour line and calendar line distances.
Note: A paper cutout polar dial is included in APPENDIX 9 which works for any latitude, however, it has Italian hours (sunset data) which were designed for and thus only valid for a specific latitude of 32.

## CASE STUDY ~ A POLAR DIAL WITH CALENDAR CURVES AND ITALIAN HOUR LINES

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ} \mathrm{N}$ | The dial plate from 7 am to 5 pm will be |
| :--- | ---: | :--- |
| location long: | $108.2^{\circ} \mathrm{W}$ | a maximum of 16 inches east to west |
| magnetic declination: | $10.6^{\circ}$ | E | | and 8 inches top to bottom (approximately) |
| :--- |

Hour lines will be shown and also Italian hour lines. Italian hour lines indicate the time since the last sunset, however common practice is to use them to show the time until the upcoming sunset. Italian hour lines, and their cousin the Babylonian hour lines showing time since sunrise, are latitude specific. In other words the trick of tilting a dial plate will correct the time of day hours for a latitude shift but such is not the case for Italian and Babylonian hour lines that rely on the geometry of the Earth's curvature which makes them latitude specific.

A spreadsheet was constructed as below, using the trigonometric formulae, however this data is also available for a gnomon relative linear height of 1 in the appendices.

POLAR DIAL - hour line distances, and below that the calendar points on the hour lines

| hour | degrees | radians | tan | GNOMON HT ~ HOUR LINE DISTANCE |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | 1 | 1.5 | 1.75 | 2 | 2.25 |
| 1 | 15 | 0.2618 | 0.2679 | 0.27 | 0.40 | 0.47 | 0.54 | 0.60 |
| 2 | 30 | 0.5236 | 0.5774 | 0.58 | 0.87 | 1.01 | 1.15 | 1.30 |
| 3 | 45 | 0.7854 | 1.0000 | 1.00 | 1.50 | 1.75 | 2.00 | 2.25 |
| 4 | 60 | 1.0472 | 1.7321 | 1.73 | 2.60 | 3.03 | 3.46 | 3.90 |
| 5 | 75 | 1.3090 | 3.7321 | 3.73 | 5.60 | 6.53 | 7.46 | 8.40 |

hour line distance $=$ style linear height $* \tan (15 * t i m e)$

| calendar data: | Gnomon height | 1.75 | Hours from noon |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | decl | radians | 0 | 1 | 2 | 3 | 4 | 5 |
| June solstice | 23.50 | 0.4102 | 0.76 | 0.79 | 0.88 | 1.08 | 1.52 | 2.94 |
| May and July | 20.00 | 0.3491 | 0.64 | 0.66 | 0.74 | 0.90 | 1.27 | 2.46 |
| August and |  |  |  |  |  |  |  |  |
| April | 12.00 | 0.2094 | 0.37 | 0.39 | 0.43 | 0.53 | 0.74 | 1.44 |
| September and March equinox | $\times \quad 0.00$ | 0.0000 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| October and February | -12.00 | -0.2094 | -0.37 | -0.39 | -0.43 | -0.53 | -0.74 | -1.44 |
| November and January | -20.00 | -0.3491 | -0.64 | -0.66 | -0.74 | -0.90 | -1.27 | -2.46 |
| December solstice | -23.50 | -0.4102 | -0.76 | -0.79 | -0.88 | -1.08 | -1.52 | -2.94 |

calendar height on hour line $=$ style linear height * tan (declination) / cos (15*time )


The spreadsheet shows that a 1.75 inch gnomon linear height would generate a dial plate of 6.53 length on either side of noon, or 13.1 inches which fits easily in the 16 inch limitation. Additionally, the largest calendar line spread would be twice 2.94 inches or 5.8 inches which also fits within the 8 inch limitation.

However, the above does not consider longitude corrections, which is about 13 minutes. Those 13 minutes came from 4 minutes per degree of longitudinal distance from the dial (longitude 108.2 west) and the legal time meridian's longitude of 105 . The distance is thus 3.2 degrees, placing the actual correction at 12.8 minutes or 12 minutes 48 seconds.

The spreadsheet below shows the new hour distances.

| GNOMON HT ~ HOUR LINE DIST |  |  |  | $\frac{1.75}{\text { AM }}$ | long corr |  | $\begin{array}{r} 12.8 \\ \hline \text { radians } \end{array}$ | long corr dec hrs |  | 0.21 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hour | degrees | radians | tan |  | hour | degrees |  | tan | PM |  |
| 1.21 | 18.2 | 0.3176 | 0.3288 | 0.58 | 0.79 | 11.8 | 0.2059 | 0.2089 | 0.37 |  |
| 2.21 | 33.2 | 0.5794 | 0.6544 | 1.15 | 1.79 | 26.8 | 0.4677 | 0.5051 | 0.88 |  |
| 3.21 | 48.2 | 0.8412 | 1.1184 | 1.96 | 2.79 | 41.8 | 0.7295 | 0.8941 | 1.56 |  |
| 4.21 | 63.2 | 1.1030 | 1.9797 | 3.46 | 3.79 | 56.8 | 0.9913 | 1.5282 | 2.67 |  |
| 5.21 | 78.2 | 1.3648 | 4.7867 | 8.38 | 4.79 | 71.8 | 1.2531 | 3.0415 | 5.32 |  |

Similarly the calendar line distances on each hour line would be different, and the spreadsheet could accommodate that also.

However, the geometric method is worth addressing and will be used in this case study, with the spreadsheet as a cross check. The geometric method will be implemented using CAD as a drafting tool.


The geometric methods which used CAD as a drafting tool, agreed with the spreadsheet on hour distances. A benefit of CAD is that the final drawing may be printed and used as a template for the actual dial.


To the right is a close-up of the above, except that lines have been drawn for 12 and 20 degrees, and for completeness the 0 degree equinox has a circle on it, albeit somewhat superfluous.

The CAD system used was TurboCAD whose deluxe version is available for around $\$ 100$, and also provides 3d modeling. This system allows lines to be grouped, and this facilitates the rest of the calendar lines because once those declination lines have been drawn, their base can be moved and calendar points drawn quickly, it saves re measuring those calendar based declination angles.

An hour line is selected, in this case 1 pm legal time.

A line is drawn from the base to the top of the style.

That line is rotated, an arc was used, down to the dial plate.

From where that arc intercepted the dial plate, a line was drawn at the declination, 23.5 in this case, and where that line intersected the original hour line, there is the point for that calendar line for that declination.



The results of shifting the group of declination angles is shown to the left.


The morning hour calendar lines have been added.

An arc is shown for 0800 standard legal time, together with the 0800 three calendar declination angle lines.


The calendar dots are drawn with a curve drawing function, in this case the Bezier function. And the entire upper half was selected and mirror copied around the equinox line.

Italian lines for the hours until sunset need to be added. Italian lines are actually hours since the last sunset, however in the real world they are used to show hours until the next sunset. This is latitude 32.75, and either the formulae may be used or the tables in the appendix.

Hour angle of rising/setting sun: hsr = arccos( tan(lat) * tan(decl) ) from noon
where
Sun Declination:

$$
\begin{aligned}
\text { decl }= & \text { degrees }\left(0.006918-0.399912^{*} \cos (\mathrm{da})+0.070257^{*} \sin (\mathrm{da})\right. \\
& -0.006758^{*} \cos \left(2^{\star} \mathrm{da}\right)+0.000907^{\star} \sin \left(2^{\star} \mathrm{da}\right) \\
& -0.002697^{*} \cos \left(3^{\star} d a\right)+0.001480^{\star} \sin \left(3^{\star} d a\right)
\end{aligned}
$$

where da = Day angle:
$\mathrm{da}=2$ * pi * $(\mathrm{j}-1) / 365$ (in radians, is an intermediate figure)
where J is the day of the year
J=1 on 1 January, J=365 on 31 December. February being 28 days.

| Jan | Feb | Mar | Apr | May | Jun |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 31 | 59 | 90 | 120 | 151 |
| Jly | Aug | Sep | Oct | Nov | Dec |
| 181 | 212 | 243 | 273 | 304 | 334 |

alternative formula:
decl $=$ DEGREES $=\left(23.45^{*} \sin (\operatorname{radians(0.9678(jd-80))))}\right.$ source: Claude Hartman

Or the spreadsheet may be used.


Either way, sunset is local apparent time of:-
1655 winter solstice standard time
1906 summer solstice time
Sunset times are local apparent time. With a dial that is not longitude corrected, i.e. showing L.A.T., then there is no problem. However if the hour lines are longitude corrected then those hour lines cannot be used as the foal points for the Italian lines unless the sunset times are also longitude corrected. Since the hour lines were longitude corrected, the sunset times for the dial's location will be later as the dial is west of the legal time meridian.

The correction is 12.8 , almost 13 minutes. So the legal time for sunset will need adjusting:-

$$
\begin{array}{lll}
1708 & \text { winter solstice standard time } & \text { (being } 1655+13) \\
1919 & \text { summer solstice time } & \text { (being } 1900+13)
\end{array}
$$

The sunset time at the equinox is $1800(6 \mathrm{pm})$ local time, but that also needs adjusting, so it is now

1813 March or September equinox (being $1800+13$ )
A line is drawn from 1708 winter, through 1813 on the equinox to 1919 on the summer solstice line, this is the sunset time. That sunset line will not fit on this dial plate. So now it is backed off one hour to:-

| 1608 | 1713 | 1819 | for the 1 hour to sunset Italian line |
| :--- | :--- | :--- | :--- |
| 1508 | 1613 | 1719 | for the 2 hour to sunset Italian line |
| 1408 | 1513 | 1619 | for the 3 hour to sunset Italian line |
| 1308 | 1413 | 1519 | for the 4 hour to sunset Italian line |
| 1208 | 1313 | 1419 | for the 5 hour to sunset Italian line |



The CAD version is then printed out and the dial plate and gnomon constructed.

A key point about Italian lines that can never be repeated too many times is that they are latitude dependent. While a polar dial, or any hour angle based dial, can be tilted to correct for a dial's actual latitude compared to its design latitude, this is not true for Italian or Babylonian hour lines. They work for one latitude only.

These Italian lines were adjusted to accommodate the longitude. They result in exactly the same placement were no longitude correction applied to both them and the dial plate hour lines. This is because they are indicating the time right now, here, until the next sunset


The dial plate was constructed. This dial plate was to be copper wire attached to a paver. The ends were of small copper pipe, each of those two end pipes had a 10 gauge copper wire attached to it, and these were set in holes drilled in the concrete paver.

However, before that the hyperbolic calendar lines and the straight equinox line were cut from copper wire and shaped. Similarly with the hour lines, and the Italian lines.

If the calendar lines are closest to the concrete paver then the hour lines would rest above them, and to keep proportions, the gnomon linear height would be from the top of those hour line wires. The shadow of the gnomon and the hour lines would be read on the concrete paver. Alternatively, the hour lines could be closest to the concrete paver with the calendar lines above them, in which case the gnomon linear height would be from those lines just above the concrete paver.


The dial was secured by four copper rods to holes in the concrete paver.

The final dial rested on the 12 o'clock pillar of a large garden analemmatic dial.

Foot note: because the tangent of 45 is 1 , and because the 3 pm or 9 am line is 45 degrees, for non longitude corrected dials one of the rules of thumb is that the gnomon linear height is the same as the sub-style to 3 pm or 9 am distance. If the dial is longitude corrected, as this one is, then the 3 pm and 9 am rule of thumb does not apply. So, printing the gnomon on the CAD layout is helpful here, especially if the layout is rescaled later.

To the right is a similarly constructed polar dial except it is suspended in mid air. The time is read by the shadows of both the gnomon as well as the hour lines on a wall. The sun face was a commonly available clay figure, and its borders were pieces of ceramic floor tile. This dial shows about 3:00 pm standard time as it is longitude corrected, but not mean time as there is no EOT correction. Designed by the author for central Phoenix, Arizona.


## CHAPTER ELEVEN

## Meridian dials are like polar dials in many ways DIALS FACING TRUE EAST OR WEST

While a true east or true west dial is less common than a horizontal sundial, it is the next step in the evolution of understanding the art and science of building and designing sundials, and they are found on many cube dials.

A true west facing wall shows the time between noon and 6pm or thereabouts. Solar noon can never show because the gnomon's shadow would parallel the wall. And times after 6 pm may happen in the summer when the sun moves across the east-west line. The point is that you should establish what hours you want to display first and then decide on the dial plate's size, and from that you can establish how far the gnomon's style must be from the dial plate. The methods are the same as the polar dial, there is no difference, except that the equatorial line slants at an angle equal to the co-latitude ( 90 degrees minus the latitude) of the dial location, the hour lines also now slant because they are perpendicular to the equinox line. And calendar curves slant also. Further, a true east facing wall is the same as a true west wall, the slope looks the opposite to an observer but exactly the same as a west dial if it were painted on glass. A meridian dial is formed when a polar dial is rotated around its style by 90 degrees.

The design for the dial's latitude is to slope it. Like the polar dial the hour lines themselves are not latitude dependant. Longitude correction is the same as for a polar dial. Empirical, geometric, or trigonometric methods may be used. The gnomon is angled at the latitude, and a protractor with a plumb-line shown in the picture to the right or an inclinometer that you have calibrated may be used to ensure correct latitude alignment.


To the right is a true west vertical dial, designed for latitude 32 degrees north and a longitude of 108 degrees west. Direct east and direct west facing dials are universal, they can be moved to other latitudes and merely tilted to adjust for the change. This is because the hour lines are not a function of latitude. Of course longitude corrections if used will complicate such a movement.
 nodus, and thus indicate solstice or declination lines.

-



Nodus and one end of the style's projection.

Other end of the style to sub-style projection. A protractor is drawn centered here for a radius of the style's linear height.

In the above picture, the style and the nodus were projected to the dial plate as can be seen by the line with little circles at each end. This projected style line then has a line drawn perpendicular to it from the nodus projection and proceeding down and to the left, this is the equatorial line, and depicts the equinoxes. The nodus shadow is a bit on the winter side of the equinox.

Geometric methods were used for the hour lines, and you will note that the projection of the style is about a quarter of an inch above the 6pm (winter) 7 pm (summer) hour line of this west facing northern hemisphere dial. In fact it represents about 12 minutes being the solar time difference between the dial's longitude and that of the legal standard time meridian. This is the result of the longitude correction. Like the polar dial, the hour lines themselves cannot just be shifted.

This dial has 15 -minute or quarter-hour lines shown between the hour lines, and not only the equinox and the solstice lines are shown but so are the other dates, so all months are represented. Of course all but December and June share declination lines. So July shares a line, actually a hyperbola, with May, and so on.


Instead of astronomical declination lines, some people like to have declination lines for the number of hours in the day, and chapter 24 covers this.

## GEOMETRIC METHOD FOR TRUE EAST OR WEST MERIDIAN DIALS

First establish how far from the dial plate the style will be, or figure out how big the dial plate must be, and work backwards to establish style's linear height. On the 6 o'clock line where the substyle is, draw a protractor of the same radius as the style's linear distance from the plate. Then proceed around the protractor at 15 degree increments and project those lines to the dial as shown.

No longitude correction


If there is no longitude correction built in, where the 15 degree line intercepts the dial plate, there is the 5 pm line (west facing), or the 7am line if east facing. Then the three o'clock pm (or 9 o'clock am) line is the 45 degree angle ( 3 times 15 degrees per hour), and its distance from 6am or 6 pm is equal to the style's linear distance from the dial plate, a clue to the trigonometric method since the tangent of 45 degrees is 1.

Latitude is managed by the style and the dial plate being rotated so they make an angle equal to the latitude and co-latitude respectively.

Longitude is corrected so that indicated solar time, called local apparent time or L.A.T., reflects the legal time at the standard time longitude or meridian (if the EOT were applied). This is done by rotating the above protractor an amount equal to the difference between the sun dial's location and the reference meridian, so for example in Phoenix AZ the longitude is 7 degrees west of the standard. Alternatively a fixed mental correction may be applied, or the longitude correction may be folded into the dial's EOT chart or table.

| Phoenix | $33.5^{\circ} \mathrm{N} 112.0^{\circ} \mathrm{W}$ | mag var $\quad 11.8^{\circ} \mathrm{E}$ |
| :--- | :--- | :--- |
| PHX | mst is at $105^{\circ}$ | PHX is $7^{\circ} 0^{\prime}$ from mst |
|  | i.e. | 28 minutes from mst |

To adjust the dial for Phoenix longitude, the dial must be rotated 7 degrees. One cannot slide the dial plate a certain amount, because the hour line spacing ratios would no longer be accurate. This is shown below, and notice that the 6am, 6 pm line is offset so that 12 o'clock will be shown earlier than it should be ...

With longitude correction

...because the legal 105 degree meridian gets to 12 o'clock before it does at the 108 degree meridian.

By the way, this exact same geometric method is used for a wall facing exactly true west (occidental) as well as exactly true east (oriental).

DECLINATION LINES: Adding calendar lines is simple. The calendar lines will be curved hyperbolae except for the equinox line which is straight. On an equatorial dial they were circles, on an armillary they were straight, but because the dial plate is now flat, geometry comes into play.


The geometric method for constructing them is to use a similar idea with the protractor that was used for the hour lines and again, is the exact same method used for the polar dial.


The equinox line is drawn and then the style is drawn upwards whose length equals the distance of the style from the dial plate, the style linear distance. Then [1] a line is drawn to the style from the selected hour line (in this case the three o'clock line) and rotated down [2] to the equinox line, and from thence a line drawn to the selected hour line [3] at an angle that matches the declination for the sun for the date in question, this would be 23.5 degrees for the solstice. The winter solstice would be above and the summer solstice would be below. It is an equal distance below because the dial plate is aligned with the Earth's polar axis perpendicular to the equator. After this process is repeated for all the hours, and the dots are joined and then a hyperbola [4] is drawn.

For the date to be shown, the style needs a knob or a cut out notch (nodus) so the location can be seen, otherwise all you would see would be a straight line. In the photo, the gnomon is cut short so the tip clearly makes the time and date visible.

## TRIGONOMETRIC METHOD FOR TRUE EAST OR WEST MERIDIAN DIALS (this is the same method as for a polar dial)

HOUR LINES: $\quad$ Whereas the polar dial had noon at its sub


The equatorial line is along "ah", and the style is "t", where "at" equals the linear height of the style above the dial plate. The angle "ath" is equal to the time in hours from 6 (am or pm) times 15 (there are 15 degrees in an hour). Longitude corrections, if any, are applied here.

$$
\begin{array}{ll}
\text { we see that:- } & \tan (a t h)=a h / a t \\
\text { therefore:- } & a h=a t * \tan (a t h) \tag{1}
\end{array}
$$

The distance along the equatorial or equinox line to the hour line is equal to the style linear height times the tangent of the time difference from 6 o'clock. The angle can be adjusted for longitude by simply adding or subtracting the difference between the dial's location and the standard time meridian before performing the tangent calculation.

If this formula looks quite different from some other published formulae, that is because the base hour in this book is 6 am or pm, whereas some other formulae use noon as the base hour.

DECLINATION, EQUATORIAL, SOLSTICE, OR LENGTH OF DAY LINES:


The distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style linear height times the tangent of the declination all divided by the cosine of the time difference from 6 o'clock. This is repeated for several of the hour lines.

## COOK BOOK SUMMARY

MERIDIAN DIAL: Geometric:
Longitude correction if desired is made by rotating the protractor


Each 15 degree line intercept to the dial plate marks an hour from solar 6 o'clock (am or pm). At 45 degrees the distance from 6 o'clock is always equal to the style's linear distance from the dial plate.

Latitude is managed by the dial plate and the style being tilted so they angled at latitude. A plumb-line and protractor or a calibrated inclinometer that you have verified may assist here. Longitude is corrected so that indicated solar time, L.A.T., reflects the solar time at the standard time longitude or meridian. That is done by rotating the protractor by an amount equal to the difference between the sun dial's location and the reference meridian. Alternatively the longitude correction may be folded into a tailored EOT table.

Calendar or declination lines are constructed thusly:


MERIDIAN DIAL: Trigonometric [the time base line is 6 o'clock, not noon]
Hour line linear distance from
the 6am/pm line
$=$ gnomon style linear height * $\tan (15$ * hour)
Calendar or declination lines are drawn by connecting points from declination points on each hour line. Each such point is measured from the equinox line as:-

Linear distance along an hour line to a declination point

$$
=\frac{\text { linear style height } * \text { tan (declination) }}{\cos (15 * \text { hour difference from } 6 \text { o'clock) }}
$$

NOTE: If these formula differ radically from other formulae, this book uses 6 o'clock (am or pm as appropriate) as the time base line, and some other published formulae use noon.

Note: APPENDIX 3 has tables for the meridian dial hour line and calendar line distances.

## A STAINED GLASS MERIDIAN DIAL

To the right is a vertical meridian dial of stained glass. There were several firings and a sandblasting phase so the gnomon's shadow would stand out. The tracing black for the hour lines, calendar lines, and the Italian hour lines (discussed elsewhere) used the same pigments used in the 13th century and was fired at 750 degrees C. On the reverse side is a silver stain of the type used in the 14th century was fired at 1250 degrees F which produces a glorious golden yellow transparent color. The hour display area was sand blasted so the gnomon's shadow would stand out. When sandblasting glass, or any compound with silica, great care must be taken to avoid the dust. The health risk is silicosis and a mask and blasting cabinet are recommended.

The dial shows about 3:00 in the afternoon summer time which when corrected for longitude and EOT was correct, and the nodus was on May, this was correct also. The Italian hours, show just under 5 hours until sunset, which again was spot on.


## A CERAMIC MERIDIAN DIAL

To the left is a true west facing meridian dial in Phoenix, Arizona. It is longitude corrected and shows standard time as a result. There is no EOT correction so it is not mean time, in other words, not clock time.

Arizona does not use summer time or daylight saving time, however this dial does have summer hours marked on the summer solstice curve just in case Arizona ever elects to participate in the daylight saving time folly.

This dial was ceramic fired to cone 6 , using slips for the color.

In summary... The vertical east and west facing dials are simple to design.

## CHAPTER TWELVE

## The horizontal dial

## THE EMPIRICAL METHOD FOR A HORIZONTAL DIAL

NOTE: Horizontal dials are usually designed using geometry or trigonometry, the empirical method is mostly educational, and tables are frequently used for horizontal dials that are not longitude corrected. Sundial computer programs are of course based on trigonometry.

The horizontal dial spends its life horizontal. The gnomon is aligned to true north/south, and is parallel to the Earth's polar axis, so the sun revolves in a circle around that style. That means that the style forms an angle with the sub-style that is equal to the latitude. The empirical method of building a horizontal dial consists of leveling a dial plate and then placing the gnomon in place. The style is at the latitude angle, and points true north. Then at 90 degrees to the style as in the bottom picture, a protractor or trigon is affixed, and the 15 degree angles projected to the dial plate.


To the left is a simple trigon designed for a gnomon about one inch thick. The rear part, not easily seen, is a set of clamps to attach the trigon to the gnomon. Then a spirit level and protractor are used to verify that the trigon is in fact still set at the latitude of the gnomon's style.

The front of the trigon is a wooden plate with a protractor marked on it, every fifteen degrees there is a mark.

A cotton thread is attached to one of two nails, one for the left side of the nodus of the style, another for the right. Thus the thickness of the gnomon is taken into account. Then the cotton thread is stretched until it touches the dial plate, and you hold it there with your thumb. And you also adjust the thread so that it is on one of the 15 degree lines. This marks one point of an hour line, the other point is where the style and the sub style meet, on the dial plate at dial center.


Latitude affects the style, and thus the hour lines. The noon hour line should be an extension of the sub-style if longitude correction is not built in. Longitude corrections made to cause the L.A.T. (local apparent time) to match time at the legal meridian result in the noon line no longer being an extension of the sub-style line, it is offset. To make the dial match mean time, the equation of time must be considered.

Longitude is easily accounted for by rotating the trigon's protractor or by displacing the cotton thread by the same number of degrees by which the dials location differs from the legal meridian.

So for example in Silver City there is about a 3.2 degree difference. Looking at the trigon's protractor, displace the cotton thread by 3.2 degrees. So the noon hour line would no longer be 90 degrees, but now it would be altered by 3.2 degrees. Similarly with all other hours. This is the case where noon is no longer north south, it is now deflected somewhat.

The picture below is what we would see if we were looking from the pole, facing the equator standing at and looking at the trigon's protractor.


Vertical south facing dials use the same rule, displace the hour lines west if west of the legal time meridian. However if you haven't figured out east and west yet, then it is a bit more complicated. West on the protractor of a horizontal dial is a shifting to the right because you are facing south. For a vertical dial we are facing north, so shifting the angle lines west means displacing them to the left. The real point to remember is that it is easy to get confused, so write down your corrections before you do them. This process adjusts the dial for longitude. Note that the longitude correction cannot be made by rotating the dial plate, even if you retained the gnomon's north alignment. This book has a template for horizontal dials so you may experiment with a paper dial before making the real one.

All that is left is the equation of time correction. A table is usually provided for that, although some dials have figure of 8 curves to enable a visual correction. Some dials have the longitude correction folded into al location specific equation of time table.

When the hour lines have been penciled in, you can verify they are correct. On a sunny day, check the shadows hourly.

To do this, look at the equation of time. If, as on November 10th, the EOT value is -16 degrees, then the sun is running fast. So you would subtract 16 minutes from the indicated time. So, at 16 minutes to the hour, the shadow should be on the hour line, because the sun is running fast. This allows the empirical hour lines to be verified before being finalized.

All along, we have discussed hour lines, these are based on the 15 degree measurements of a protractor. Hour lines themselves are not 15 degrees apart except for equatorial and armillary dials, and the bi-filar dial. Half hour lines can be drawn, they would be every 7.5 degrees on the protractor, and quarter hour lines can be drawn using 3.75 degrees on the protractor. To beat a dead horse, the protractor has 15 degree lines, the dial plate does not. The dial plate's hour lines are a function of latitude, and of time, but they are not the simple 15 degrees per hour.

On a horizontal dial, the east and west lines are the $6 p m$ and $6 a m$ lines. Offset of course if longitude corrections were made. But what about hours before 6am or after 6pm? The 6am and earlier lines or $6 p m$ and later lines cannot be drawn with a simple trigon, that is because the trigon's angles would parallel the dial plate. A geometrical or trigonometric approach is easier in that case.

All dials whose gnomon is of appreciable thickness, must take into account that thickness. Two results come from this. One is that the dial plate becomes split, i.e. noon has two hour lines, one for just before, and one for just after the noon hour, or it has a single but thick one!

## A horizontal dial with a wide gnomon has 2 noon hour lines



The other result is that when hour lines are included for before 6 am or after 6 pm , then the base of those hour lines is switched to the other side of the gnomon. The black circles show where the sun hits the style and thus why the hour line uses a different base. The dial has two dial centers!

Solar hour angle dials have hour lines based on 15 degrees per hour but they are not 15 degrees themselves. Their intra-hour-line angle varies, and increases as they depart from noon, the exception being the equatorial, armillary, and bi-filar dials. This is easily seen in the geometric method of dial construction, and readily apparent when trigonometry is employed.

Some of the old sundials were used perhaps more for the date than for the time. The sun is above or below the equator by an amount called the sun's declination. Not to be confused with magnetic declination (which pilots and mariners who use it all the time call magnetic variation), nor should it be confused with a vertical wall's displacement from true north or south, also called declination. To paraphrase Mark Twain's comments on spelling, "it is a poor language that cannot find many uses for the same word". In fact the 500 most common words in the English language average about 28 different meanings each. But that is a digression.

Look back at the trigon, it used an oblong or square protractor. It was set at 90 degrees to the style, and if extended would meet the dial plate as a straight line. The 90 degree angle was used because this is when the sun is on the equator, it is also 90 degrees from the polar axis i.e. our latitude-angled style. So, the line where the trigon based hour-points were drawn is a straight line, and is where the tip of the gnomon's shadow will be at the equinox. The east-west line resulting from connecting those dots is a calendar or declination line, and in this case it is called the equinoctial or equinox line.

The trigon's protractor plate could be turned sideways, and then lines marked up and down by 23.5 degrees for the winter and summer solstices.


For simplicity, the above is for a horizontal dial with no longitude correction.
While the solstice lines of longest and shortest day and the equinox lines where the daytime equals nighttime are most common, other declinations can be used. For example the first, tenth, and twentieth of each month might be noted, or the fifth, fifteenth, and twenty fifth could be used. If the dial became too cluttered, then that calendar information might just be marked on the noon line. Most declinations have two dates associated with them, the equinox is a perfect example because March 21 and September 23 or thereabouts, share that line.

Lines of declination might be drawn for the day's length. Namely, identify what declination is associated with a 5 hour day, a 6 hour day, a 7 hour day and the like. In this case the declination lines would not be marked with a date, but the day's length.

Other lines might be drawn, however they don't usually use a trigon. Such lines would be the Italian hour lines, which in essence indicate how many hours remain until sunset. To the purist they indicate how many hours since the last sunset, but most dialists subtract that value from 24 and provide helpful information for gardeners, namely how long before one must put away the tools and retire to the kitchen for a cup of tea.

Babylonian lines may also be drawn, they indicate how long since sunrise. The variations are endless. On one of my vertical dials I have a secondary dial that indicates when to plant tomatoes, corn, and peppers. When the last and first frost is, when is a bad time for pollen sufferers, and roughly when summer time comes into play.

Are there other empirical methods that can be used to build a horizontal sundial? Yes, hour points or lines can be marked when the shadow falls there based on the formula:-

$$
\text { hour point on an hour line }+ \text { EOT => legal time to make the mark }
$$

Thus, if the EOT is say -5 , then at L.A.T. (sundial time) 1200 , the time will be 1155 . So Subtract the EOT from the standard clock time, set the alarm, and rush outside and mark the hour points.

## THE GEOMETRICAL METHOD FOR A HORIZONTAL DIAL

Consider a dial plate $A B C D$. Draw a noon local apparent time line $X Y$. From $X$ draw a line $X Z$ that makes an angle with XY equal to the latitude. In essence this is the gnomon's style rotated 90 degrees lying flat on the surface of the dial plate as if the wind blew it over, resting peacefully.

Draw a line perpendicular to the style that goes to $Y$, that point on the style is labeled $Z$, or, from Y draw a line to $Z$, the angle $X Y Z$ being the co-latitude, and thus intercepts the style at 90 degrees.

Our gnomon is WXZ when rotated back vertically. ZY is the path the sun's rays travel at the equinox at solar noon. ZY would be where the trigon we used in the empirical method would lie.

Rotate $Y Z$ to the top as YR, and then draw a semi-circular protractor LYM centered on R .


Then mark off 15 degree marks on the protractor LYM, rotating them by the longitude difference is so desired, and extend the protractor's 15 degree lines to AB. A longitude correction with the protractor rotated west or clockwise is shown. One rotates the protractor west (clockwise) when the dial's location is west of the standard legal time meridian. Finally, the hour lines are drawn from the dial center up to where the protractor lines meet the west to east line $A B$.

Notice that the noon line is offset to the west because the longitude correction was for a location west of the standard meridian. Another way of verifying this is that noon on the legal standard meridian happens before noon on the dial's meridian when the dial is west of the standard. So, noon must be shown before solar noon where the dial is installed. Finally, the equation of time, EOT, is managed mentally by a printed table for the sundial observer to consider.

As always, the longitude correction need not be incorporated as above, but could instead be blended into an EOT table that would then be location specific.

## One technique for marking hours further away from noon

The previous geometric method works well, however it can be seen that when the hour line is three or four, or more, hours from noon, the angles from the protractor get unwieldy. And 6 o'clock is unmanageable. There are a couple of techniques that take this into account.

One technique for hours some time off from noon is summarized below. To simplify the pictorial, no longitude correction is shown, otherwise it is the same example as on the previous page.


An oblong extends east to west covering only the three hours either side of noon. From point $Y$ (the 12 o'clock local apparent time point) to the bottom right, point $D$ a line is drawn. Then three intersections appear, they are labeled E, F, and G.

Then centered on G, measure GF, and in the opposite direction mark point FF the same distance from G. Thus GF equals GFF. And draw a line from dial center through it, this is the 4 o'clock line. And similarly measure GE and define point EE such that GE equals GEE, this is the 5 o'clock line. And similarly for the opposite side, morning hours.

Appendix 9 has a template to facilitate this method.

## Another technique for marking hours further away from noon

Extend the 45 degree line RB and the plate bottom line CD so they meet at J, similarly on the other side to meet at K .

Or, make lines XK and XJ equal in length to XR .
Drop perpendiculars from $A$ to $C$ and from $B$ to $D$ so that intersections for the early and late hours can be drawn on lines $A C$ and $B D$.

At those new centers K, and J, draw a protractor and mark off the 15 degree arcs and have them intercept BD and AC.


The protractor process is done on both sides. Or it could be done on one side and horizontal lines drawn parallel to $C D$ to transfer the points from $B D$ to $A C$ if no longitude correction is desired.

This provides the 4,5 , and 6 pm lines, even 7 pm and 8 pm . Similarly the 4 am , 5 am , $6 a \mathrm{~m}, 7 \mathrm{am}$, and 8am lines.

As will be seen later, the horizontal dial methods work for the pure vertical true south facing dial.
This technique has the benefit of being able to draw hour lines whose distance along "AB" would be excessive using the simple single protractor approach.

Appendix 9 has a template to facilitate this method.

## Another geometrical method for a horizontal dial

A method discussed by Sundials And Roses Of Yesterday by Alice Morse Earle in the early 1900's uses a technique of interest.


Two circles are drawn on the dial plate center "A" where the gnomon is based, one has a radius of the style's length $A C$, the other of the sub-style's length $A B$. From A draw 15 degree lines, these are not the hour lines, they are construction lines only. Then draw horizontal construction lines on the outer circle where the radial lines meets it, and draw vertical lines on the inner circle where the radials meets it, as shown in the offset to the lower left above. Where the horizontal and vertical construction lines meet is the hour line point, a line from which is then drawn to dial center, point A. In the example above, 30 degrees left of noon is drawn which represents 10am, since there are 15 degrees per hour. Of course, the left hour lines must be shifted left, the right shifted right, to account for the gnomon's thickness.

Longitude corrections are made by selecting an angle smaller or larger than 15 degrees, or its multiple, the change being the number of degrees of longitude separation between the dial and the legal standard time's reference longitude.

For all sundials, the dial plate may indicate only the hours, or it may indicate other information, called dial furniture. Lines that show the sun's declination are helpful for they show the dates for that declination. Declination lines can show the length of the day, when to sow, plant, reap, and when to get allergy medicine! To find the declination for a specific date you might use a table such as in appendix 2. It may also be calculated using a formula discussed in appendix 8 covering formulae.

## Declination or calendar curves for the horizontal dial using geometry.

A horizontal dial with three hour lines shown is shown below.


First, draw a gnomon for the dial center "C". From the nodus draw the equinox line ( 90 degrees to the style), and from that the solstice lines (approx 23.5 degrees on either side). The three lines (equinox and the solstices) intersect the gnomon's base line extended, or the noon line, at points $\mathrm{a}, \mathrm{b}$, and c . These three points whose distances from the dial center are $\mathrm{Ca}, \mathrm{Cb}$, and Cc are then transcribed to the dial plate (right pictorial to left pictorial).


The equinox line is then drawn perpendicular to the noon line, and it produces equinox intercepts for those additional hour lines, d and e. Distances Cd and Ce are then located from the left dial plate to the right hand picture on its equinox line. This produces two more hour lines on the right hand side picture, Cd and Ce . Those hour lines on the right hand side do not have angles that match their hour lines on the dial plate, and this is because this is a projection.

Now that there are two more hour lines, or as many as you choose, this produces intercept points for the solstice lines, namely points $f, g, h$ and $i$, see below.


Points $\mathrm{f}, \mathrm{h}, \mathrm{g}$, and i are now transferred back to the dial plate, from the right projection pictorial to the left picture.


When this process is completed for as many hour lines as desired, the dots are connected and then the declination lines drawn.


The following protractor layout from appendix 9 may be helpful when drawing declination line curves. The left quarter protractor is for the hour lines, the right protractor is for the gnomon, the style being on the top, so the sub-style is angled below it. Two sets of solstice/equinox/solstice points are drawn in the example below, they are on the noon and the 9am or 3pm line.


## THE TRIGONOMETRICAL METHOD FOR HORIZONTAL DIAL HOUR LINES

Consider CSP to be the gnomon, CS being the style, CP the sub-style. From $S$ extend a perpendicular to a new point Q on the East-West line. As the sun moves by "ha" degrees from noon, a new hour line is drawn whose angle is H .

(1) $\quad \tan (h a)=x / y$
(2) $x=y$ * $\tan (h a)$
tan of hour angle ( 15 degrees * hour )
(3) $\quad \tan (H)=x / z$
(4) $\quad \sin ($ lat $)=y / z$
(5) $y=z * \sin$ (lat) thusly
tan of the hour line angle
sin of latitude or $\varnothing$ thusly

$$
\begin{align*}
H=\operatorname{atan}(x / z) & =\operatorname{atan}\left(\left(y^{*} \tan (\mathrm{ha})\right) / z\right)  \tag{6}\\
& =\operatorname{atan}\left(\left(z^{*} \sin (\operatorname{lat}) * \tan (\mathrm{ha}) / z\right)\right) \\
& =\operatorname{atan}(\sin (\mathrm{lat}) * \tan (\mathrm{ha}))
\end{align*}
$$

So, to calculate any hour line from the sub style:

$$
H=\arctan (\sin (l a t) * \tan (h a))
$$

The hours used in the hour line formula may be adjusted for longitude. If you use a spreadsheet then trigonometric functions use radians, and must be converted back using degrees. The formula is then something like:-

> DEGREES(ATAN(TAN(RADIANS(15*time))*SIN(RADIANS(latitude))))

## Tables and nomograms can be used which are trigonometric in nature

Appendix 3 has tables with hour line angles calculated for various latitudes, it also has nomograms for hour line angles. The next chapter which covers the vertical true north/south dial, and this book's introduction also showed the use of tables to design a dial that is not corrected for longitude. Either the tables or nomograms in appendix 3, or the spreadsheet provided with this book will provide tables of hour angle by latitude. As mentioned, nomograms may be used and are included in the appendices, and chapter 32 discusses such earlier methods for dial design.

## Declination or calendar curves for the horizontal dial using trigonometry. The following technique can also use simple geometry.



Winter solstice for example

Equinox

Summer solstice for example

SUN'S ALTITUDE METHOD: The distance from the foot of the style, point T, and not from the dial center C , to a given hour line's declination point (TD) is determined by:-

```
distance TD = style height * cotan (sun's altitude)
sun's altitude = arcsin ( sin (dec) * sin (lat) + cos (dec) * cos(lat) * cos(time) )
```

How is the declination found? The tables in appendix 2 may be used, or their formula can be used directly:-

$$
\begin{aligned}
\text { Sun Declination: decl }= & \text { degrees }\left(0.006918-0.399912^{*} \cos (d a)+0.070257^{*} \sin (\mathrm{da})\right. \\
& -0.006758^{*} \cos \left(2^{*} d a\right)+0.000907^{*} \sin \left(2^{\star} d a\right) \\
& -0.002697^{*} \cos \left(3^{*} d a\right)+0.001480^{*} \sin \left(3^{*} d a\right)
\end{aligned}
$$

where:

$$
\mathrm{da}=2 * \mathrm{pi} *(\mathrm{j}-1) / 365 \quad \text { (Day angle: in radians, is an intermediate figure) }
$$

SUN'S AZIMUTH METHOD: An alternative to using the sun's altitude hence nodus shadow distance which relies on the nodus height, the sun's azimuth can be used from the foot of the style, point T , and not from the dial center C , to a given hour line. The intersection is the declination point for that azimuth and hour line.

## ADDITIONAL THOUGHTS

Supplemental Shadows has some expanded notes on this topic.
The nodus indicates the calendar lines, the style indicates the hour line shadow. A nodus allows a larger gnomon that would otherwise work on a dial plate. Dial plates and gnomons must be aligned as discussed. Turning a dial plate or bending a gnomon to correct a horizontal (or vertical dial) will upset all the angular relationships. However, a horizontal dial may be tilted if the dial was for a different latitude. A wedge that tilts the dial plate north or south has the effect of making the dial act as if it were at another latitude, thus old antique dials may be thusly made to believe they are at their old latitude, and once again live to see the sunlight.

Chapters 23 and 24 cover calendar and other lines in detail, with analemmas discussed in chapter 25 . Chapter 20 shows a ceramic horizontal dial as part of an 8 inch cube dial.

## COOK BOOK SUMMARY

HORIZONTAL DIAL: Geometric
Hour lines:


Calendar or declination lines: see chapter 23
HORIZONTAL DIAL: Trigonometric
Hour lines: hour line angle $=\arctan (\sin$ (latitude) $* \tan (15 * h o u r)$ )
Calendar or declination lines: see chapter 23
HORIZONTAL DIAL: Tabular
Appendix 3 tables A3,1a, A31.b, A3.1c provide the hour lines by latitude.
NOTE: Horizontal dials are usually designed using geometry or trigonometry, the empirical method is mostly educational, and tables are frequently used for horizontal dials that are not Iongitude corrected. Sundial CAD programs are of course based on trigonometry.

NOTE: the photo to the right of a horizontal dial has a deliberately wide gnomon to show how gnomon width is managed.

NOTE: arctan is the angle whose tangent is, sometimes written as atan, atn, or $\tan ^{-1}$

Appendix 9 has a template for laying out a horizontal dial, and on the next page that template is used to facilitate drawing the dial plate.


HORIZONTAL DIAL: Method: Using Tables: [appendix 9 has the template used here]


Appendix 3 table A3.1b for latitude 40 provides the following hour line angles assuming no Iongitude correction.

> 8 am and 4pm 48.07 degrees 10am and $2 \mathrm{pm} \quad 20.36$ degrees

> 9am and $3 p m \quad 32.73$ degrees
> 11am and 1 pm 9.77 degrees etc

The hour lines before and after noon are drawn. The gnomon is drawn and moved to dial center. The longitude correction of 4 degrees (at 4 minutes per degree) is 16 minutes, and is west of longitude 105 thus we must add the time. The longitude correction of 16 minutes is added to the equation of time, producing a longitude corrected revised EOT table:-

| Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| +25 | +30 | +25 | +16 | +13 | +16 | +22 | +20 | +11 | +2 | +1 | +11 |

The dial is aligned to true north, and the shadow mentally corrected with the revised equation of time (EOT). Alternatively the hour lines could be adjusted individually for longitude, however the dial would be less portable. If the hour lines were longitude adjusted, they would not simply be rotated, but the angle recalculated for the longitude offset one hour line at a time. The equinox line is derived by extending the gnomon from the nodus to the sub-style line extended, and perpendicular to the style. See the longitude corrected ceramic dial to the right.

Longitude corrections of more than 4 degrees cause the resulting combined EOT table to always have the same sign. That is because 4 degrees is 16 minutes, which is the maximum EOT deviation.


## CHAPTER THIRTEEN

## The vertical dial facing the equator (or the pole)

Assuming the dial faces true south (or true north in the southern hemisphere), as in small cube dials, then all the methods of the horizontal dial apply, the exception is that the method uses the co-latitude, rather than the latitude. If the dial declines somewhat east or west from the north south meridian, as on building walls, then refinements are needed which are covered in the "vertical decliner" and "great decliner" chapters 16 and 17.

The geometrical or the trigonometric methods discussed so far for horizontal dials are used as-is for the true south vertical dial, the sole exception is to use co-latitude (90-latitude) in place of latitude.

Since the methodology is the same, there is no point in duplicating those methods here. However, there are tables and nomograms for hour lines in appendix 3 should you choose the trigonometric method, and it may save much time. One such table is extracted here...


The chart above covers horizontal dials of latitudes 30 to 39 or vertical dials of 51 to 60 degrees. This means that latitude 32 of a horizontal dial has hour lines that spread out which match a vertical dial's hour lines for latitude $58 \sim$ this is because of the latitude/co-latitude relationship between the vertical and the horizontal dial.

Don't work on vertical dials when the ground is wet beneath you, if the style is at a height that might cause impalement. I merely mention this fact without my reasons for such knowledge. Having designed your sundial, you might choose to use software such as SHADOWS to simulate the shadow's movement by the hour or through the year. Of course, SHADOWS can also design the sundial for you, however the hand crafted design work is more satisfying, is it not?

## Proof a vertical dial design matches a horizontal design for the co-latitude

Consider the following diagram of the planet Earth. At latitude 20 degrees for example there is a vertical dial plate. For simplicity, the polar axis is shown as vertical.


Angle ecb is the latitude, and we set angle dce to 90 degrees, so angle acd is the co-latitude.
The angle $x$ is the vertical dial's dial-plate angle with the sun. In the horizontal dial, angle w is also by definition equal to $x$. Thus the hour lines for the vertical dial at latitude $x$ match those of a horizontal dial of latitude $90-x$. By definition the reverse is also true.

The lower figure is the same as the upper figure except that the horizontal dial is now rotated around the polar axis to be at the same time or longitude as the vertical dial, angle bcd being the co-latitude.


This is why a horizontal dial may be used for a vertical dial at the co-latitude. However, as the next page shows, this similarity is only skin deep.

## Problems with a vertical dial design matching a horizontal design

The only things a vertical and horizontal dial have in common is the relationship between latitude and co-latitude and the hour line angles for dials not corrected for longitude. There are more issues not in common than there are similarities.

First, in the northern hemisphere, the shadow moves clockwise for a horizontal dial, counter clockwise for a vertical. In the southern hemisphere the reverse is the case. This also means that hour line numbering must also be reversed.


Second, if longitude is considered when designing a dial, a horizontal dial's offset for an hour line is opposite to a vertical dial's offset, this is for the same reason as the change in rotation of the shadow between the two dials mentioned above.

If the dial was west of the legal time meridian then in the northern hemisphere both horizontal and vertical dial hour lines would deviate to the west by varying degrees so the shadow would hit an hour line earlier.

However if you took the horizontal dial plate and rotated it to be a vertical dial, that hour line correction would now be in the wrong direction.

Third, for a horizontal dial the winter calendar curves are farther away from dial center than the summer curves, whereas for a vertical dial, the winter curves are closer to dial center. Chapter 23 discusses this.

Fourth, if a figure of eight analemma was placed on the plate, the analemma would thus have to be reversed left to right about its axis because of the second reason above. And because of the third reason above, the analemma would also have to be flipped top to bottom. Chapter 25 discusses this. In essence a $180^{\circ}$ rotation of the analemma.

Not excessively relevant to the above discussion, but an interesting special case for two matching horizontal dials that ought to be in the book somewhere

While we have shown that a horizontal dial has the same design as a vertical dial for the co-latitude, the case of the reclining dial facing the equator introduces an interesting twist should you have two horizontal dials sitting around doing nothing. Chapter 15 discusses reclining dials.

If you have two horizontal dials that do not have longitude corrections applied, and do not yet have the hour lines labeled, then you can place them such that they are aligned as shown to the right, and both will read the same time.


## Declination (calendar) lines for a vertical dial.

The geometrical processes described for the horizontal dial in chapter 12 are applied directly to the vertical dial. The horizontal dial's summer solstice line becomes the vertical dial's winter solstice line, and the horizontal dial's winter solstice line becomes the vertical dial's summer solstice line.

When using the protractor layout template from appendix 9, please double check that the substyle and style are correctly placed on the top right hand side of the template.


In the enthusiastic burst of energy arising from completing the declination lines, it is possible to mistakenly place the sub-style where the style is supposed to be on that template. Should this be done, and the declination lines completed erroneously, then that enthusiasm will be replaced by feelings of distress. I merely mention this without further comment.

To the right is a vertical dial of stained glass. There were three firings and a sandblasting phase so the gnomon's shadow would stand out.

The dial shows about $3: 45$ pm between the winter solstice and the equinox and when the EOT and longitude corrections were added, this was correct.


## Relevant thought for vertical recliners

The concept of a vertical dial being the same as a horizontal dial for the co-latitude and vice versa, namely that a horizontal dial design matches the design of a vertical dial for the co-latitude, is important.

First, it allows one table, or one spread sheet to work for both dial types. Second, when vertical dials that face pure south are reclined, or tilted backwards or forwards, three simple special cases result, these are discussed elsewhere.

## A VERTICAL DIAL FACING THE POLE (in this case North)

Some dials on columns have a dial on their north facing surface. Some north facing walls also have them. However they do not work for six months in the year, and then only for hours before 6 am or after 6 pm . The gnomon's style points upwards and parallels the north south polar axis. in the northern hemisphere it points very close to Polaris, the north star.


The gnomon is drawn first as XBQ , and QHX being 90 degrees, a circle with radius $R Q$ is then drawn to match the length QH [ QH because co-latitude is used here]. Then lines RK, RJ are drawn to intercept the extension of CD. A line AB is drawn parallel to KJ, and perpendiculars AC and $B D$ drawn where points $A$ and $B$ intercept RK and RJ. These perpendiculars are to hold intersection points for the early and late hours drawn from points J and K . The rest of the process uses those 15 degree lines radiating from J and K , or any other vertical dial design discussed thus far.



A true north facing dial. The hours are marked with circles within the body of a skink, a cross between a lizard and a snake.

There is no 12 o'clock hour line, for it would be midnight. North facing dials show hours from summer sunrise for a few hours, then nothing until a few hours before sunset. Dials placed at the poles would be an exception.

## A VERTICAL DIAL FACING THE EQUATOR

Draw a gnomon as shown below the right, take distance CB as a distance to dial center, and draw a circle of radius $A B$ as shown below left. This process is exactly the same as the horizontal dial design in chapter 12 except this uses co-latitude for dial center whereas the horizontal dial used latitude. Also, the Calendar or Declination lines use the same method used for horizontal dials, except winter and summer are reversed.


VERTICAL DIAL FACING THE EQUATOR: Trigonometric
Hour lines: hour line angle $=\arctan (\cos$ (latitude) * $\tan (15 *$ hour) )
Calendar or declination lines: Except winter and summer are reversed, use horizontal dial techniques.

## VERTICAL DIAL FACING THE EQUATOR: Tabular

Appendix 3 tables A3,1a, A31.b, A3.1c provide the hour lines by latitude. Vertical dials are on the bottom of the chart because of the co-latitude relationship. A worksheet or template such as is used on the next page can simplify dial layout, and can actually be put together and tested with the sun before completing the final dial.

NOTE: arctan is the angle whose tangent is, sometimes written as atan or tan ${ }^{-1}$
NOTE: declination lines for a vertical dial have the winter solstice nearest the dial center, whereas they are furthest from dial center on a horizontal dial. Similarly a vertical dials summer solstice line is farthest from dial center whereas it is closest on the horizontal dial.

Appendix 9 has the template for laying out a vertical dial which is demonstrated on the next page.

## VERTICAL DIAL: Method: Using Tables:



Appendix 3 table A3.1c for latitude 40 (vertical dials use the bottom not top index of those tables) provides the following hour line angles assuming no longitude correction.

$$
\begin{array}{llll}
8 \mathrm{am} \text { and } 4 \mathrm{pm} & 53.0 \text { degrees } & 9 \mathrm{am} \text { and } 3 \mathrm{pm} & 37.5 \text { degrees } \\
\text { 10am and } 2 \mathrm{pm} & 23.9 \text { degrees } & 11 \mathrm{am} \text { and } 1 \mathrm{pm} & 11.6 \text { degrees }
\end{array} \text { etc }
$$

The hour lines before and after noon are drawn. The gnomon is drawn and will be moved to dial center. The longitude correction of 4 degrees (at 4 minutes per degree) is 16 minutes, and is west of longitude 105 thus we must add the time. The longitude correction of 16 minutes is added to the equation of time, producing:-

| Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| +25 | +30 | +25 | +16 | +13 | +16 | +22 | +20 | +11 | +2 | +1 | +11 |

The dial plate is vertical, the gnomon style is true north and at latitude and note that the style:substyle angle is thus co-latitude for a vertical dial. The shadow is mentally corrected with the revised equation of time (EOT). Alternatively the hour lines could be adjusted for longitude, the lines cannot just be rotated, but an angle must be used for a time corrected by 16 minutes for each hour line individually. The equinox line is derived by extending the gnomon from the nodus to the sub-style line extended, and perpendicular from the style.

Longitude corrections of more than 4 degrees cause the resulting combined EOT table to always have the same sign. That is because 4 degrees is 16 minutes, the maximum EOT deviation. This paper dial can be tested in the actual sun, before the final dial is built.

IMPORTANT NOTE. While a vertical dial derives easily from a horizontal dial, some points must be observed. First, if you rotate a horizontal dial to become a vertical dial, the shadow rotation reverses, thus the shifting of hours to correct for longitude and for EOT also reverse. Second, the solstices also reverse. This becomes important when a calendar line or an analemma is considered, chapters 23 and 25 respectively have to consider this.

## CHAPTER FOURTEEN

## The General Model for almost all hour angle dials

For a sundial, the sun rotates around the Earth, and as there are 360 degrees in a circle and 24 hours in a day, that means the sun rotates around the Earth's polar axis 15 degrees per hour. Those 15 degrees define solar time, and as we have seen, it varies from clock time due to many factors, one being that the Earth's annual orbit around the sun is elliptical, not circular.


The sun moves north from December to June, south June to December as shown above. If we made a shadow casting device (called a gnomon) with a tip or other mark on it (called a nodus) that caused a clearly defined point or tip of a shadow, and at the same solar (not clock) time each day marked the shadow's tip, over a year a line would be drawn by that shadow tip, an hour line, see above right pictorial. The hour line results from the base of a triangle, solstice to solstice, of shadow lines for that solar hour. If clock time were used as opposed to solar time, then that line would look like a figure of eight, and demonstrate the equation of time. Some sun dials have those figures of eight on their hour lines, and it is called an analemma, and that figure of eight is not symmetrical.

In the picture below the hour shadow triangles are in one hour increments for simplicity, they are triangular because, as above, they show the range of the hour's shadow tip throughout the year.


One hour increments are common on sun dials, and each hour from winter to summer solstice builds a set of shadow ranges. Each is triangular. Each is 47 degrees at the apex, being twice the sun's journey of 23.5 degrees (approximately). And the apex, as far as shadows go is called a nodus on a shadow casting device or gnomon).

To the left is planet Earth shown rather large, and to its right is the sun, shown rather small. And some triangular shadow rays are shown from 10 am to 2 pm each being 15 degrees from its neighbor.

Each of the rays show the winter and summer solstice limits for a given solar (not clock) time. Those limits form an hour line when they hit a surface. And each shadow ray triangle thus shows where each hour line will be.

The surface holding those hour lines (called a dial plate) can be at any angle to those rays, horizontal and vertical are the two most common. And vertical dials may be southerly or they may face east or west. The pictures below show a horizontal, then a vertical south facing dial, then a west facing vertical dial.


To the left is a dial plate for a horizontal dial. The solid lines are for 10 am and 2 pm , as on the previous page. The dashed line is the limit of the summer shadow tips, it marks the summer solstice, around June 21. The hour lines do not center on the base of this columnar gnomon, they have a "dial center" elsewhere. And if a line were drawn from the dial center to the nodus, the tip in this case of the gnomon, it would be called the style, and the angle it would form with the dial plate would be the dial's latitude.

To the left is a dial plate for a vertical south facing dial. The solid lines are for 10 am and 2 pm , as before. The dashed lines are the limit of the solstice shadow tips. The top "U" shaped curve is the winter solstice, around December 21, and the lower " $\cap$ " marks the summer solstice curve, around June 21. The hour lines do not center on the base of this columnar gnomon, they have a "dial center" elsewhere. And if a line were drawn from the dial center to the nodus, the tip in this case of the gnomon, it would be called the style, and the angle it would form with the dial plate would be the dial's co-latitude. Of course the angle it would form with the horizontal would be the latitude.


To the left and right are dial plates for the vertical dial which is facing true west. The pure east or west facing dial center is at infinity! Solid lines are hour lines, dotted lines are annual limits. The upper curve is the winter solstice curve, the lower curve is for the summer.


Any dial plate shape may be modeled in this manner, such as the armillary, equatorial, or any irregular shape.

## A General geometric model of the previous dials

We have now seen the geometric methods for designing the armillary and equatorial dial, the polar and meridian dials (true east or true west), as well as the vertical (true south) and the horizontal dial. We can blend all those concepts into one easy to remember model. This is intended to show a natural symmetry in geometric dial design.


The above general geometrical model shows a symmetry in the design of the polar, vertical, and horizontal dials in the context of their gnomon's style. For a polar or a meridian dial, CD is the linear height of the style's nodus above the dial plate. In this model, CD does different things depending on dial type.

For a horizontal dial, this model is based on the ratio of the DC to CB. For a vertical dial the ratio is based on DC to CA. In all cases, DC is the equinoctial ray going to the dial plate at C from nodus D .

The model on the previous page was developed by Albrecht Durer in 1525 and is pictured to the right in perspective. The sun is at the top left, the planet Earth is at the bottom right showing the polar axis and equator, with the dials shown in between.


Any dial not susceptible to the methods described here can be designed because that dial has a horizontal or vertical dial that matches it somewhere else on the planet. The French dialists Sagot and Savoie developed such a model.


Dials may be designed for portability, or for a specified location.

Latitude adjustments can be simplified by tilting a dial.
Longitude adjustments can be made by either considering the longitude difference when the hour lines are designed, or the difference can be included into a tailored equation of time.

Portable dials should not have the longitude correction built in since it would limit their portability. If a dial has no longitude correction built into it, then it is simpler to design, and it can be relocated. The correction for the EOT is simply modified to include the longitude correction.
e.g.


The longitude correction of say - 8 minutes that would have been built in to the dial, now becomes one correction table when you deduct the fixed 8 minutes from each EOT entry.
e.g.


A review of the common hour angle dials other than the decliner and the inclined decliner which are covered in their own chapters. Additionally, appendix 1 in the separate appendices shows the planet with several of each dial types on it at several latitudes, latitude $0,30,60$ and $90^{\circ}$.


Regardless of where a dial is, in space or on the planet, when the sun's rays shine they will make shadows, and the west to east range tells the time, while the north to south direction shows the date.

To the left is shown the sun and it's light hitting a conical gnomon, and 5 different hours are shown as five triangular shadows. They are triangles because of the solstice through equinox to solstice annual solar travel.

This pictorial shows the north or south shadow extremes reflecting the date, and the west to east range reflecting the time. The time comes from the sun's angle, or hour angle, around the polar axis. The date comes from the sun's declination above or below the equator. The pictorial above thus shows how both the date and the time are determined.

## ARMILLARY DIALS

The armillary dial to the right, named for the likeness to arm bracelets, uses the 15 degrees per hour directly. The picture shows the sun's rays for several hours, and for each of those hours the solstice extremes are shown. The dial plate parallels the north south polar axis. The date has a tangent relationship and the complete annual cycle can be seen on the dial plate.


## EQUATORIAL DIALS

The equatorial dial has a dial plate at 90 degrees to the armillary, thus it parallels the equator. Just like the armillary dial, there is no complicated geometry. It is 15 degrees per hour for the time. The date gets a little out of hand at the equinoxes, in fact at the equinox, the shadow recedes to infinity. Like the armillary, this is a tangent relationship, however the alignment of the dial plate is what causes the problem at the equinox.


## MERIDIAN

The meridian dial to the left has a dial plate coming from the polar axis proceeding out and beyond. The hour lines begin close in at 6 am or 6 pm , but as the time moves from those points, the hour lines increase rapidly away. They recede based on the tangent of the time in hours from 6 am or 6 pm , multiplied by 15 . That 15 being the 15 degrees per hour rotation of the sun around the Earth.

The polar dial to the right is like the meridian dial however the dial plate is rotated 90 degrees from it, it parallels the north south polar axis. The hour lines are also based on the tangent of the hour multiplied by 15 , from noon.

HORIZONTAL


The horizontal dial to the left has a dial plate that lies flat on the Earth's surface. For clarity that plate has been lifted up a bit.

VERTICAL
The vertical dial to the right is like a horizontal dial except it is 90 degrees to the surface. The vertical and horizontal dials have a latitude to co-latitude relationship.


In all these cases the 15 degrees per hour applies. Geometry comes into play when the display on a dial plate is considered. As a rule polar and equatorial dials are a simple 15 degrees, and the rest of these hour angle dials have hours lines based on the tangent of the hour angle. And if latitude is used, then a factor based on the sine (or cosine) of the latitude.

To clarify the pictures on this and the preceding page, the gnomons on these two pages are cones in space, whose tip is the shadow caster. In other words, there are no styles nor sub-styles depicted.

## CHAPTER FIFTEEN

## The vertical recliner

## INCLINING OR RECLINING DIALS

## Facing true north, south, east, or west South

First case: a vertical recliner directly facing the equator. The vertical recliner exists on slopes such as roofs, or, on walls where I live that were not quite vertical! There are three generic types, and all three have simple solutions.


- Cases where the roof's pitch is the same as the latitude.
- Cases where the roof's pitch is the less than the latitude.
- Cases where the roof's pitch is the more than the latitude.

Second case: a true east or west facing dial that reclines, this has a simple conversion to a vertical decliner.


In the pictorial below, there is one dial plate on the left which shows how an inclined dial at latitude 20 becomes a decliner at latitude 70. As another example, the other two separate dials show a dial plate on the equator or latitude 0 that is inclined about $50^{\circ}$, and it becomes vertical at the pole or latitude 90, but now declines by that inclination angle.

This vertical dial at latitude $20^{\circ}$ reclines backwards by about $25^{\circ}$, and
becomes a vertical dial at latitude $70^{\circ}$. with a declination of $65^{\circ}$


The dial on the equator or latitude $0^{\circ}$ reclines backwards by about $40^{\circ}$, (inclines $50^{\circ}$ ) and
at latitude 90 that same dial plate is now no longer inclining or reclining backwards and it is now vertical but declining by $50^{\circ}$.

NOTE: after studying this chapter, look at chapter 13 section "problems with a vertical dial design matching a horizontal design" which has an interesting case for two horizontal dials with no longitude correction.

## A VERTICAL RECLINER, FACING THE EQUATOR

To clarify a few terms


And a slope that leans forwards "proclines".

Vertical recliners facing the equator typically exist on the roofs of buildings aligned on true north. In many housing areas, this doesn't happen often.

There are three classes of vertical recliners whose dial plate faces the equator.

1. the slope's angle is the same as the location's latitude, this is a polar dial.
2. the slope's angle is the less than the location's latitude, then this is a horizontal dial for a latitude equal to the difference between the latitude and the slope's pitch. For example, a $20^{\circ}$ pitched roof in latitude $30^{\circ}$ calls for a dial designed for $10^{\circ}$.
3. the slope's angle is more than the latitude, then this is a vertical dial for a latitude equal to the co-latitude of the roof and latitude angular difference. Vertical and horizontal dials share a co-latitude relationship already discussed.

## The slope's pitch is the same as the latitude ~ use a polar dial

When the slope's pitch is the same as the latitude, this is exactly what polar dials live for, the hour lines are based solely on the time, not latitude, as a result these are universal dials since they can be moved to any latitude and merely tilted, the hour lines don't get upset by latitude shifts. Hour lines are based on the tangent of the hour multiplied by 15.


Trigonometrically, using the above figure, $\tan (15)=x / y$, thus the distance from the sub style " $x$ " is given by: $x=y * \tan (15)$, or, hour line distance = style height * tan(hours*15). One hour would be 15 degrees, two hours would be 30 degrees and so on.

Longitude correction can be made by adding or subtracting the dial's longitude from the legal time's longitude and adding or subtracting that difference from the 15 degree radials, however the dial is then no longer universal because location has been factored in.

## Slope's pitch is the less than latitude $\sim$ use horizontal dial of lesser latitude

It was mentioned earlier that a horizontal dial designed for another latitude can be used at a new latitude by the simple application of a wedge. That wedge causes the gnomon to once again be parallel to the Earth's polar axis. A roof is no different, except of course that a roof that does not decline, i.e., one that has its ridge running true east to true west, is perhaps not common.

The design is merely a construction using methods already discussed. For example, a roof of 40 degrees pitch in latitude 70 is shown exaggerated below, this calls for a dial designed for $30^{\circ}$.


Design a gnomon whose style height is equal to latitude (70) minus roof-pitch (40), in other words a style angular height of 30, pointing north parallel to the Earth's polar axis. The hour lines are then designed for a latitude of the dial's angular style height, in other words the gnomon's angle (30) when placed upon the slope. Longitude could be considered as usual.

## Slope's pitch is the more than latitude ~ use vertical dial of greater latitude



In this case we use a vertical dial with a style height of roof pitch (40) minus the latitude (10), being a 30 degree style height. For vertical dials, the style height is the co-latitude, so, the vertical dial's design latitude would be $60^{\circ}$. A vertical dial and a horizontal dial share a co-latitude relationship. Thus this is the same as a horizontal dial for latitude 30, except the dial center points to the pole, not the equator.
use horizontal-dial designed for: roof - latitude or
use vertical-dial designed for: 90 - (roof - latitude)
(reverse am/pm hours, and reverse longitude correction)
(normal am/pm hours and normal longitude correction)

## A VERTICAL RECLINER, FACING TRUE EAST OR TRUE WEST

Recall that a horizontal dial at latitude "x" has the same design as a vertical dial at latitude "90-x" (the co-latitude), with am and pm hours switched, in other words the dial rotated around the vertical axis. Similarly, a vertical true east/west meridian dial has the same design as a vertical south dial at the co-latitude. And a reclining meridian becomes a declining vertical.


The actual lines of one dial are the design lines of the other dial when moved to a new location $90^{\circ}$ away. The inclination of one is the declination of the other. A sloping true east or west dial at one latitude is the same as a declining dial facing north or south at a latitude equal to $90^{\circ}$ plus the original dial's latitude. Since that takes one over $90^{\circ}$, we rotate the design-dial $180^{\circ}$ around the pole, which produces the formulae:

The latitude of the "design dial" is:- $\quad=90$ - the latitude of the actual dial and
The declination of the "design dial" is:- = 90 - the reclination of the actual dial or $=$ the inclination of the actual dial

and
The dial plate was rotated $90^{\circ}$ in latitude, so on the new dial noon is horizontal, not vertical
and
The dial was rotated $180^{\circ}$ around the pole, so on the new dial the hours lines are mirror imaged
Thus the simple formula for converting true east or true west facing sloping dials to a dial that is true south or north facing but declining facilitates such a dial design. And the "design" then obviously uses the vertical decliner techniques. Some cross checks are in order. First, the noon line is horizontal. Second the dial center is towards the equator. Always build a model to validate your calculations, the case study that follows shows this process. Remember the rules for the definition of a declining dial:-


An example of a CAD empirical design for a reclining dial for latitude 33 north.
Front south facing design: Here a pyramid has been made. The front face is a south facing recliner. The face angles are $30^{\circ}$ from vertical or $60^{\circ}$ from the horizontal. The final gnomon style is set at $33^{\circ}$ for the dial's actual latitude.

Using the techniques from chapter 30 , we design a vertical dial for the colatitude of $27^{\circ}(60-33)$, i.e. $63^{\circ}$. And for a design latitude of $63^{\circ}$ we derive from the formulae or spreadsheet the following hour line angles.

| Latitude | 63 | Hour line angles for |
| :---: | :---: | :---: |
| TIME | DAY | simple dials |
| am | pm | VERTICAL |
| 12.00 | 12.00 | 0.00 |
| 11.00 | 1.00 | 6.94 |
| 10.00 | 2.00 | 14.69 |
| 9.00 | 3.00 | 24.42 |
| 8.00 | 4.00 | 38.18 |
| 7.00 | 5.00 | 59.45 |
| 6.00 | 6.00 | 90.00 |



The table to the left is used for the south face. Of course for the southern hemisphere, this would be a north facing dial plate, with the same hour line angles, however the am and pm hours would be reversed.

The side east facing dial reclines and thus needs to be converted to a south facing declining design-dial, with its final dial center pointing south (for northern hemispheres) and with the noon line horizontal.

The dial plate reclines $30^{\circ}$ from vertical at latitude $33^{\circ}$. Using the formulae from the preceding page, this generates a design dial that is vertical, at latitude $57^{\circ}$ (90-33) that declines $\mathrm{S} 60^{\circ} \mathrm{E}$ (being $90-30$ ). Or more simply, the design dial's declination is the actual dials inclination. Using the spreadsheet for vertical decliners, the following figures are produced.

| Lat $57^{\circ}$ | $\begin{aligned} & -D E C \\ & S x x E \end{aligned}$ |
| :---: | :---: |
| hh | Dec $60{ }^{\circ}$ |
| 6.00 | -36.9 |
| 7.00 | -32.3 |
| 8.00 | -28.2 |
| 9.00 | -23.9 |
| 10.00 | -18.9 |
| 11.00 | -11.9 |
| 12.00 | 0.0 |
| 13.00 | 25.5 |
| 14.00 | 75.6 |
| 15.00 | -67.4 |
| 16.00 | -51.2 |
| 17.00 | -42.6 |
| 18.00 | -36.9 |
| SD | -29.4 |
| SH | 15.8 |

The final noon line is horizontal, morning hours are above it, afternoon hours below. Use the am and pm hour line angles directly from the spreadsheet. The west facing dial is an opposite of the east dial plate.

Below are the south and the east facing dial plates seen perpendicularly from the plate. The SD specifies where the sub-style will lie and the SH gives the angle from the sub-style for the resulting gnomon. Use software such as SHADOWS to double check, and always make a model to verify your resulting deductions.


## Some observations and rules of thumb

In the picture below, there are two sets of sun rays covering 0600 hourly until noon. The noon line, the lowest of all the lines, is horizontal. The matching sun ray lines are connected with dashed lines, and they converge at the dial center, which in this case is not on the dial plate.


The same picture but with angles measured on the 60 degree slope using CAD, that are very close to those calculated by the spreadsheet. Another example that using CAD to derive complicated angles is practical.

Some rules of thumb:
For an east facing dial, the following are rules of thumb. For a shallow slope, less than $45^{\circ}$, the upper hour lines (0600) are spread out, becoming closer at the lower end (1200). For steep slopes, more than $45^{\circ}$, the upper hour lines (0600) are more tightly packed than the lower ones (0600). For $45^{\circ}$ slopes, the lines are spread out at the 0600 and 1200 end, and more compact in the middle, 0900. Similarly for a west facing slope. The noon line, the
 lowest of all the lines, is horizontal.

One question sometimes asked is that the resulting SD or SH do not seem to reflect the final dial's actual latitude. In this case the dial is at latitude 33, but the SD is 29.4 and the SH is 15.8 , surely this cannot produce a style aligned with the poles at latitude 33. It does, because both the SD and the SH are used together, in math terms, a vector addition. The SD angles the sub-style up from the noon line, and the SH being perpendicular to the dial plate, which is sloped, makes the style itself even further up. The result is a style correctly set at latitude. Before making the final dial, a paper dial plate and gnomon mockup was made and the result was sun tested. The styles in the mockup photo to the right are parallel.


NOTE: Vertical reclining dials are usually designed using geometry or trigonometry, the empirical method is less frequently used. Sundial programs are of course based on trigonometry.

## COOK BOOK SUMMARY FOR DIALS FACING THE EQUATOR OR EAST/WEST

VERTICAL RECLINER whose slope is the same as the latitude
use a polar dial

Hour line linear distance from the noon line = gnomon style linear height * tan (15 * hour)
Linear distance along an hour line to a declination point = linear style height * tan (declination) / cos (15 * hour)

VERTICAL RECLINER whose slope is less than the latitude
use a horizontal dial of lesser latitude = latitude minus slope
hour line angle $=\arctan (\sin ($ latitude $) * \tan (15 *$ hour $))$
or use the tables in appendix 3 tables A3,1a, A31.b, A3.1c - horizontal index
VERTICAL RECLINER whose slope is more than the latitude
use a vertical dial of co-latitude =90-(slope - latitude)
(normal am/pm hours, and longitude correction) or a horizontal dial of latitude = slope - latitude with dial center to the pole
(but reverse am/pm hours, and longitude correction)
hour line angle $=\arctan (\cos$ (latitude) * tan (15 * hour) )
or use the tables in appendix 3 tables A3,1a, A31.b, A3.1c - vertical index

## EAST AND WEST RECLINERS

The latitude of the "design dial" is:- $=90-$ the latitude of the actual dial and
The declination of the "design dial" is:- =90-the reclination of the actual dial or $=$ the inclination of the actual dial and The dial plate is rotated $90^{\circ}$ in latitude, so on the new dial noon is horizontal, not vertical and The dial is rotated $180^{\circ}$ around the pole, so on the new dial the hours lines are mirror imaged

## Atkinson's Theorem

It has been mentioned that a horizontal dial designed for a latitude other than the dial's location can be corrected by rotating the dial up or down on the east west axis line, for example by placing a wedge at the north end or the south end. And, just as this was the basis for the south facing reclining dials, the question arises about longitude corrections.

It has been emphasized that rotating hour lines does not correct for longitude except in the case of the equatorial and armillary dials. All other dials that need a longitude correction to derive standard time, need their hour angles to be adjusted before the design is done, and this is done hour by hour.

Atkinson's Theorem states that a longitude correction can be made by rotating a dial around the polar axis, in other words, rotating the entire dial around the style of the gnomon.

## CASE STUDY ~ AN INCLINING MERIDIAN DIAL, LONGITUDE CORRECTED

The objective of this dial was to have a true east, true west meridian dial that inclined up, or reclined back. This dial was for latitude $33^{\circ}$ degrees, and longitude corrected for longitude $108.2^{\circ}$ with a legal meridian of $105^{\circ}$.

A DeltaCAD macro was used and its results tested against other programs such as the Excel spreadsheets.

- An inclining true east/west dial has the same design hour line angles as a vertical decliner
- whose latitude is the co-latitude of the meridian dial,
- whose declination is the inclination of the meridian dial,
- whose dial plate is rotated by 90 degrees so noon is horizontal not vertical,
- whose vertical axis is rotated so the am hours of one are the pm hours of the other

So, these dials for about latitude $33^{\circ}$, inclined up by $45^{\circ}$, should match a vertical dial for latitude $57^{\circ}$ with a declination of $45^{\circ}$.


East face

. 44.3 - 36.1 - $29.6-23.8-17.9-11.1-2.3$

| Lat: | 57.3 | Long: | 108.2 |
| :--- | :--- | :--- | :--- |
| SD: | -24.5 | $\mathrm{SH}:$ | 22.5 |

Above to the left is the DeltaCAD programmed drawing for the 45 degree incliner at latitude 32.7, and above to the right is a vertical decliner for latitude 57.3 declining 45 degrees. The vertical decliner hour line angles from vertical match the inclining meridian dial's hour line angles from horizontal. This is the results of the latitude:co-latitude relationship. The hour lines are mirrored around noon, this is because the dial when moved $90^{\circ}$ of latitude is on the other side of the planet, or $180^{\circ}$ and thus mirrored. The noon line is a bit above the horizontal line for the east facing dial as this dial is west of the legal meridian, so noon must be indicated earlier.

The dial plates were printed on normal $81 / 2$ by 11 inch paper and cut out, and the dial center part ignored so that the dial plate area would be better used.


The dial on the left side of the adjacent picture shows the east facing or morning dial.

The dial on the right side of the adjacent picture shows the west or afternoon dial. And the noon line is not shown, it being below the horizontal line. Of course, the dial could show that noon line, however the total dial plate size was limited since it had to fit on an $8 \times 8 x 8$ block. The reason the noon line was below the horizontal was the same reason the east facing dial had the noon line above the horizontal. Namely, that as this physical dial was west of the legal meridian, legal noon happens earlier that at the westerly location. Thus, noon must be depicted earlier because this dial was longitude corrected.

Tiny holes were punched for the sub style line (SD), the style line (SH), and for the hour lines. The reason that the sub style line is marked is obviously because it is needed to complete the dial. The reason that the style is marked, albeit with shorter and smaller lines, is so that the gnomon template will be valid after the clay has been dried, bisque fired, and glazed, all of which introduce shrinking.

The dial plates have the design latitude marked for obvious reasons. The longitude is marked to indicate that these are longitude corrected. EOT correction is still required.

Each dial has a vertical line marked close to the edge for alignment when the dial plate is affixed to the final sloped surface.

Slips used were Bermuda green, buttercup yellow, and robin's egg blue, all on a whitish or cream slip of titanium dioxide. A woodland brown was used to highlight the hour lines and the Roman numerals marking them. A sky blue dark slip was used to highlight the latitude and longitude data. The bisque firing was at cone 06 (not cone 6). The glaze used was a clear transparent satin glaze, fired at cone 6.

The dial faces rest on an $8 \times 8 \times 8$ which was cut symmetrically on one corner. To the right is the final dial after assembly looking from the south west to the north east, showing the afternoon dial plate. The clay dial plates took less than 3 hours including the CAD design and printing. They took several days to dry, and two firings which take about a day each. Where the dial plate halves were joined, consideration was made for the size of that joint. This was done by pairing down that diagonal cut when the clay plates were leather hard.

Final assembly of the dial plates with grout, and mortaring the $8 \times 8 \times 8$ block to a $2 \times 12 \times 12$ concrete paver took less than a couple of hours, a fitting addition to a secluded garden paradise.


## CHAPTER SIXTEEN

## The vertical declining dial, mostly facing the equator or pole

## hint: read this chapter quickly first, then re read it

- A vertical decliner mostly facing the equator or the pole.
- The first technique discussed uses an auxiliary horizontal dial. After some discussion on the gnomon, another method using an auxiliary equatorial dial is discussed in detail.
- The concept for vertical decliners of rotating the gnomon's sub-style so it is no longer vertical, and why one would want to do that.
- Geometrical and trigonometric methods are shown.


The vertical dial can decline a little, or a lot. If the declination is small then an auxiliary horizontal dial can be used to facilitate the vertical decliner dial design. Tables from appendix 5 can also provide the information.

The geometrical design method first discussed here uses an auxiliary or surrogate horizontal dial. The gnomon is vertical for design purposes of the hour lines. The style is obviously polar axis aligned. The hour lines on the vertical decliner dial are matched to align with those of a horizontal dial turned away by the wall's declination.

Another method uses an auxiliary or surrogate equatorial dial whose $15^{\circ}$ lines are used directly, however this method requires the gnomon's sub-style to be rotated by an angle SD (style distance). This second method offers some benefit over the first, and is discussed after a discussion of rotating a gnomon's sub-style by a distance SD.

While a gnomon can have a vertical sub-style which simplifies the vertical decliner dial geometric design method using a surrogate horizontal dial, in practice the gnomon is almost always rotated so the sub-style is no longer vertical. The gnomon's style still points to the pole, and is still angled at latitude, however the sub style is rotated around the dial center by an angle called the style distance, SD, making the base of the sub-style the shortest distance from the nodus which makes calendar line design simpler. When this rotation is performed, the gnomon's angles are no longer latitude and co-latitude, they use a "style height" angle, SH , instead.

NOTE: Measuring true north/south is doubly important for vertical decliner dial design because not only does the installation itself need accuracy, but the hour line angles themselves depend on wall declination. Chapter 6 discusses finding true north.

## FOR LARGER ANGLES OF WALL DECLINATION WHERE THE DIAL CENTER IS NOT READILY ACCESIBLE SEE THE GREAT DECLINER CHAPTER 17

## SELECT A DESIGN METHOD ~ the following pages describe most of these options



## A VERTICAL DECLINER MOSTLY FACING THE EQUATOR (or pole)



Look at the portable sundial to the left. While all sun dials including vertical decliners may be designed using a surrogate equatorial dial, what if a south wall isn't quite south? While we can use a projection of the equatorial dial, in some cases it may actually be simpler to use a horizontal auxiliary dial.

Looking at the dial pictured to the left, while the circular shapes of the vertical dial plates may obscure the fact, nonetheless it can be seen that were you to extend the hour lines of the vertical dial then they would meet those of the horizontal dial assuming they shared a common gnomon.

Imagine further that the vertical dial was twisted (declined) out of alignment about the vertical dial's vertical axis. And imagine that the horizontal dial was extended to meet that twisted or declined vertical dial, then the hour lines would still have to meet, assuming they still shared a common gnomon.

In the figure below a vertical dial such as a wall of a building has not been aligned with true south. Where I live, the early builders used true north, then magnetic north (which varies by the year), and then whatever they felt like. Interesting, yet challenging for a dialist.


The degree of wall twist is called the wall's "declination". It is always measured from south or north, with the declination being what you would face with your back to the wall. This wall declination has nothing to do with magnetic declination (which both aviators and mariners wisely call variation), nor the sun's declination above or below the equator.

There are many right ways to design the hour lines, and early writers developed complicated methods which became their pet way of doing things. In this book the intent was to show only the intuitive or interesting methods. For that reason, we focus on using a surrogate horizontal dial which is more intuitive than using a surrogate equatorial dial.

## The vertical decliner empirical model.

The empirical method for vertical decliners is to place the gnomon vertical and with the style polar aligned. Attach a trigon as discussed in the early chapters, and from thence run 15 degree lines to the wall, and that point of intercept to the dial center becomes the associated hour line.

## A GEOMETRIC METHOD TO DERIVE THE VERTICAL DECLINER'S HOUR LINES USING A SURROGATE HORIZONTAL DIAL.

The simple model is shown below. Everything is in one pictorial. First draw the gnomon DEC as shown on the right, angle ECD is the latitude. In the right hand figure below, measure the lengths of ED and EC as they are used in the left hand pictorial below.


1. draw a gnomon for the latitude (top right)
2. transfer DE to the vertical decliner's vertical line (top left).
3. transfer EC but rotate it from the vertical by the declination
4. draw the surrogate h -dial hour lines for latitude 32.75, for example, 0900 and 1500 use 28.41 degrees (from a spreadsheet, no longitude correction applied, but it can be), points C9 and C3 are the points of intersection, EC is the surrogate dial's north south line.
5. draw lines from the vertical decliner's dial center D to the intersection points and measure their angles, they are 46.47 and 37.72 , and by calculation they should be 45.52 and 37.25 , close for a rough draft

Angle DEC on the left hand side is a projection of the 90 degrees from the gnomon, its depicted angle is irrelevant. For example, the design method of a pure south vertical dial is the same as this except the wall declination is zero degrees. The depicted angle DEC would show as 180 degrees, a straight line, which is what it would appear to be if both dials were aligned properly and you looked at them from true south, even though DEC would still actually be 90 degrees. Actual hour lines for 3 hours from noon are shown, with 12 noon as implicit in the gnomon line CE. These are not 15 degree lines, they are hour lines of the appropriate angle for the horizontal dial's latitude. They may or may not have been adjusted for the dial's longitude.

This is an intuitive process. This method does not use raw $15^{\circ}$ solar hour angle radials since they were considered when the surrogate horizontal dial was designed. This method uses the wall declination directly. The style is aligned at latitude with true north/south. For the purposes of this design method, the sub-style is vertical.

Once the dial plate's hour lines are drafted, it is common practice to rotate the gnomon's substyle from the vertical by what is called the sub-style (angular) distance (SD). This is done for a variety of reasons which simplify dial furniture such as calendar information. And for the angle between the style and sub-style uses the style height (SH), and not the dial's co-latitude. SD and SH are discussed in the following pages.

## A GEOMETRIC METHOD TO ROTATE THE SUB-STYLE BY THE STYLE (ANGULAR) DISTANCE, SD. (Optional for surrogate h-dial method.)



The method shown above develops the sub style's rotation (style distance) from the vertical by moving $C$ to $Q$. The benefit is simpler dial furniture construction later on.

Pictured to the right, draw the gnomon (side view left). Show the top view (right) angled by the declination CBD. This process moves point $C$ to $Q$ so $Q$ is the shortest distance from nodus point A. $C Q=X Y$ is the horizontal displacement used, and angle CBQ is the gnomon rotation angle (SD).


Our objective is to rotate the gnomon so the sub-style is no longer vertical by placing the nodus the shortest distance from the dial plate (moving point $C$ to point Q ), retaining the style's polar alignment, and creating a gnomon with no need for chamfering on the edge BC.

1. draw a normal gnomon, style $A B$, e.g. latitude 32.75 , side view is left side
2. draw the wall's declination as $B D$, e.g. declination 15 , top view is right side
3. mark on $B D$ point $Y$ such that $B Y=A C$, this is the horizontal part of the gnomon that is neither the style nor sub-style
4. from $Y$ draw a line $Y X$ where $Y X$ meets $B C$ extended at 90 degrees, this is the linear horizontal displacement needed.
5. take line $X Y$, the horizontal displacement, back to the original horizontal line of the gnomon as CQ, not because will alter the nodus to dial pate line, but rather because it, along with the vertical, will form the angle of gnomon rotation, or style distance SD
6. draw a line from $B$ through $Q$, and this angle is the style angular distance of rotation, $S D$. Measure it, it shows 21.55 degrees of rotation. The actual result should be 21.92 , close for a quick drafting. BQ is in fact the new sub-style.

If you rotate the gnomon which most people do, then you must also derive new angles in place of the latitude and co-latitude, the new angle needed is the style height or SH discussed next.

## A GEOMETRIC METHOD TO FIND THE STYLE HEIGHT (SH), THE ANGLE BETWEEN THE STYLE AND THE SUB-STYLE. This is no longer co-latitude. If you developed the style distance (SD), then this part is mandatory.

This style height derivation is part and parcel of rotating the gnomon's sub style by a distance SD. If you rotate the gnomon's sub style from the vertical line, by the previously derived angle SD, then you must also adjust the gnomon's angles such that latitude and co-latitude are no longer used. Consider the figure below:-


Having rotated the gnomon from the vertical, by an amount called the SD, or style distance, the gnomon's geometry is changed. Latitude and co-latitude used in a normal gnomon no longer apply. In their place a new angle is needed, this is called the style height, an angular measure, abbreviated to SH . Line BQ is now the new sub-style, angle QBC is the style distance or SD.

1. Draw a line from $Q$ whose length is equal to $B X$ to a new point $R$ until it touches the semicircle, in fact angle BQR will be $90^{\circ}$ and if not, you made a mistake.
2. Angle QBR is the new style (angular) height, " SH " and is the angle between the style (polar latitude aligned) and the sub-style (wall attachment). This angle replaces what was previously the co-latitude. This shows $54.27^{\circ}$, which compares well to a calculated $54.33^{\circ}$
3. Line $B R$ is the new style, and $B Q$ is the sub-style attached to the wall, offset from vertical by linear distance QC, giving the previously derived style distance SD, angle QBC.

For latitude 32.75, declination 15, the derived SD of 21.55 and SH of 54.27 are very close to the calculated SD of 21.92 and SH of 54.33

| lat | 32.75 | decl | 15 |
| :--- | ---: | :--- | ---: |
| SD | 21.92 | SH | 54.33 |

That is all there is to the gnomon rotation.

## A GEOMETRIC METHOD OF USING AN EQUATORIAL DIAL $15^{\circ}$ RADIALS TO DEVELOP THE VERTICAL DECLINER'S HOUR LINES. This technique requires a gnomon rotated by SD and with SH in place of co-latitude.

Drawing the hour lines is as follows. First, BN is the part of the gnomon, sub-style, attached to the vertical yet declining wall. Angle NBZ is the style angular height, termed style height or " SH ". MBN is the style angular distance termed style distance or "SD". Here, height and distance are angular, whereas in some other cases they may sometimes be linear.


1. Having established the rotated gnomon NBZ, draw a line from N (an equinox ray of light), towards BZ , intercepting it at $90^{\circ}$ at place E . So, BE is a style.
2. Draw a line ZNF perpendicular to line BN, BN is the extension of the sub-style. This new line ZNF is the equinox line if one is desired.
3. From $N$ take the line $N E$ and rotate the point $E$ around $N$ to intercept the extension on $B N$ at point J.
4. Draw a circle meeting the line $Z N F$ at point N , whose radius is JN which is equal to EN . This is an equatorial dial.
5. Draw a line from the circle's center to meet the noon vertical line ( $B M$ extended to $K$ ), this is the line JK. Line JK is the base line for noon on the circle or equatorial dial. You may now adjust JK for longitude correction if so desired.
6. Draw 15 degree lines on either side of JK, these being 11 am and 1 pm hour points when extended to meet line ZNF, which in turn provide the hour lines when connected to the dial center "B".
7. The 1 pm line is shown. Noon line on the circle is JK, thus 15 degrees forward is JS, and the line SB is the hour line for 1 pm for the vertical decliner. Other hour lines are found similarly.

The above wall is declining east which is why the gnomon is set to the west part of the dial. The hour line angles may be checked with a table or spreadsheet and they will be found to match closely.

## THE TRIGONOMETRIC METHOD OF SUB-STYLE ROTATION BY STYLE DISTANCE (SD), WITH A NEW STYLE HEIGHT (SH), AND WITH HOUR LINES AS WELL.

This trigonometric method rotates the gnomon's sub-style from the noon vertical by an amount termed the Style Distance, or SD. The vertical noon line is still used as the base for the hour lines. The extended rotated sub-style of the gnomon can be used to determine the equinox line's slant since with declining dials, it is no longer horizontal line. Appendix 5.1 has the tables for the hour lines for various latitudes. Below is an extract fine tuned for latitude 32.75.

| LATITUDE: | 32.75 |  | This table gives the hour line angles from the vertical. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| TIME | DEC |  | South xx degrees East ~ wall faces south east |  |  |  |  |  | SOUTH |  |
| hh.hh | -45 | -40 | -35 | -30 | -25 | -20 | -15 | -10 | -5 | 0 |
| 6.00 | $\begin{aligned} & -65.5 \\ & -37.7 \end{aligned}$ | -67.5 | -69.7 | -72.2 | -74.8 | -77.6 | -80.5 | 83.6 | 86.8 | 90.0 |
| 9.00 |  |  |  |  |  | \% 8 | -37.3 | -37.9 | 38.9 | -40.1 |
| 12.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| STYLE:SD | -47.7 | -45.0 | -41.7 | -37.9 | -33.3 | -28.0 | -21.9 | 15.1 | -7.7 | 0.0 |
| STYLE:SH | 36.5 | 40.1 | 43.5 | 46.7 | 49.7 | 52.2 | 54.3 | 55.9 | 56.9 | 57.3 |

For a wall declining South 15 degrees East, the 0900 line has an angle with the vertical of minus 37.3 degrees. The SD and SH are -21.9 and 54.3 respectively. Here, negative angles are west of the vertical, positive angles are east of the vertical, however, use common sense to cross check your work. For proof of the following formulae, see later in this chapter as well as in appendix 7.

Note that both SD (style distance) and SH (style height) are angular and not linear distances. The trigonometry formulae are:-

Style height in place of co-latitude:
The gnomon sub style distance:
The hour line angle from vertical:

```
sh \(=\operatorname{asin}(\cos (\mathrm{lat})\) * \(\cos (\mathrm{dec})\) )
sd = atan( sin(dec) / tan(lat) )
\(z=\operatorname{atan}(\cos (l a t) /(\cos (d) \cot (h a)+\sin (d) \sin (l a t))\)
```



This model allows the sub-style to be rotated by a style angular distance (SD) which helps with subsequent calendar lines. Once rotated, the co-latitude is not used, in its place the derived style angular height (SH) is used. This model is the most general, covers wall declinations of any magnitude, and is the most practical for computer programs. This is the trigonometric version of the geometric method described earlier.

## PROOF OF DECLINER/GREAT DECLINER SD \& SH ANGLE (Tables A5.1, A5.2)

In the proof and pictorial that follows, please note where the right angle is. Also, in the math that follows, "dec" (wall declination) is the normally defined wall declination using the SxxxW terminology.


TCB is a vertical gnomon whose style is latitude and north/south.

TQB is a rotated gnomon whose style is latitude and north/south.
[SD]

$$
\tan (S D)=C Q / T C
$$

Style Distance
(formula A.22)

Style Height
$[\mathrm{SH}] \quad \sin (\mathrm{SH}) \quad=\quad \mathrm{QB} / \mathrm{TB} \quad$ Style Height
thus $\quad \mathrm{SH}=$ style height $=\operatorname{asin}(\mathrm{QB} / \mathrm{TB})$
and $\quad \cos ($ lat $)=\mathrm{CB} / \mathrm{TB}$ thus $\mathrm{TB}=\mathrm{CB} / \cos$ (lat)
and $\quad \cos (\mathrm{dec})=\mathrm{QB} / \mathrm{CB}$ thus $\mathrm{QB}=\mathrm{CB} * \cos (\mathrm{dec})$
and given $\quad \mathrm{SH}=$ style height $=\operatorname{asin}(\mathrm{QB} / \mathrm{TB})$
then $\quad \mathrm{SH}=\operatorname{asin}(\mathrm{CB} * \cos (\mathrm{dec}) /(\mathrm{CB} / \cos (l a t)))$
so $\quad \mathrm{SH}=\mathrm{asin}(\cos (\mathrm{dec}) \quad$ * $\cos (\mathrm{lat})$ )
(formula A.23)

NOTE: The above is the Illustrating Shadows derivation of the hour angle and SD and SH for a vertical decliner. This formula differs from the more common formula on the next page which summarizes the vertical decliner techniques, however the results are the same.

## PROOF OF ONE FORMULA FOR DECLINING DIAL HOUR LINE ANGLES

In the stylized figure to the right below, the triangle "h-dial" has sides "b", and "c" and angle "h". Side "b" is the horizontal dial's sub-style and a selected horizontal dial's hour line angle "h". Triangle "v-dec dial" has two named sides, with angle " $n$ " being the vertical decliner's equivalent hour line angle that is associated with the horizontal dial's angle " $h$ ". Both dials share a style that connects the h-dial and v-dec dial centers, shown by a depicted dashed line. The vertical decliner's sub-style is not depicted in the figures below, and their "SD" and "SH" (style angular distance and angular height) are derived elsewhere. Declination is "d" and " $\varnothing$ " is latitude.

$\mathrm{a}=\mathrm{b} * \tan (\varnothing)$
$\tan (n)=e / a \quad$ thus $\ldots$
so $\quad n=\operatorname{atan}(e / b * \tan (\varnothing))$
$e / \sin (h)=b / \sin (180-(h+(90+d)))$
so $\quad e / \sin (h)=b / \sin (90-h-d)$
thus
$e=b * \sin (h) / \sin (90-h-d)$
$n=\operatorname{atan}(e / a)=\operatorname{atan}\left[\frac{b * \sin (h) / \sin (90-h-d)}{b^{*} \tan (\varnothing)}\right]$

SO
$\mathrm{n}=\operatorname{atan}$
thus $n=$ atan

$\left[\frac{b^{*} \sin (h)}{\sin (90-h-d) * b^{*} \tan (\varnothing)}\right]$

$$
\begin{equation*}
\left[\frac{\sin (h)}{\sin (90-h-d)^{*} \tan (\varnothing)}\right] \tag{6}
\end{equation*}
$$

[ 2 ] desired
[ 4 ] law of sines
[ using 2, 5, 1 ]
and using

$$
\mathrm{h}=\operatorname{atan}(\sin (\varnothing) * \tan (\text { sun hour angle }))
$$

and since from the prior page

$$
\mathrm{n}=\operatorname{atan}(\sin (\mathrm{h}) / \tan (\varnothing) * \sin (90-\mathrm{h}-\mathrm{d}))
$$

[ from 6]
then using the sun's hour angle as opposed to a surrogate horizontal dial's hour line angles
then

$$
n=\operatorname{atan}\left[\frac{\sin (\operatorname{atan}(\sin (\varnothing) * \tan (\text { sun hour angle }))}{\tan (\varnothing) * \sin (90-d-\operatorname{atan}(\sin (\varnothing) * \tan (\text { sun hour angle })))}\right]
$$

Hence, considering a spreadsheet or a procedural program implementation of a vertical dial that declines, it has hour line angles " $n$ " equal to:-
i.e. $\quad n=$

DEGREES(ATAN( SIN(RADIANS((DEGREES(ATAN( TAN
(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat)) ))))) /
(TAN(RADIANS(lat))*SIN(RADIANS(90-dec-
(DEGREES(ATAN( TAN(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat))
)) $)$ )) $)$ )
where the hour itself, and longitude corrections are all considered.

The results match the formula usually published which is:-

$$
\mathrm{n}=\quad \operatorname{atan}(\cos (\varnothing) /(\cos (\mathrm{dec}) \cot (\mathrm{ha})+\sin (\mathrm{dec}) \sin (\varnothing) \quad)
$$

The Illustrating Shadows formula derived above is used in one of the vertical decliner sheets in:-
illustratingShadows.xls
and the standard common formula is also used in another vertical decliner sheet. The index of sheets as well as the individual sheets themselves clearly state which formula is used.

## PROOF OF DECLINER/GREAT DECLINER DL ANGLE (Tables A5.1, A5.2, A5.3)

DL has not been discussed until now, and this is somewhat jumping ahead, and is not used in this chapter. It is included here for subsequent completeness. The preceding pages developed SD and SH for a vertical decliner. In this context, SD is the angle from the gnomon base to local noon, and relates to "DL" which is an equivalent difference in longitude that would generate a dial plate shift, as discussed in chapter 25.

SD is in essence the place where a surrogate horizontal dial's local noon line would be aligned. And as such is an offset for that surrogate horizontal dial's hour lines so that the hour lines of the surrogate horizontal dial would match those of the vertical decliner. Along with that procedure comes the correct alignment of declination or calendar curves, and also the analemma, this is discussed in chapter 25.

Given that for a vertical decliner:-

$$
\begin{equation*}
\text { SD } \quad=\quad \operatorname{atan}(\sin (\operatorname{dec}) / \tan (\text { lat })) \tag{formulaA.22}
\end{equation*}
$$

and
$\mathrm{SH} \quad=\quad \operatorname{asin}(\cos (\mathrm{dec}) \quad * \cos (\mathrm{lat}))$
(formula A.23)
where SH is the latitude for a surrogate horizontal dial, and SD is an angle between the surrogate horizontal dial's local noon and the desired noon placement.

The surrogate horizontal dials hour lines are thus:

```
shdhr = atan ( sin(SH) * tan (hour angle))
```

and for local noon on the vertical decliner, which is some unknown hour on the surrogate horizontal dial:-
$\mathrm{SD}=\quad \operatorname{atan}(\sin (\mathrm{SH}) * \tan ($ hour angle $))$
substituting SD we get:-

```
atan (\operatorname{sin}(\textrm{dec})/\operatorname{tan}(\textrm{lat}))=\operatorname{atan}(\operatorname{sin}(\textrm{SH})*\operatorname{tan}(\mathrm{ shd hr angle) )}
```

thus we can remove atan on both sides, thus:-

```
sin(dec) / tan(lat) = sin(SH) * tan (shd hr angle)
```

substituting SH we get:-

```
sin(dec) / tan(lat) = sin(asin ( cos (dec) * cos(lat) ) )*tan (shd hr angle)
```

thus:-
$\tan ($ shd hr angle $)=\frac{\sin (\mathrm{dec}) / \tan (\text { lat })}{\sin \left(\operatorname{asin}\left(\cos (\mathrm{dec})^{*} \cos (\mathrm{lat})\right)\right)}$
$=\frac{\sin (\mathrm{dec})}{\sin \left(\operatorname{asin}\left(\cos (\mathrm{dec})^{*} \cos (\mathrm{lat})\right)\right)}{ }^{*} \tan (\mathrm{lat})$
thus:
shd hr angle $\quad=\quad \operatorname{atan}\left\{\frac{\sin (\mathrm{dec})}{\sin (\operatorname{asin}(\cos (\mathrm{dec}) * \cos (\mathrm{lat}) \quad) \quad * \tan (\mathrm{lat})}\right\}$
given:-

$$
\text { shd hr angle } \quad=\quad \operatorname{atan}\left\{\frac{\sin (\mathrm{dec})}{\sin (\operatorname{asin}(\cos (\mathrm{dec}) * \cos (\mathrm{lat}))) * \tan (\mathrm{lat})}\right\}
$$

and that the hour angle (shd hr angle) is equal to the difference in longitude that would cause this surrogate horizontal dial to displace the local noon so it would match the original vertical decliner's local noon. In other words, one hour (15 degrees) of hour angle is also the same as 15 degrees of longitude difference:-


NOTE: The above formula "DL" will ensure that local noon on a surrogate dial will be overlay local noon on the vertical decliner. If legal noon is desired, then the vertical decliner and the surrogate horizontal dial must consider dial longitude, so, the dial longitude difference from the legal meridian would be considered in both cases, as discussed in chapter 25.

NOTE: The DeltaCAD macro that draws analemmas for a dial has a limited range of analemmas, so if that DeltaCAD macro is used, you should reduce the absolute value of DL so that DL is:

$$
15<\mathrm{DL}>-15
$$

and this is only to ensure the DeltaCAD produced analemmas are usable, and for no other purpose. In other words it manages a weakness in our DeltaCAD analemma macro. Again, this is discussed in chapter 25.

## VERTICAL DECLINER DESIGN METHODS (PREDOMINANTLY SOUTH FACING DIAL)

Empirical: Build a gnomon whose style aligns true north and at latitude.
The gnomon's sub-style affixed to the wall is vertical.
Place a trigon on the style and project 15 degree arcs to the wall.
Where the point is on the wall, join to dial center.
That line is an hour line.
Geometric: (1) Draw a line below the horizontal, angled at wall declination Draw an auxiliary horizontal dial angled to meet that declined line Extend the horizontal hour lines to the new dial or ...
(2) Rotate the gnomon thus establishing a... ...style distance (angular) and style height (angular)
Project the gnomon sub-style line and form a circle The circle's center meets the vertical noon line and a line perpendicular to the gnomon's style
Intersections of 15 degree radials form the vertical decliner's
hour lines - this is an auxiliary equatorial dial method
Trigonometric: (1) Use of the tables in Appendix 5.1 for the latitude
(2) Use the formulae:

The hour line angle from vertical:

$$
z \quad=\operatorname{atan}(\cos (l a t) /(\cos (d) \cot (h a)+\sin (d) \sin (l a t)))
$$

The gnomon style distance:
SD $\quad=\operatorname{atan}(\sin (\mathrm{dec}) / \tan (\mathrm{lat})$ )
The style height:
SH $=\operatorname{asin}(\cos (l a t) * \cos (\mathrm{dec}))$


## CASE STUDY ~ A SOUTH DECLINING WEST VERTICAL DIAL

A predominantly south facing wall was in need of a vertical dial so that the passers by could determine the time.

The dial location was $32.75^{\circ}$ north, and west $108.2^{\circ}$ with a legal time meridian of $105^{\circ}$. The first objective was to determine the wall's declination, and three methods were selected.

## Astro-compass determination of wall declination

The process is simple, the sun's hour angle assuming $15^{\circ}$ per hour was calculated for the time of observation. Then the longitude correction plus the EOT set to degrees (EOT divided by four) was subtracted from that sun's hour angle. This is the opposite of the correction made when reading a sundial. The astro-compass was set, and placed on a plane such as a wooden plank. The astro-compass was rotated until the shadow was centered. The angle of the astro-compass axis was projected onto the wood, I used a projection of the leveling mechanism. The angle that the projected line made with the wooden plank's edge paralleling the wall determined the wall declination.

In practice the illustratingShadows.xls spreadsheet or the table A2.2a in the appendices simplifies this process.

For the wall in question several readings showed a wall declination of $S 5^{\circ} \mathrm{W}$. Chapter 6 expands on this method of true north determination.


## Magnetic compass determination of wall declination

The magnetic compass was placed on a plane such as a wooden plank, in this case some concrete pavers were used since they were there. The compass was set to project a perpendicular line to the wall being measured.

The compass needle showed about 355 degrees. The dial location had a magnetic declination (variation to navigators and pilots) of 10.2 degrees easterly. Thus the true bearing of the wall's perpendicular was 365 or 005 degrees. The reciprocal bearing of $005^{\circ}$ is $185^{\circ}$. Thus the wall declined S $5^{\circ} \mathrm{W}$.

An earlier chapter expands on this method of true north determination.

This agreed with the astro-compass method.

## Measured versus calculated azimuth method



The illustratingShadows.xls spreadsheet allows a time in hours and minutes to be entered together with a nodus shadow $x$ and $y$ coordinates. The "x" coordinate is used to derive the detected azimuth. If desired, the " $y$ " value is used for a graphical display of the shadow tip as a double check.

Chapter 6 expands on this method of true north determination.
This method showed a little under $5^{\circ}$, which was in general agreement with both the astro-compass method as well as the magnetic compass process.


## Points to consider when determining wall declination

Chapter 6 provides some insights on matters that can cause confusion when determining wall declination. In essence, it is good practice to make calculations before hand, develop a script, and follow it. It is possible to confuse the sense of a wall declination if care is not taken to ensure magnetic declination is understood.

The final decision was made to assume a South $5^{\circ}$ West wall declination.
The next step was to determine the hour line angles and the SD (style distance) and SH (style height). The illustratingShadows.xls spreadsheet does this, so do various programs on this book's web site. Programs on the web site cover DeltaCAD, C, FORTRAN, Pascal, Visual Basic, Basic, and java. And all of them have free compilers available which the website directs you to, except for DeltaCAD. In this case a DeltaCAD macro was used.

To the right is shown the DeltaCAD depiction using the vertical decliner macro. The hour lines are shown as well as SD and SH.

The next step was to make a mockup dial, and affix it to the proposed wall, and test it.



Hour and hour line angle VERTICAL DECLINER

$$
\begin{array}{lllllllllllll}
6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18
\end{array}
$$

83.0 -79.2-61.5-44.9-29.7-15.7-2.7 $09.922 .635 .950 .366 .0 \quad 83.0$

| Lat: | 32.8 | Long: | 108.2 | Dec: | -5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| SD: | 07.7 | $\mathrm{SH}:$ | 56.9 |  |  |
| SW l* $^{*}$ |  |  |  |  |  |

The dial tested correctly. This test is a wise idea because it is somewhat disheartening to build a dial and find out that there was an "opportunity for improvement" after hours of dial plate construction.

Next, a DeltaCAD macro was used to draw the calendar curves. This program is nothing more than the horizontal dial with calendar curves, except it is tailored just to calendar information.

The "latitude" is replaced with the style height (SH). There is no longitude correction. And the style height is taken from the first dial plate. The drawing has some hour lines which are meaningless, except they can be used with their intersection points on calendar curves, to transcribe those points to the final dial.

Here the "noon" line of those calendar lines is aligned with the style distance (SD) or the regular dial plate, as seen on the next page.


Calendar curves using SH (style height) 56.9
Align central hour X here, with final dial's SD.
The 'hours' on this plate, other than ' X ', are meaningless. they may be used for rescaling curves on paper.
Declinations used are: $23.5 \quad 20.0 \quad 11.0$

Then, the real dial plate is transcribed to a final sheet of paper, and the calendar depiction is rotated so that its noon line matches the style distance of the first plate, and that information transposed also.


The hour lines on the calendar plate are meaningless as far as hour lines go. However, they are very useful in transcribing as well as rescaling the calendar curves. When transcribing those curves, remember that the "hour lines" on the calendar depiction do not match hour lines on the actual dial plate. Note that the nodus location is critical and specific to the depicted calendar curves. The gnomon (style) may be of a different size provided the nodus is marked with a notch or other means of depicting the nodus shadow on a calendar line.

The main thrust of this web site and these books is educational, the focus is on showing the "how" of dial design. If you desire a more turnkey operation, consider using SHADOWS (which while not affiliated with this web site and series of books) which provides an excellent depiction of final dial plates.

The end result is a paper sheet the size of the final dial plate. This will be used to make impressions upon the soft clay.

This dial plate will fit in a surround of miscellaneous pieces of clay that were fired but for one reason or another never used.

For aesthetics, a strip and a crown


A slab of clay was cut from supplies and rolled. This dial plate area measured 11 inches vertically, and 15 inches horizontally.


The upper half of the dial plate has a diagonal cut along the style distance.

The lower half has the diagonal cut along the noon line.

Then some extra pieces were cut so they would look nice while fitting with kiln firing dimensions.

The equinox line, hour lines, and Italian lines will, along with their lettering, be made with a slip or an under-glaze.

The Italian lines were drafted based on two data points, which is conventional. One data point was that sunset happens on the equinoxes at 6 pm local apparent time (L.A.T.), and that the solstice times may be found in the appendices.

This dial plate was longitude corrected, the dial being at longitude 108.2 W and the legal time meridian being at 105W, the difference was 3.2 degrees or 12 minutes 48 seconds.

Since the dial plate was longitude corrected, the sunset times should also be longitude corrected. Thus the equinox sunset time would be $6: 12 \mathrm{pm}$. And the winter solstice time of $5: 57 \mathrm{pm}$ would become 5:09 pm. Fro these two times the sunset line is drafted, and those times backed off by an hour form the 1 hour before sunset time Italian hour line. And so on for the other hours.

The bisque or slip firing was done at around $1800^{\circ} \mathrm{f}$. After the glaze firing which was over $2000^{\circ} \mathrm{f}$, but less than cone 6, the assembly was placed on foam board with foam board adhesive and the same adhesive used to fix the board to the wall. Stucco was then applied.


It just didn't look right however, this was because the foam board was offset out from the wall. The solution was to distract the eye.

So a vine was painted as a surround, this added cohesion to the objects on the wall, and removed the effect of the foam board pushing out those tiles from the wall.

## THE SAME VERTICAL DECLINER BUT NORTH FACING ~ S $175^{\circ} \mathrm{E}$ or N5 ${ }^{\circ}$ E

The north side of the same building can be used for a vertical dial, useful during the summer months. In the northern hemisphere the EOT for a pure north dial would be in the range of $+/-7$ minutes. The relationship of the $\mathrm{S} 5^{\circ} \mathrm{W}$ and $\mathrm{N} 5^{\circ} \mathrm{E}$ vertical decliners is shown below.

VERTICAL DECLINERS POLE FACING

VERTICAL DECLINERS SOUTH FACING


A variation of the vertical declining DeltaCAD program was used, this dial declines $\mathrm{S} 175^{\circ} \mathrm{E}$ or N $5^{\circ} \mathrm{E}$, the latitude and longitude being the same.


This modified program simplifies dial SD: 07.7 SH: $\quad-56.9$ Rotated 180 around the clock plate orientation and construction, however, the normal vertical decliner methods can be used with minor human manipulation.

The angles of the hour lines to the horizontal were measured using DeltaCAD's measurement tool showing how they match the displayed hour line angle data.

Those angles also match other available software popular with diallists, such as SHADOWS and ZW2000.


The earliest and latest hours would be sunrise and set, see the appendices. The latest morning and earliest afternoon hours would be when the azimuth at the summer solstice crosses the east west line, tables A4.2 book 1. For latitude 32 that would be 0500 to 0900 mst and 1500 to 1900 mst . Although daylight savings time might be better choice, the author has a bias towards standard time. Wall declination and shading of course affect the useful hours displayed.

To the right is "The Skink Dial", between a lizard and a snake, showing 4pm winter time being of course 5pm daylight saving time, a north vertical declining dial.


To the left is shown a CAD vertical dial with the equator facing plate on the lower half, the pole facing plate on the upper. This shows how the SD is deflected to the right (for this wall's declination) for both plates when viewed looking at the plate from the sun's side and how noon or midnight hour line is deflected to the left (for this longitude) for both plates when viewed looking at the plate from the sun's side.

The plate was tested in the morning and afternoon hours and was correct. Then the print out was placed on a slab of clay, and lines traced into the clay with a blunt pencil. The plate was fired as before.


## CHAPTER SEVENTEEN

## Vertical great decliners, face mostly east or west great decliners are ones who declination angle is large and frequently have a dial center that is not easily available

This chapter covers the following topics:

- The great decliner
- What they are
- The gnomon design process
- The hour lines


The great decliner is the vertical dial more closely aligned true east or west than south. Rather than calculate just where else on the planet this dials might exist as a true cardinal dial, or be truly vertical or horizontal, a method is shown of calculating just how to design the gnomon and the hour lines.

As the wall declination approaches a direct east or direct west facing, an auxiliary horizontal dial becomes less and less practical for designing the dial, and an auxiliary equatorial dial becomes more and more practical for constructing hour lines. The dial center moves away, far away, and special techniques are needed to calculate the hour lines to be placed on the dial plate.

The extreme case is when the dial is true east or true west, and that uses exactly the same methods as the polar dial, namely an auxiliary equatorial dial.

The great decliner uses exactly the same techniques as the normal vertical decliner in so far as the gnomon angles go, namely style distance (from vertical) and style height (style to sub-style). However the dial center itself may be miles away from the actual dial, infinity miles if the dial plate faces true east or true west. Thus the only new technique needed is how to project those hour lines and the gnomon from that far away place, the dial center.

NOTE: Vertical declining dials are usually designed using geometry or trigonometry, the empirical method is sometimes used. Sundial programs are of course based on trigonometry.

## GREAT DECLINERS ~ dial center often not accessible

What if the walls declination is excessive, an obvious case would be the meridian dial, namely an oriental or occidental dial facing true east or true west that is off by just a little bit.


Wall viewed from above
The method of using an auxiliary horizontal dial becomes impractical. One method is to build a dial for true east or west and use a wedge to align it with true north. Another method is to build the style, then use a trigon and empirical methods to complete the task. The style installation requires alignment on the wall, at a slope that is less than or equal to the latitude. In the figure below are shown two gnomons. The upper one is aligned with true north and with latitude.

The lower one is a projection, and would be the same gnomon were this dial on the equator. As the angle of latitude increases, the various angles with the wall vary proportionally. Simply put, a style's angle when fabricated with a wall is not the same as the wall's declination. It depends on the wall's declination as well as the latitude. Similarly with its slope.

We must be very specific about the wall's
 declination angle when we construct formulas.
The walls below are $\mathrm{N} 0^{\circ} \mathrm{W}, \mathrm{N} 0^{\circ} \mathrm{E}$, and N 85 W , and also $\mathrm{N} 30 \mathrm{~W}, \mathrm{~S} 60 \mathrm{~W}, \mathrm{~N} 60 \mathrm{E}$, and S 30 E . The declination is named for the direction we face with our back to the wall.

However, when developing our own formulas from first principles, it may be more expedient to use the $15^{\circ}$ rather than the $85^{\circ}$ angle, not because of the size, but because that angle may simplify the math in the formulae, and thus reduce the chances of errors.

## TRIGONOMETRIC OR TABULAR CONSIDERATIONS

The tables in appendix A5.2 use exactly the same formulae as in appendix A5.1, the sole difference is that their declination angles are greater than 45 degrees. Some rules of thumb are:-

VERTICAL DECLINERS
NORTH FACING Gnomon points up, hour lines radiate from bottom


Gnomon points down, hour lines radiate from top


A design for South $x x$ degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. If longitude correction is applied, care must be applied as hour lines shift. The North facing decliner gnomons are inverted, and the vertical is midnight.

Also hours are clockwise when north declining, whereas they are anti clockwise when south declining.

Normal vertical decliner methods, formulae and tables are usable with no change, merely larger angles are used, hence why table A5.2 exists which is otherwise exactly the same as A5.1

The sole complication with a great decliner is that the dial center may literally be miles away from the actual dial plate. Since a dial of such size would increase a person's property taxes, there needs to be a simple way of constructing only a part, a useful part of a vertical great decliner.

The great decliner design can be truly mathematical or geometric, using the methods or formulae of the normal vertical decliner.

## THE GREAT DECLINER WHOSE DIAL CENTER IS NOT ACCESSIBLE THE GENERAL, CORRECT, AND ACCURATE METHOD.

The process in overview is straight forward, merely more time consuming than previous dials. The other factor in the design of a great decliner is that human judgment is needed in order to select the dial plate dimensions, this judgment cannot easily be made into a specific procedure.

1. Define the latitude and the wall's declination.
2. Use tables in Appendix 5, such as 5.1 or 5.2 (or formula A8.21 to A8.24) and develop
a. the hour line angles
b. the style distance (the angle the sub-style is rotated from the vertical noon hour, just as with a declining dial)
c. the style height (the angle between the style and sub-style (part affixed to the wall or surface just as with any declining dial)
3. Draft a dial such as in the lower figure on the next page
4. Determine where in that larger picture the final dial plate will be

You may find that a spreadsheet can simplify the repetitive work of deriving hour line coordinates.
5. Determine the endpoints on the final dial plate of the gnomon's sub-style. This uses very simple trigonometry.
6. Using Pythagoras's theorem as well as trigonometry, determine the dimensions of the gnomon that sticks out of the wall, Pythagoras provides the direct distance from the unreachable dial center, although trigonometry could have been used. And knowing that distance from dial center, trigonometry can be used to find the two distances separating the style from the sub-style.
7. Determine the endpoints of each hour line, just as you did for the gnomon in step 5, this uses very simple trigonometry. This can be done before step 5 if desired.

NOTE: Pythagoras's Theorem, contrary to the scarecrow's words in the Wizard of Oz, states that for a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the two remaining sides.


In the above, hypotenuse squared $=3$ squared plus 4 squared, or 9 plus 16 , or 25 , thus the hypotenuse itself is the square root of that 25 , in other words 5 . The scarecrow got it wrong, he said it was an isosceles triangle, useful trivia (Wizard of OZ).

The following values are derived using standard vertical decliner methods of chapter 16. The appendix tables A5.1 can be used for wall declinations of about 45 degrees, and tables A5.2 can be used for wall declinations of more than 45 degrees, ones that approach true east or true west.

STEP 1: For example, a south 85 degrees west wall, latitude 32 degrees north, derive figures.

2. All needed information is provided. Since the wall faces mostly west, the am hours were dropped. On a large piece of paper, or using a CAD program, draw the gnomon and hours lines.

STEP 3: Near the dial center the hour lines are very bunched up, and the gnomon would be very long if we built the entire complete dial. However, if just a part of the dial were selected, then all would be well with the world. This is where a paper or CAD drawing helps out. Visually select a section of the dial that you believe would be usable.

STEP 4: Here is a visually selected area that we will work with for the final dial.


From the dial center, we selected an area about 8 units to the right of, and about 5.5 units below the dial center. Each hour line, and the gnomon sub-style also, can be defined as two points. So, for each line, all we need is those two points.

## STEP 5: THE GNOMON COORDINATES

The gnomon's sub-style is at an angle SD (style distance) of 57.9 degrees from the vertical. Looking at the chart, about 9 units to the right of dial center.


- tan $($ angle $)=$ horiz $/$ vert
- vert $=$ horiz $/ \tan$ (angle)
- horiz = vert * tan(angle)

note: the $x$ and $y$ box dimensions are not drawn to scale.

Assume the gnomon's sub-style begins say 9 units horizontally from the dial center (1 unit from our selected dial plate area), an arbitrary starting point, the vertical distance for the gnomon's sub-style, for the first point will be:-
vert $=9 / \tan (57.9)=5.65 \quad$ coordinate thus $=$ horiz=9 vert=5.65
as the top left dial plate coordinates are horiz=8 and vert=5.5, the relative position of the first gnomon sub-style coordinate is found by subtracting 8 and 5.5 respectively, thus:-

1 unit right from the top left (the 9 units minus 8 units)
0.15 units down from the top left (the 5.65 units minus the 5.5 units)

Similarly we select the lower right part of our gnomon's sub-style. Taking a horizontal value of, say, 10.5 units, the vertical drop would be: $10.5 / \tan (57.9)$ or 6.59 units down. Thus the relative position from the top left of our selected dial plate area is thus found by subtracting " $8,5.5$ ", giving:-
2.5 units right from the top left (the 10.5 units minus 8 units)
1.09 units down from the top left (the 6.59 minus 5.5 units)

Hence the sub-style can be drawn. And the style will protrude from that sub-style making an angle with the wall of SH (style height), namely 4.21 degrees. It will be then pointing true north and parallel to the Earth's polar axis. However, the rest of the linear dimensions of the gnomon are still needed.

The above figures match closely the figures on the spreadsheet in a couple of pages that can aid the dialist. Remember, different calculators and spreadsheets may produce slightly different figures due to rounding and other approximations.

## STEP 6: THE GNOMON STYLE DISTANCES FROM THE SUB-STYLE

The top left of the gnomon sub-style was 9 and 5.65 units from dial center, so the hypotenuse distance is the square root of the sum of 9 squared and 5.65 squared, or, 10.6 units


With a hypotenuse of 10.6 units from the dial center, and a style height of 4.21 degrees, the top left end of the gnomon is found by geometry or trigonometry. The 10.6 was the hypotenuse of the first triangle, but is now the adjacent side of the new triangle.
$\tan (4.21)=$ dimension of top left edge of gnomon / 10.6
dimension of top left edge of gnomon $=10.6 * \tan (4.21)=0.78$ units
And the bottom right edge of the gnomon is similarly found:- The horizontal dimension was 10.5 , the vertical dimension drop was 6.59, so that hypotenuse would be:- 12.39 units. And thus the edge dimension is $12.39 * \tan (4.21)=0.91$ units.

The sub style is the difference between 12.39 and 10.6, in other words:
1.79 units

Thus the final gnomon is:-


The gnomon will thus be angled at the style distance of 57.9 degrees if our calculations were correct. The above figures match closely the figures on the spreadsheet on the next page that can aid the dialist. Remember, different calculators and spreadsheets may produce slightly different figures due to rounding and other approximations.

## STEP 7: THE HOUR LINE COORDINATES

Each hour line is similarly found by selecting two points, and connecting those points. In fact, a simple spreadsheet can be made that looks at the points for each hour line and the gnomon:-


This spread sheet was built by entering the horizontal coordinates of the selected final dial plate on the wall, and entering the hour line angles and their associated times from the great decliner tables, and the "down units" formula was simple trigonometry (using tan hour or gnomon SD angle), or Pythagoras's Theorem (for radial distance from the far away dial center).

To avoid negative vertical or down figures, the spread sheet lets you displace a horizontal position, such as for the 3, 4, and 5pm lines. The gnomon sub-style was also displaced. An east facing great decliner can use this spreadsheet, however you pretend you are using a mirror.

This 7-step technique is the most general, and works for all cases of great decliners.
If the dial is North declining east or west, then the vertical line is not noon, but midnight, and the gnomon points upwards not downwards.

Remember, different calculators and spreadsheets may produce slightly different figures due to rounding and other approximations.

The spreadsheet is available on the web site in the reference page.

## COOK BOOK SUMMARY

## GREAT DECLINER - SOMEWHAT EAST OR WEST DIAL

Determine the latitude and the wall's declination. The latitude established the gnomon's style angle with the horizontal, the wall's declination affects the sub-style angle with the wall as well as with the horizontal. If the wall's declination is trivial, and if you are not feeling energetic, then you may empirically design the dial beginning with the gnomon first.

If the wall's declination is still mostly easterly or westerly, then the formula or tables for a normal vertical decliner are used. They are in appendix A5.2 for large angles such as these, and use exactly the same formula as in A5.1, the sole difference being the wall's declination angle.

Calculate two points for each hour line and for the gnomon, based on the final dial plate's distance from the far far away dial center. Connect the dots for each hour line and thus draw the final hour lines. If calendar lines will be used then obviously the hour lines extend from the solstices with the equinox calendar line in between.

You may find that a spreadsheet can simplify the repetitive work of deriving hour line coordinates and gnomon shape.

1. Define the latitude and the wall's declination.
2. Use tables in Appendix 5, such as 5.1 or 5.2 and develop
a. the hour line angles
b. the style distance (the angle the sub-style is rotated from the vertical noon hour, just as with any declining dial)
c. the style height (the angle between the style and sub-style,
just as with any declining dial)
3. Draft a dial plate that has a dial center
4. Determine where in that larger picture the final dial plate will be
5. Determine the endpoints of the gnomon's sub-style. This uses very simple trigonometry.
6. Using Pythagoras's theorem as well as trigonometry, determine the dimensions of the gnomon that stick out of the wall, Pythagoras provides the direct distance from the unreachable dial center, although trigonometry could have been used. And knowing that distance from dial center, trigonometry can be used to find the two distances separating the style from the sub-style..
7. Determine the endpoints of each hour line, and for the gnomon. This uses very simple trigonometry. This can be done before step 5 if desired.

Declination lines are discussed in chapter 23.

## VERTICAL DECLINER: Trigonometric based tables

A vertical decliner is to be built for Phoenix, Arizona. The wall is south 85 degrees west.

| Table of cities with useful data |  |  |  | North is positive |  |  | West is positive |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| City | Country, State, City |  |  | Hemisphere |  | Lat |  | $\begin{aligned} & \mathrm{Ma} \\ & \mathrm{~g} \end{aligned}$ | $\begin{aligned} & \text { Tim } \\ & \mathrm{e} \end{aligned}$ | Long |
| id |  |  |  | +n-s |  | var | ref | corr |
|  | UK |  | Weymouth |  |  |  |  | 50.6 | 2.5 | $\begin{aligned} & 3.4 \\ & w \end{aligned}$ | 0 | 10 |
| PHX | USA | AZ | Phoenix |  |  | 33.5 | 112.0 | $\begin{aligned} & 11 . \\ & 8 \mathrm{e} \end{aligned}$ | 105 | 28 |

If we use table A5.2b in appendix 5 for latitude 32 degrees, we find that for S85W:-

| 1pm | 44.8 | 2pm | 51.3 | 3 pm | 54.0 | 4pm | 55.7 | 5pm |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SD: style | istanc |  | is 57.9 degrees (note: close to co-latitude) |  |  |  |  |  | 57.9 | SD |
| SH: style height: |  |  | is 4.2 degrees (note: close to wall co-declination) |  |  |  |  |  | 4.2 | SH |

The gnomon can be cut, it has an angle of 4.2 degrees, and set on the wall, at an angle of 57.9 degrees from vertical. Use trigonometry to get gnomon dimensions between sub-style and style. An equation of time table is needed, which could include a longitude correction of 28 minutes, gleaned from the city tables.


While declination lines may take more thought for great decliners, if the gnomon is properly designed and the style correctly set at latitude pointing to true north, then the shadow of the style will parallel the hour lines. This fact simplifies cross checking additional declination or calendar lines. Declination or lines are discussed in chapter 23.

NOTE: Another value, DL, is sometimes calculated, this is a "difference in longitude" and is a value used when special features are added such as the analemma (chapter 25), and provides for alignment of special features with the vertical decliner dial's hour lines.

## CASE STUDY ~ A VERTICAL GREAT DECLINER - S $85^{\circ}$ East (Rather like an arrow head)

Since the building already has a $\mathrm{S} 5^{\circ} \mathrm{W}$ and $\mathrm{S} 175^{\circ} \mathrm{E}\left(\mathrm{N} 5^{\circ} \mathrm{E}\right)$ vertical decliner on the south and north walls respectively, the decision was made to add an east facing vertical decliner as well.

Using the DeltaCAD macro for the vertical great decliner, the layout [1] below was generated. It provided a style distance (SD), a style height (SH) and the hour lines. The SH was $4.2^{\circ}$ so next the calendar curve DeltaCAD macro was used to generate calendar curves for an SH of $4.2^{\circ}$, as shown in picture [2] below.


The SD line was extended from the top right figure [1], as was the SH line, using good rulers. Next, the calendar curve layout [2] was moved until its SD and SH lines matched, as shown in the picture [3] above.

Then the hour lines were extended [3] using a metal ruler from the hour line picture to the calendar picture. At the same time, some vertical and horizontal grid lines were transferred as well. Alternatively, the calendar curves can be produced with $1^{\circ}$ angled lines, and the actual hour line angles minus the style distance can be used to transpose those lines.

The resulting dial plate was cross checked with other software such as SHADOWS (not affiliated with this web site). Since it matched, a mock dial plate was created and taped to the wall, aligned vertically or horizontally, and with a gnomon added. This was checked the next morning by using times adjusted for the EOT.

The dial plate mockup was found to be off. This is why a mock dial as a cross check is beneficial. The correct shadow for the times (EOT adjusted) was marked on the mock dial, and the decision made to re-calculate the wall declination which showed that the building was not square. The correct declination for the east facing wall was $\mathrm{S} 84^{\circ} \mathrm{E}$, in other words one degree of error. This resulted in about a $1^{\circ}$ difference in the 1000 hour line, and about a $2^{\circ}$ difference in the 1100 hour line. This matched the mock dial's marked 1000 and 1100 hour line. The mock dial paid for itself.

So, contrary to this section's heading, this is a South $84^{\circ}$ East dial, not South $85^{\circ}$ East one! The new SH was $5.0^{\circ}$ (not 4.2), the new SD was $57.1^{\circ}$ (not 57.2 ) and the hour lines angles changed by about $2^{\circ}$ at 1100 am.

When making the final clay dial plate, as always remember that when the clay tile is cut and later set, the joints will have a width larger than the transposed drawing. In other words, ensure that cut lines in the clay remove some clay equal to the eventual grouted gaps.

The dial plate shape suggested an arrow as an outline, appropriate for a south western cultural heritage.

Then the dial plate was made from clay using slip for the dial furniture, as discussed in the earlier case studies. The clay dial plate was bisque fired first, glazed next, and then the copper gnomon was attached as usual to a clay piece with an epoxy. When the clay piece with the epoxied gnomon was cured, the entire dial plate was set on the wall with an exterior mastic (Versabond) to act as
 a clay sealer as well as to hold the pieces in place. Then grout was applied, two coatings to minimize cracks, and then sealed.


A question often asked is why go to all the bother of using two pictorials for one dial when available software has both pictorials, calendar and time, merged.

The answer is that the focus of the Illustrating Shadows series of books is not just at the rote, understanding, or application level of learning, but rather at the correlation level. The whole point is to correlate calendar curves to the style nodus, and all of that to the dial plate with its hour lines. The intent is not simply a turnkey rote activity.

## CHAPTER EIGHTEEN

## THE INCLINED DECLINER ~ DIALS THAT FACE ANY WHICH WAY

PART 1 ~ INTRODUCTION, AND THE STEEP DIAL


There are two classes of inclined decliners. One is the steep inclined decliner whose slope exceeds design latitude, more like a vertical dial, and is shown to the left. The dial center is on the upper part of the slope, and hour lines radiate out from the pole towards the equator.

The other is a shallow inclined decliner whose slope is less than latitude, more like a horizontal dial, and is shown below. The dial center is on the lower part of the slope, and hour lines radiate out from the equator towards the pole.

Actually, the difference between steep and shallow dials is more a test of latitude compared to the slope of the dial plate/gnomon intercept, in other words where the dial center is. Shallow dials have the dial center below, steep dials the dial center above.

Steep inclined decliners are shown first, and shallow inclined decliners next. The methods of designing inclined decliners, or reclined decliners, or proclined declined dials, are discussed
 using techniques such as:

- A CAD designed declining recliner as an introduction
- using CAD - this time a more involved version
- using available software programs for the same dial
- using trigonometry - the formulae
- using spreadsheets for the same dial which use the trigonometric formulae
- using geometry for the same dial in step by step mode
- A review of the geometric method in larger steps
- Modifications to geometry to consider longitude
- Actual construction of the final dial

Various models exist for dials that incline while declining. The real world dial of this sort would be a roof on a building not aligned with the cardinal points of the compass.

## THE STEEP INCLINED DECLINER

The steep inclined decliner is closer to the vertical dial than the horizontal. The dial center is towards the top of the dial plate. The pitch of the inclination exceeds the design latitude.


Angle CSD is the latitude, and object CSD" is the gnomon actually seen on the roof, being what is visible if a full gnomon CSD constructed that actually went through the roof. $D$ is the perpendicular drop from the top of the style (dial center) down to the base surface below the roof. Thus hour lines can be drawn from D, for example 3pm above, 1pm to 3pm below.


The pictorials above and to the left are the STEEP inclined decliner, the simplest variant, because the hour line intersections with the roof bottom are simple. They use a surrogate horizontal dial. The only major issue is the gnomon and its dial center being the point of intersection " C ". The STEEP inclined decliner is demonstrated first. And like the vertical decliner variant that used a surrogate horizontal dial, the gnomon here also does not need to be rotated using SD and SH.

The SHALLOW variant is more involved. It is examined after the simpler STEEP inclined decliner.
Geometric, 3d-CAD, and trigonometric methods are demonstrated.

If the base of the roof on which SD"D lies was a horizontal dial then, for a steep inclined decliner, the hour lines become obvious. The below is a stylistic version of the lower pictorial on the preceding page.
the line DS is north south


The hour lines can be drawn from point C on the roof to the base of the roof for the 1,2 , and 3 pm lines. The issue is locating point C .

This is actually easier said than done. The horizontal plane is easy to draw geometrically, and it is the usual horizontal dial. Point $D$ will need some effort to ascertain. Then the sloping roof must be collapsed to touch the base of the roof while retaining the point C , and then the lines can be drawn.


Remember that point $C$ above is vertically above point $D$, the above is a projection only. Variations can arise such as when the roof angle is less than the latitude, equal to the latitude, or greater than it. Remembering that the angle in question is not the angle of the roof at the edge, but rather the angle obtained in the plane CSD, namely the projected roof angle of CD"D.

Having shown the general method, it is easily seen that longitude corrections are simple since they affect merely the horizontal dial lines.

Finally, this of all dials has many opportunities for screwing it up, so having drafted the real roof hour lines, it might be worth constructing the final gnomon and marking the hour lines in an almost invisible marker, then testing them empirically.

The following pages demonstrate the two classes of these dials, steep and shallow, and include a case study. The steep inclined decliner is the simplest and demonstrated first, the shallow inclined decliner is more involved and demonstrated second.

## GEOMETRICALLY DESIGN A STEEP $65^{\circ}$ INCLINED $45^{\circ}$ DECLINER



This method works when the slope exceeds the actual latitude, i.e. a steep slope. It requires that there be a new dial center on the upper part of the inclined declining surface "C", that can be connected to the horizontal surrogate dial's center "S". The first phase has six steps:-

1. draw the roof profile, i.e. the sloped surface $B C$ inclined ( $65^{\circ}$ in this case) at angle NBC
2. draw a line which is the declination ( $45^{\circ}$ in this case) of the base of the roof, radiating from N in the direction of what will be point $S$ ( S is not yet located)
3. from $N$ draw a line at $90^{\circ}$ from line $S N$ which will meet the arc in the next step at $A$ which is not yet located
4. rotate the point C on the roof (inclined decliner's dial center) using an arc radius NC through to NA (now locating point A) so the vertical drop from the roof to its floor is projected, NA=NC
5. from point A (the projected inclined decliner's dial center laid flat), draw a line at co-latitude from NA to intersect the declination line NS at point $S$ (now locating point S), the surrogate horizontal dial's center. " S " is now located, and a gnomon built, in this case for $32^{\circ}$
6. connect the surrogate dial center $S$ to the bottom of the roof $B$, this provides an intermediate angle NSB measuring $14.88^{\circ}$ in this case, and used in the next figure on the next page.

We have taken the 3d model of the inclined declining object and generated three triangles. The gnomon is SNA, its style is SA , and point A is the projection of point C . The roof profile is NBC.

Important figures that will be used in the next diagram are:-

- angle NSB - the difference from noon line SN (surrogate dial center to the north) and SB which is the line from the surrogate dial center to the place on the bottom of the inclined declining slope where the shortest line from the inclined decliner's dial center meets the bottom of that slope.
- SB - the direct line from surrogate dial center to the base of the roof B
- $\quad B C$ - the shortest distance direct line from the base of the roof $B$ to the new dial center $C$

1. A noon line beginning at point $S$ is drawn vertically upwards, but it does not terminate anywhere in particular.
2. From that line is offset the line SB whose length equals $S B$ in the preceding pictorial, and whose angle is NSB also from that same diagram.
3. From point $B$ is drawn an east west line purely for clarity, and from it is also drawn the bottom of the inclined declining dial, the bottom of the roof line, it is declined by the same declination as before, in this case 45 degrees. At 90 degrees to the base of the inclined dial, is drawn a line BC whose length equals BC in the preceeding figure. We have exactly copied the angle NSB, and the line lengths $S B$ and $B C$, however, the angle between $S B$ abd $B C$ is not preserved.


4. A surrogate horizontal dial has its dial center placed at $S$, its noon aligned, and the hour lines projected out to the declination line (see circles on the 1500, 1600, and 1700 hour lines). Where those hour lines intersect the declination line, lines are drawn to the inclined decliner's dial center C. They were measured from noon

|  | figure to | fig 5 in a few |
| :---: | :---: | :---: |
|  | the left | pages provides: |
|  | ( $0^{\circ}$ base) | (68.12 ${ }^{\circ}$ base) |
| 1pm | 6.92 | 6.92 (-61.20) |
| 2pm | 12.19 | 12.21 (-55.01) |
| 3 pm | 17.24 | 17.00 (-51.12) |
| 4pm | 22.11 | 22.09 (-46.03) |
| 5pm | 28.20 | 27.68 (-40.44) |

A good match. Note that figure 5 used a SE face while this is SW facing. However symmetry exists because the declination is $45^{\circ}$.

This method is simple to follow. It is simple because it uses a surrogate horizontal dial whereas in subsequent geometric models a surrogate equatorial dial of 15 degrees radials is used. This will increase the complexity of the diagram, but save generating an intermediate horizontal dial. This evolution is similar to the evolution of the vertical decliner methods discussed in chapter 16.

This method assumes a steep slope, does not work for shallow slopes, and uses the horizontal dial's gnomon, thus the gnomon on this method's inclined decliner is not perpendicular to the inclined decliner's dial plate. That gnomon is perpendicular to the surrogate horizontal dial plate, thus there is no SD/SH (style distance, style height) which makes calendar curve design more complicated.

## 3D CAD EMPIRICAL DESIGN OF A STEEP $65^{\circ}$ INCLINED $45^{\circ}$ DECLINER

These were once common as a demonstration of skill. Some diallists suggest an empirical approach by locating the horizontal line, align a horizontal plate along it, establish the gnomon, drop the gnomon's points to the inclined declined surface below, build that gnomon, and mark the hours as the sun shines. Others offer a geometric or trigonometric technique. One author suggests this is a problem for the diallist entirely of their own making and easily avoided. Rather than repeat by rote such alternatives, the suggestion here is to use CAD by applying equatorial dial radials to the surface.


We could use the same method used in chapter 30 with the wine glass using the 3d intersect tool, however, for flexibility, we shall use the 3d subtract tool instead.

Figure 1 above has a style set at latitude oriented to the pole, and two sets of radials. There are 7 radials, covering noon to 6 , each being 15 degrees apart and perpendicular to the style. In CAD this is done by making one radial and then radial copying it. The two radial sets and the style are combined with 3d-add. Two sets of radials are needed in order to draft hour lines. This dial is sloped $25^{\circ}$ off vertical, declining exactly $45^{\circ}$.

Then build the slope, here a $65^{\circ}$ recliner that declines $45^{\circ}$, i.e. South $45^{\circ}$ East. This is shown in figure 2. Then additional radials perpendicular from the dial plate to the style, that will establish the style distance and height angles in a later step. This is done by setting the work plane to the sloped surface, also visible in figure 2, drawing two cylinders and bringing them up to the style. They are not 3d-added to the style because when the 3d subtract happens it would complicate drafting the hour lines in a subsequent step.



The next step is to use the 3d subtract tool to subtract the hour lines from the dial plate. Figure 3 shows the dial plate and the resulting holes.

Then, with the work plane on the slope of the dial plate, draw a horizontal line and then lines from each pair of pair of holes made by the equatorial radials, as shown in figure 4. Understanding work-planes is crucial in CAD work such as this.

Having drafted the hour lines, they can then be measured with the angular dimension tool. The accuracy of this system depends on the accuracy of the draft hour lines, which in turn depend on the accuracy of the equatorial radials. Figure 5 shows the resulting hour lines, they are using 6 am as the base line.


Fig 5
measuring the hour lines

The SD and SH values were compared with calculated values and were within a degree.

The northwest face is a mirror of the northeast face similarly, however because this particular dial inclines, the north east and west faces differ from their southern counterparts. One of the northerly faces must be

The hour line angles were compared with calculated angles, and were within about a degree.

The style distance can be measured at the same time as the hour line angles, however the style height requires the work plane to be correctly set up, since angles are measured based on the work plane. Work planes are one of the key secrets to effective CAD usage.

Figure 6 shows the angles for the gnomon. This establishes the south east face. The southwest face is a mirror image as this dial is declining 45 degrees, mid way between 0 and 90. designed in addition to those southerly faces.



Fig 7

Figure 7 has a style and equatorial radials and the dial face is now facing northeast. In figure 8 the 3d subtract method has again been employed, however there is a difference. Only one set of radials was used, and only one post to hold the style up from the sub-style.

The reference for style height is simply the dial plate itself. The reference for style distance and the hour lines is a horizontal line from where the style itself intersects the dial plate. The hour line angles were measured with the angular dimension tool, see figure 8. Then in figure 9 the style height and style distance were measured also.

The hour line angles and style distance and height were within a degree of calculated values. It is critical that the style be accurately placed, failure here will cause variances of several degrees. As always, these final values should be verified with a model tested in sunshine.


Fig 8


Thus, the north faces have been designed, they mirror each other because of the declination being 45 degrees

The set of four faces may then be applied to a base, this would be one with $65^{\circ}$ slopes.

A spreadsheet was used to compare the CAD derived values with those produced by other means, such as by calculation or by using software designed for sundial design. The only purpose of this spreadsheet was to show the accuracy obtainable.

SOUTH EAST FACE
A completed dial is pictured below, and it truly is a problem of the diallists own making and easily avoided!

## SOME RULES OF THUMB

Some useful rules of thumb are that in the northern hemisphere, north facing dials indicate clockwise, south facing dials indicate counterclockwise.


NORTH EAST FACE

|  | math | CAD | DIFF |
| :---: | :---: | :---: | :---: |
| 6 | 22.6 | 23.2 | -0.6 |
| 7 | 11 | 10.8 | 0.2 |
| 8 | 1.2 | 1.6 | -0.4 |
| 9 | 14.7 | 15.6 | -0.9 |
| 10 | 30.2 | 30.4 | -0.2 |
|  |  | fig 8 |  |
| SD | 22.34 | 22.3 | -0.1 |
| SH | 50.1 | 50.1 | 0.1 |
|  |  | fig 9 |  |

While in the real world, such a CAD technique may not be used very often, it does show the power of CAD systems, and demonstrates yet again the equatorial dial basis for most hour angle dials.

## CHAPTER NINETEEN

## THE MORE INVOLVED SHALLOW INCLINED DECLINER

This dial plate will be designed using several techniques. First, using CAD, or computer aided design. The 3d CAD systems are nothing more than computerized reality, a computerized version of what people did on the good old days when they used trigons, protractors, and leaded weights on strings. Of course, a CAD derived dial is only as good as the measurements that went into using it, and three of those measurements are:
dial alignment with the cardinal points
dial plate declination (how far off from true north etc)
dial plate inclination (how far off from the horizontal)
Another key point is the definitions and what is intended or really meant by some words.
declination - here the rotation about the vertical axis of a dial plate, the term S 10 W means look south, and turn 10 degrees west.

inclination - some texts mean the movement from the vertical, some mean the movement from the horizontal. For standardization purposes, this text will mean inclined up from the horizontal, which is the complement of... reclination - the angle made backwards from the vertical.

inclining up $20^{\circ}$
shallow

inclining $70^{\circ}$ up
steep

And, software such as SHADOWS will be reviewed and how it can verify a dial design. Also, the use of spreadsheets will be examined, a very useful tool but one needing some degree of intuition. And, a geometric method will be reviewed, step by step. Also, the trigonometric formulae will be portrayed in normal form as well as in Excel format for the typical spreadsheet, and finally a DeltaCAD macro will be demonstrated.

What do the masters say of this dial? Mayall had but one enquiry in twelve years and presents a geometric solution. Rohr shows a geometric and a trigonometric method. Drinkwater says this is a problem of the diallist own making and easily avoided, and continues with a long discussion of why these dials should be avoided. Waugh focuses on getting the gnomon correct, and then, considering the equation of time, mark the hours on the hour and getting on with your life. Some books may have a typographical misprint, so double check the formulae before relying on them.

## 3D CAD EMPIRICAL DESIGN ~ SHALLOW INCLINED $20^{\circ}$ S45W DECLINER

In the example in the first few pages of this chapter the 3d TurboCAD deluxe program was used. In this example, TurboCAD Professional was used. The difference as far as dial design goes is that one can draw lines on a work-plane on a surface depicted in solid color and with intersecting radials hidden at their point of intersection. In other words, one can skip the 3d intersect or the 3d subtract phase and do things even more visually.

First, select a work plane and build a very thin yet tall cylinder. This will be a radial. Then do a radial copy to get, in this example, 6 morning and 6 afternoon hours.

Then do a 3d add so they are one object, you could also do a "group" which has the advantage that group members can have different colors, however it has the disadvantage that a 3d subtract or intersect operation may not do what you want. These are the radials at the equinox, and a set can be built for any declination of the sun using the same process, they become cones either side of the equinox whose radials form a plane.


This is the starting place. A gnomon is added, and an angular measure also. Another advantage of TurboCAD professional is that text and text attached to angular measures, and so forth, all appear in 3d rendered mode without the need to "explode" them and make the text specific 3d objects. If the measure looks as if it is not measuring the slope of the gnomon fear not, the angular measure was done a little bit to this side of the gnomon for clarity.

In the next picture, the final dial plate was inclined by 20 degrees.
Then rotated. But that rotation would be problematic because as soon as it is declined, the axes change. So to counter that, a new cube was created and selected with the tilted dial plate, this resets the rotational axes, and the two together can be declined, and the temporary cube is then deleted.

In addition to the final dial plate being rotated (declined) and tilted (inclined), the style and its radials were also tilted to latitude.

NOTE: The next three pages may make less sense unless you have and use TurboCAD.

At this point the benefits of the TurboCAD professional program allow some steps to be skipped. In the first example at the start of this chapter the 3d subtract or intersect operation was employed to clearly see just where the equatorial radials intersected with the dial plate.

With TurboCAD professional, the model can be rendered as you work which means that you do not see the radials below the dial plate. Thus the
 hour lines can be drawn directly and they will be seen because such lines are rendered in the professional version of this software even though they are not solid objects.

All this is done by first setting the work plane to the sloped rotated (inclined declined) dial plate, using the work-plane by facet function. Then lines are simply drawn from the dial center to the place where the equatorial radials vanish into the dial plate.

The work-plane can then be set to the gnomon, using the "by facet" function, and the style to substyle angle determined. That is the height of the style from the sub-style for this vertical gnomon, which is why the words "style height" were not used. Style height is a term typically reserved for a gnomon perpendicular to a dial plate, and at this point the gnomon is perpendicular to the horizon.

While it is not normal, there is nothing wrong with using a vertical gnomon except that for some declining inclining plates it just isn't practical.

A gnomon perpendicular to the dial plate is simpler to construct and stronger. So, how is the style distance (SD) and the final style height (SH) determined?


With the work-plane set to the dial plate, a cylinder is drawn and pulled up. The entire figure manipulated until the bottom and top circles of that cylinder are merged, this means that the observer is perfectly perpendicular to the dial plate. At this point, a line can be drawn from the dial center following the style, which is truly the sub-style of normal parlance.

Again, simple with TurboCAD Professional. With the more common version the process uses an extra step or two and requires the dial center point and the equatorial radial points, and the sub-style intersection line to be made visible by using the 3d subtract tool.

A gnomon can be drawn from the place where the style distance line appears and the final style height measured angularly. Other techniques exist also.

At this point, the angles can be measured. From the previous page the style distance SD was determined as from the noon line as:

$$
\text { SD }=4.56
$$



From the picture above the hour line angles from noon are:-

|  | CAD | Spread sheet <br> calculated |
| :--- | :--- | :--- |
| 0900 | 23.80 | 22.89 |
| 1000 | 12.29 | 12.16 |
| 1100 | 5.97 | 5.27 |
| 1200 | 0 |  |
| 1300 | 4.48 | 4.64 |
| 1400 | 9.13 | 9.29 |
| 1500 | 14.19 | 14.60 |
| 1600 | 21.76 | 21.59 |
| 1700 | 32.86 | 32.55 |

Designed for: latitude 32.65, long=105 legal=105, dec=S45W, inc=20

NOTE: Always check whether noon LAT (local apparent time) or noon adjusted for longitude is what is employed.

The final verification is for the style distance and height, SD and SH. In this case the CAD method measured SD from noon.


| The CAD provided |  |
| :--- | :--- |
| SD | 4.56 |
| SH | 18.50 |

The spreadsheet deduced
SD 4.55
SH 17.76

Why was the style height off? Because it was the SH for a vertical gnomon whereas we need the SH for a gnomon adjusted by the style distance. The technique to measure the SD for the adjusted gnomon was simple. The work-plane was set to the dial plate, a cube placed on it, and that cube rotated to parallel the style distance line. Then the work plane was switched to the vertical surface, so angular measures would truly measure the gnomon perpendicular to the surface rather then the gnomon that was vertical to the horizon. The angle was then measured properly, and found to be:

| The CAD provided | The spreadsheet deduced |  |  |
| :---: | :--- | :--- | :--- |
| SD | 4.56 | SD | 4.55 |
| SH | 17.41 | SH | 17.76 |



The SD (style distance) and the hour line angles were from noon.


The CAD system measured the noon to horizontal distance to be:

### 47.37 degrees

The spreadsheet calculated that angle to be:

| Noon:v LAT | -43.2 | Vertical \& noon LAT line difference |  |
| :--- | ---: | ---: | ---: |
| Noon:h LAT | 133.2 | 46.8 | Horizontal \& noon line difference (90+ and 90-) |

46.8 degrees

SD may be calculated from the vertical as opposed to the horizontal. In this case the SD value of 4.56 from noon when added to the 47.37 degrees noon is from the horizontal provides a SD from horizontal of:

| 51.93 | SD from horizontal |
| :--- | :--- |
| 38.07 | SD from vertical |

At this point, all data has been deduced and found to match closely values from a spreadsheet.
The next step would be the paper mock up to validate the design.
The final step would be the actual dial plate design itself.
The lessons learned using CAD are a good understanding of using work planes and their coordinate systems.

## CASE STUDY ~ declining dial - S $45^{\circ} \mathrm{W}$ inclined a SHALLOW $20^{\circ}$ [software]

There are a number of software programs available for sun dial design. One in particular is called SHADOWS which is not affiliated with this book. The SHADOWS system was provided the longitude, latitude, declination and inclination.


The data plate showed a style distance of:-

$$
\text { SD }=38.57
$$

however, this was measured from the vertical. The CAD system provided a style distance from noon of 4.56 degrees, and when the angle of the noon line was considered, SD from horizontal was 51.93 , and its complement was 38.07 which is within half a degree of the SHADOWS calculated version.

The SHADOWS style showed a 25 mm long style and a 17 and 25 mm high style. This means that the vertical distance was 8 mm compared to a horizontal distance of 25 mm . This provides the angle using simple trigonometry of the SH .

$$
\text { SH }=\operatorname{atan}(8 / 25)=17.74
$$

And this compares well to the spreadsheet as well as the CAD derived data.

Finally the hour lines must be evaluated.

|  | CAD | XLS | SHADOWS |
| :--- | :--- | :--- | :--- |
| 0900 | 23.80 | 22.88 | 23.26 |
| 1000 | 12.29 | 12.16 | 12.32 |
| 1100 | 5.97 | 5.27 | 5.44 |
| 1200 | 0 | 0 | 0 |
| 1300 | 4.48 | 4.64 | 4.48 |
| 1400 | 9.13 | 9.29 | 9.62 |
| 1500 | 14.19 | 14.60 | 14.47 |
| 1600 | 21.76 | 21.59 | 21.96 |
| 1700 | 32.86 | 32.55 | 32.85 |

NOTE: While SHADOWS asks for the dial's longitude, it does not consider that longitude when generating hour lines unless specifically instructed to consider it.

Thus far, the dial plate for a dial declining 45 degrees to the west, inclined upwards by about 20 degrees matches closely with CAD modeling, with spreadsheet usage (which is trigonometric), and with available software such as SHADOWS. SHADOWS is available from

## A declining dial - S $45^{\circ} \mathrm{W}$ inclined a SHALLOW $20^{\circ}$ - [trigonometry]

Formulae for inclining decliners may be found on the British Sundial Society (BSS) web site, as well as in a few other publications.

Some books may need careful review by the reader before reliance is placed on the formulae espoused.

Before producing the formulae here, they were tested and debugged, the mechanism being through a spreadsheet.

NOTE: There are limitations on these formulae, so a reasonableness check is warranted, as well as validation by a model in the real sun, before proceeding to a final design implementation.

Style Distance from noon

$$
\begin{aligned}
\text { SDn }=\quad \operatorname{atan}( & (\sin (\mathrm{dec}) * \sin (\mathrm{inc}) * \\
& ((\sin (\mathrm{inc}) * \cos (\mathrm{dec})-\tan (\mathrm{lat}) * \cos (\mathrm{inc})) / \\
& (\cos (\mathrm{inc})+\tan (\mathrm{lat}) * \cos (\mathrm{dec}) * \sin (\mathrm{inc})))))
\end{aligned}
$$

Style Height SH

```
SH = asin( (cos(lat) * sin(inc) * cos(dec) ) -
```

    (sin(lat) * \(\cos (i n c))\) )
    The SD and hour line angles are from the noon line. They are adjusted with the angle from the horizontal to noon distance, and the resulting number subtracted from 90 degrees provides the same data with the vertical as a reference.

Vertical to noon

$$
=\quad \operatorname{atan}(\tan (\mathrm{dec}) * \cos (\mathrm{inc}) \quad)
$$

The hour line angles from the vertical are:-

$$
\begin{gathered}
=\quad \operatorname{atan}\left(\quad \left(\left(\left(\begin{array}{l}
(\cos (\mathrm{lat}) * \sin (\mathrm{inc}))- \\
\\
{\left.\left.\sin (\mathrm{lat})^{*} \cos (\mathrm{inc})^{*} \cos (\mathrm{dec})\right)^{\star} \tan (\mathrm{ha})\right)+}^{\left.\cos (\mathrm{inc})^{\star} \sin (\mathrm{dec})\right)} / \\
\\
\left(\cos (\mathrm{dec})+\sin (\mathrm{dec})^{\star} \sin (\mathrm{lat})^{\star} \tan (\mathrm{ha})\right)
\end{array}\right)\right.\right.\right.
\end{gathered}
$$

NOTE: Always check whether noon LAT (local apparent time) or noon adjusted for longitude is what is employed.

## A declining dial - $\mathbf{S} 45^{\circ} \mathrm{W}$ inclined a SHALLOW $20^{\circ}$ - [spreadsheet]

The spreadsheet of choice was Excel. Excel has its rules and quirks. The spreadsheet discussed here is nothing more than an implementation of the formulae for declining incliners.

Since the spreadsheet has been discussed in the CAD and SHADOWS discussions of declining incliners, this section will focus on the spreadsheet and its formula.

The spreadsheet is available on the CD that accompanies this book, and is kept current on the web site as:-
www.illustratingshadows.com/illustratingShadows.xls
Excel and Open Office

The spreadsheet begins with raw data.

| Declination [ +SW - SE ] | $\mathbf{4 5}$ | <90 | SOUTH WEST |
| :--- | :--- | ---: | ---: |

The dial plate declination is entered, positive being in this case South West. The inclination from horizontal is entered next, and finally the latitude. The spreadsheet then deduces trigonometric values for the angles, and derives the inclination from the vertical. Recall that Excel needs radians as opposed to degrees.

Conditional formatting was used to highlight some useful information, such as confirming the declination as southwest, and the inclination as shallow.

| SOUTH WEST |
| :--- |
| SHALLOW |

which measures the inclination to 45 degrees. The words SHALLOW or STEEP confirm the nature of the slope as a human double check. This spreadsheet highlights a declination of 90 degrees as inappropriate also using conditional formatting, for which the solution is to use the east or west decliner system.

The spreadsheet then provides the style distance (SDn) from noon but more importantly, from the vertical also. And the style height SH is also provided. In this case $\mathbf{3 8 . 7}$ and 17.7 degrees.

| SDn | 4.6 |
| :--- | ---: |
| SDv | $\mathbf{- 3 8 . 7}$ |
| SDh | 128.7 |
| USE THIS |  |
|  | $\mathbf{- 1 7 . 7}$ |

SDn (style distance) => angle from NOON
SDv + noon angle $=>$ angle from VERTICAL
SDh + noon angle => angle from HORIZONTAL
SH (regular angular style height)

The spreadsheet takes the SD from noon, and the noon from horizontal values and derives several figures. This is in essence a simple addition of the horizontal to noon, and noon to SD as well as their complement. Rather than provide highly complex formulae, several figures are produced, and it is left to the wisdom of the user to select the most likely usable information.


| SDV | -38.7 USE THIS SDv + noon angle $=>$ angle from VERTICAL |
| :--- | ---: |

A table of hour line angles is produced, there are two sets, one for hours before, and one for hours after noon. Again, the human has some selecting to do to determine the hours to use.

| HOUR LINE ANGLE FROM VERTICAL |  |  | angle from horizontal | angle from noon |
| :---: | :---: | :---: | :---: | :---: |
| HR |  | hr.ln.angl |  |  |
| 6 |  | 10.47 | 79.53 | 53.69 |
| 7 |  | 52.31 | 37.69 | 95.53 |
| 8 | AM | -86.63 | 176.63 | -43.41 |
| 9 | HOURS | -66.10 | 156.10 | -22.89 |
| 10 |  | -55.38 | 145.38 | -12.16 |
| 11 |  | -48.49 | 138.49 | -5.27 |
| 12 |  | -43.22 | 133.22 | 0.00 |

USE THESE

| HOUR LINE ANGLE FROM VERTICAL |  |  | angle from horizontal | angle from noon |
| :---: | :---: | :---: | :---: | :---: |
| HR |  | hr.In.angl |  |  |
| 12 |  | -43.22 | 46.78 | 0.00 |
| 13 |  | -38.58 | 51.42 | 4.64 |
| 14 | PM | -33.93 | 56.07 | 9.29 |
| 15 | HOURS | -28.62 | 61.38 | 14.60 |
| 16 |  | -21.63 | 68.37 | 21.59 |
| 17 |  | -10.67 | 79.33 | 32.55 |
| 18 |  | 10.47 | 100.47 | 53.69 |

The formulae used in the spreadsheet are:-
Style Distance from noon

$$
\begin{aligned}
& \text { SDn }=\text { DEGREES }\left(\text { ATAN } \left(\left(\operatorname { s i n } ( \mathrm { dec } ) ^ { * } \operatorname { s i n } ( \mathrm { inc } ) ^ { * } \left(\left(\sin (\mathrm{inc})^{*} \cos (\mathrm{dec})\right.\right.\right.\right.\right. \\
&\left.\left.\left.\left.\left.-\tan (\mathrm{lat})^{\star} \cos (\mathrm{inc})\right) /\left(\cos (\mathrm{inc})+\tan (\mathrm{lat})^{\star} \cos (\mathrm{dec})^{*} \sin (\mathrm{inc})\right)\right)\right)\right)\right)
\end{aligned}
$$

Style Height SH = DEGREES(ASIN((cos(lat)* $\left.\left.\left.\sin (\mathrm{inc})^{\star} \cos (\mathrm{dec})\right)-\left(\sin (\operatorname{lat})^{\star} \cos (\mathrm{inc})\right)\right)\right)$
Horizontal to noon = DEGREES(ATAN(tan(dec) * cos(inc)))
The hour line angles from the vertical are:-

$$
\begin{aligned}
& =\text { DEGREES(ATAN( ((( (cos(lat)*} \sin (\mathrm{inc}))- \\
& \\
& \left.\left.\left.\quad \sin (\mathrm{lat})^{\star} \cos (\mathrm{inc})^{\star} \cos (\mathrm{dec})\right)^{\star} \tan (\mathrm{ha})\right)+\cos (\mathrm{inc})^{\star} \sin (\mathrm{dec})\right) \\
& \\
& \left.\left(\cos (\mathrm{dec})+\sin (\mathrm{dec})^{\star} \sin (\mathrm{lat})^{\star} \tan (\mathrm{ha})\right)\right)
\end{aligned}
$$

And as mentioned earlier, these formulae do have some limitations.

PPRELUDE TO A CASE STUDY ~ A southwest declining dial $\sim \mathbf{S} 45^{\circ} \mathrm{W}$ inclined up a
SHALLOW $20^{\circ}$ using geometry - step by step first
[STEP 1] TO DETERMINE BY GEOMETRIC CONSTRUCTION THE ANGLE BETWEEN THE NOON LINE AND THE HORIZONTAL.


Noon line for a dial inc $20 \operatorname{dec} 45$
to find angle between noon and the horizontal

THE NOON LINE AND ITS OFFSET: First [1] a vertical line (ab) is drawn, and then second [2] a horizontal line (fe) is drawn, which can be the approximate linear height of the proposed style, but that is not important. Third [3] a line (eg) is drawn downwards at the angle of the dial plate's inclination up from the horizontal. Downwards for gently reclining slopes, but upwards if the dial proclines which is very uncommon.

That reclining line (eg) meets the vertical line at (g). And fourth a [4] perpendicular to that slope is drawn a line which meets that original vertical line at ( J ), and it has a length. That line is rotated [5] until it meets the vertical line at the top (k) from whence a line is drawn down at an angle equal to the dial plate's declination [6] where it will intersect with a horizontal line drawn (LJ). This intersection ( L ) has a line ( Lg ) drawn from it [7] down to point $(\mathrm{g})$ on the original vertical line (ab). And that line ( Lg ) makes an angle equal to the angle of the noon line with the horizontal, and it was measured as:-

```
geometric
the spreadsheet provided:
CAD
SHADOWS
SHADOWS
```

47.33 degrees

Which places all figures in the ball park.
This line from point " L " to and beyond point " g " is the noon line on the final dial plate.
[STEP 2] TO DETERMINE BY GEOMETRIC CONSTRUCTION THE DIAL CENTER.


## Dial center for a dial inc 20 dee 45

THE DIAL CENTER: From point (f) draw line (fm) which is perpendicular to line (Lg). This line will later be extended.

On the vertical line (ab), make line (fo) equal to line (fm), an arc is used for this. Then draw a line from point (o) to the earlier point (e) as line (oe). See expanded insert below to the left.


Depending on the values of inclination and declination, the dial center may be above to the left as opposed to the bottom and right, or in fact any where.

## [STEP 3] TO FIND THE SUB-STYLE, AND THUS SUB-STYLE DISTANCE (SD)

To determine by geometric construction the style distance (SD) and style height (SH), from the dial center point (C) draw a line through the existing point (f). This line (Cf) is the sub-style, and when measured from vertical it was found to be 38.97 degrees which is within one tenth of a degree of other figures.

[STEP 4] TO FIND THE STYLE HEIGHT (SH), through point (f) draw a line ( xy ) which is perpendicular to the recently drawn sub-style.

Make a line (fs) along line ( xy ) equal to the existing line (fe), an arc was used for this.
Finally draw line (Cs) which is the style, the angle between (Cs) and (Cf) was measured at 16.95 degrees which is within a degree of the other values derived in this section, this is the style height (SH).

Note that line (Cs) is not associated with line (Je), their closeness is coincidental. Note that line ( xy ) is not associated with line ( fmp ), their closeness is coincidental. This suggests great care with geometric constructions, as confusion can generate errors.
[STEP 5] TO FIND THE HOUR LINES by geometric construction the hour lines and their angles.
On the line (Cs) draw a line (fn) perpendicular to it from point (f). The intersecting point is ( n ), the " $n$ " signifying the nodus.


Extend the line (Cf) to point (t) such that the length (ft) equals the length of line (fn). An arc was used, however the arc centered on point (f) through points (nt) is not associated with the arc centered on point (f) through point (s), their proximity is co-incidental. This in essence is taking the circle of radials and projecting them from the nodus to the dial plate.

Centered on point (t) draw an arc to hold the radials, this arc is of any size. A line is drawn from the center (t) to where the noon line intersects the line (xy), point ( $m$ ) on the (LC) line. And beginning with the line (tm), draw a set of 15 degree hour angle depictions from the sun.


|  | CAD | XLS | SHADOWS | GEOMETRIC |
| :--- | :--- | :--- | :--- | :--- |
| 0900 | 23.80 | 22.88 | 23.26 | 20.30 |
| 1000 | 12.29 | 12.16 | 12.32 | 10.76 |
| 1100 | 5.97 | 5.27 | 5.44 | 4.84 |
| 1200 | 0 | 0 | 0 | 0 |
| 1300 | 4.48 | 4.64 | 4.48 | 4.34 |
| 1400 | 9.13 | 9.29 | 9.62 | 8.75 |
| 1500 | 14.19 | 14.60 | 14.47 | 13.51 |
| 1600 | 21.76 | 21.59 | 21.96 | 20.28 |
| 1700 | 32.86 | 32.55 | 32.85 | 31.03 |
| angles measured from noon |  |  |  |  |
|  |  |  |  |  |



The radials from the center of the circle with the 15 degree hour lines are extended to ensure they meet the line (xy).

From the dial center (C), lines are drawn and these are the final hour line for the dial plate.

Finally the hour line angles are measured, for consistency they were measured from noon. Of course the angle of the noon line with the horizontal or vertical was previously determined on the first of the geometric worksheets.

The hour lines agreed closely on the afternoon side, less so on the morning hours.

This particular combination of declination and inclination was such as to produce a number of angles that were small making for longer distances on some lines which in turn can increase the errors.

However in spite of those drafting errors, the method can be seen to have significant merit.

Having decided on a dial whose plate inclined by 20 degrees while declining 45 degrees to the west, it became apparent that a declining dial of 50 degrees would be more pleasing for the final location.

This meant recalculating the angles, and the geometric method was again used, albeit condensed so the "big picture" would be more apparent. The major phases in the geometric method are:-

- dial center and noon line
- style distance and height
- hour lines


## CASE STUDY ~ A southwest declining dial ~S 506 inclined up a SHALLOW $20^{\circ} \sim$ using

 geometry three major steps ~ noon and dial center, SDISH, and the hour linesNOON LINE AND ITS OFFSET: A vertical line (ab) is drawn, and then a horizontal line (fe) is drawn, which can be the approximate linear height of the proposed style, but that is not important. Then a line (eg) is drawn downwards at the angle of the dial plate's inclination up from the horizontal. Downwards for gently reclining slopes. That reclining line (eg) meets the vertical line at (g). And then a perpendicular to that slope is drawn a line which meets that original vertical line at (J), and it has a length. That line is rotated until it meets the vertical line at the top ( $k$ ) from whence a line is drawn down at an angle equal to the dial plate's declination where it will intersect with a horizontal line drawn (LJ). This intersection (L) has a line (Lg) drawn from it down to point ( g ) on the original vertical line (ab). And that line ( Lg ) makes an angle equal to the angle of the noon line with the horizontal, and it was measured as 41.86 , or 48.14 back from the vertical.


DIAL CENTER: From point (f) draw line (fm) which is perpendicular to line (Lg). This line will later be extended. On the vertical line (ab), make line (fo) equal to line (fm), an arc is used for this. Then draw a line from point (o) to the earlier point (e) as line (oe). From point (m), extend the line away from point $(f)$ to a new point $(p)$ where the length of ( mp ) is equal to the length of line (oe). Connect point ( $p$ ) to earlier point ( L ) making a new line, ( Lp ). Back off from line ( pL ) by the latitude of the dial, and draw that new line extended, so it meets the extension of line (Lg) at a new point (C) which is the dial center.

SUB STYLE AND THUS SUB STYLE DISTANCE (SD): from the dial center point (C) draw a line through the existing point (f). This line (Cf) is the sub-style, and when measured from noon it was 5.43 degrees or 47.29 from horizontal, or 42.71 from vertical since the noon line was 41.86 from horizontal or 48.14 from the vertical, which is within one tenth of a degree of some other figures.

## Style distance (SD) and Style Height (SH)

a

SH is 18.43 , spreadsheet is 18.87 noon line of final dial

p
b


STYLE HEIGHT (SH): through point (f) draw a line (xy) which is perpendicular to the recently drawn sub-style.

Make a line (fs) along line ( xy ) equal to the existing line (fe), an arc was used for this.
Finally draw line (Cs) which is the style, the angle between (Cf) and (Cs) is the style height (SH) and was measured at 18.43 degrees which is within a degree of the other values derived in this section.

Note that line ( xy ) is not associated with line (fmp), their closeness is coincidental.

HOUR LINES: On the line (Cs) draw a line (fn) perpendicular to it from point (f). The intersecting point is ( $n$ ), the " $n$ " signifying the nodus.

Extend the line (Cf) to point (t) such that the length (ft) equals the length of line (fn). An arc was used, however the arc centered on point (f) through points ( $n t$ ) is not associated with the arc centered on point (f) through point (s), their proximity is co-incidental. This in essence is taking the circle of radials and projecting them from the nodus to the dial plate.

Centered on point (t) draw an arc to hold the radials, this arc is of any size. A line is drawn from the center (t) to where the noon line intersects the line (xy), point ( $m$ ) on the (LC) line. And beginning with the line (tm), draw a set of 15 degree hour angle depictions from the sun. The radials from the center of the circle with the 15 degree hour lines are extended to ensure they meet the line (xy). From the dial center (C), lines are drawn and these are the final hour line for the dial plate.

## Final hour lines



Finally the hour line angles are measured, for consistency they were measured from noon. Of course the angle of the noon line with the horizontal or vertical was previously determined on the first of the geometric worksheets.

The hour lines were measured as shown and were:-

|  |  |  |  |  | $1 p m$ | $2 p m$ | $3 p m$ | $4 p m$ | $5 p m$ | $6 p m$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 0900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 | 1700 | 1800 |
| geometric | 24.26 | 12.74 | 5.31 | 0 | 4.86 | 10.11 | 15.93 | 23.36 | 33.79 | 53.20 |
| spreadsheet | 25.01 | 13.24 | 5.70 | 0 | 4.98 | 9.91 | 15.46 | 22.63 | 33.55 | 53.63 |

And these are in very close agreement. Some of the angles were larger thus having points of intersection closer in, which allowed for increased accuracy compared to a 45 degree declining 20 degree inclined dial.

Reiterating the prior data:-

| noon line from horizontal | 41.86 | from vertical | 48.14 |
| :--- | :--- | :--- | :--- |
| SD 5.43 from noon | 47.29 |  | 42.71 |
| SH | 18.43 |  |  |

## Longitude considerations

A major question is whether to design this dial to consider the longitude of the location, compared to the location of the legal time meridian.

As a rule, dials that are portable, or may be relocated later, should not have the longitude correction built into the dial plate, rather, there should be a tailored equation of time that blends the equation of time (EOT) with the difference in time between the dial design location and that legal meridian.

However for large dials that are designed and built in place, with no intention of subsequent movement, then the longitude correction can be built in.

The choice is left to the diallist. However if there are several dials on the property, it may be simplest to have them all designed the same way.

The next section shows how the geometric method easily accommodates the longitudinal correction.

There is no change to the location of the noon line for that is the local noon, and similarly the angles from the noon line to the vertical and horizontal do not change. The dial center does not change, nor does the style distance or style height.

All that is needed is a simply rotation of the $15^{\circ}$ radials and the re-drafting of those final hour lines themselves.

```
CASE STUDY CONTINUED ~ A southwest declining dial ~ S 50}\mp@subsup{}{}{\circ}\textrm{W}\mathrm{ inclined up a SHALLOW
20
```

HOUR LINES CONSIDERING LONGITUDE: the process is exactly the same as in the last step with one exception. That exception is that while the original noon line is retained since that is local solar noon and the basis for all construction of hour lines, the $15^{\circ}$ radials and consequently their associated hour lines are affected. Those $15^{\circ}$ radials are rotated using a very simple algorithm.

## Final hour lines considering dial longitude



In the northern hemisphere the $15^{\circ}$ radials are rotated clockwise when west of the legal time zone, counter clockwise if to the east. The opposite direction for the southern hemisphere. The amount of the rotation is the longitudinal difference. For the design location for this dial that meant a clockwise rotation of 3.2 degrees.

Finally the hour line angles are measured, for consistency they were measured from noon. Of course the angle of the noon line with the horizontal or vertical was previously determined on the first of the geometric worksheets.

Reiterating the prior data that did not change with a longitudinal consideration:-

| local noon line from horizontal | 41.86 | from vertical | 48.14 |
| :--- | :--- | :--- | :--- |
| SD 5.43 from noon | 47.29 |  | 42.71 |
| SH | 18.43 |  |  |

The hour lines were affected by longitudinal considerations, the new angles were:-

|  |  |  |  |  | 1pm | 2pm | 3pm | 4pm | 5pm | 6pm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 | 1700 | 1800 |
| geometric | 28.41 | 14.50 | 6.68 | 1.44 | 4.10 | 8.86 | 13.95 | 20.62 | 30.46 | 47.80 |
|  |  |  |  | left | righ |  |  |  |  |  |

These angles show a displacement that is appropriate, and the new noon will happen earlier than solar noon, which is correct.


The angles can be checked by using the spreadsheet, as before, except that the base time would need to be adjusted by the longitudinal difference, being $1^{\circ}$ for 4 minutes. In this case, the SHADOWS software was used and the angles found to be in close agreement. Since the geometric method uses the local solar noon as the base, and SHADOWS does not, it is important to add into the SHADOWS depiction the local solar noon, and measure angles from thence.

A cardboard dial plate and gnomon were made, and the site prepared. The column to the right of normal clay brick, was put together without mortar, but the top had been leveled and the dial plate set with a compass after considering magnetic declination. The style alignment and dial plate inclination was also checked. One common level available in hardware stores has a magnetic base thus when compass readings are taken, it must be removed.



Every hour the shadows were checked on the cardboard dial plate, with consideration to the equation of time. If for example the EOT table said 10 minutes was to be added to sun time, then on the hour it would actually be 10 minutes past, so the dial would be checked 10 minutes after each hour.

The mock dial plate worked so the final dial plate was constructed. Copper wire 12 gauge was used and straightened by rolling with a block of wood, then placed on the mockup dial plate and soldered. The gnomon was soldered to the dial plate, and the entire plate then secured to the concrete slab with an epoxy that would survive the elements.

The site for the dial just looked a bit too close to the picket fence, so it was moved a couple of feet, and then the column built. The slope was managed by cutting an $8 \times 8 \times 8$ block at 20 degrees.


| location long: | $108.2^{\circ} \mathrm{W}$ |
| :--- | :--- |
| location lat: | $32.65^{\circ} \mathrm{N}$ |
| magnetic declination: | $10.6^{\circ} \mathrm{E}$ |
| (variation) |  |
| Dial declines | S 50 W |
| inclines | 20 |
| S 50 W true is | S 39.4 W mag |
| 230 true | 220 magnetic |

To get the final dial plate declining 50 degrees to the west, $230^{\circ}$ magnetic, the $8 \times 8 \times 8$ sloped cap was rotated with a compass until the alignment read a tad short of $220^{\circ}$ magnetic. That wedge was secured with mortar placed in the wedge, and with some excess mortar, the final dial plate was mounted.

The top and bottom of the dial plate was checked for level and alignment with the cap it rests on, as well as with the magnetic alignment.

When cured, some quarter round wood was cut and secured with epoxy on the upper part of the wedge and the dial plate's underneath, this would secure the dial plate from accidental jarring. The dial shows local sun time corrected for the longitudinal difference to the legal standard time meridian, thus all that is needed is the EOT correction.

For winter months, the gnomon's position or style's length may be such that the shadow is too far north of the dial plate for its shadow to touch the hour lines furthest from noon. You may need to extend the gnomon or style towards the equator to better show those hours. This is an artistic or aesthetic decision.


## A shallow inclined decliner - S $50^{\circ} \mathrm{W}$ inclined a SHALLOW $20^{\circ}$ - DeItaCAD

A DeltaCAD macro was built using some of the code and functions from the horizontal dial macro. The formulae from the spreadsheet: illustratingShadows.xIs on this book's CD and web site were coded, and with a few refinements, the inclined decliner macro became functional.


The same parameters were used as for the final inclined decliner just constructed.


Inclined Decliner dial. Check hour naming if inc/dec angles excessive.

| Lat: | 32.8 | Long: | 108.2 | RefLong: | 105 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Inc: | 20 | Dec: | -50 |  |  |
| 12 LAT:v: | -48.2 | 12 LAT:h: | 41.8 | Noon:v: | -49.4 |
| SDv: | -42.9 | SDn: | 05.3 | SH: | -18.9 |

The program shows a pictorial of the final dial plate. The style distance from the vertical is 42.9 degrees, compared to 42.71 in the preceding construction. The Style height is 18.9 degrees compared to 18.43 degrees in the previous method.

The hour lines are shown both from noon as well as from the vertical. The DeltaCAD macros offered the following values from the noon line.

| 0900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 | 1700 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 28.5 | 15.3 | 7.1 | 1.1 | 3.9 | 8.8 | 14.2 | 20.9 | 30.7 | and a spurious line |

Whereas the preceding geometric method provided the following information from noon:-

|  |  |  |  | 1 pm | 2 pm | 3 pm | 4 pm | 5 pm | 6 pm |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0900 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 | 1700 | 1800 |
| 28.41 | 14.50 | 6.68 | 1.44 | 4.10 | 8.86 | 13.95 | 20.62 | 30.46 | 47.80 |

The difference of the noon line and noon as depicted is that one is L.A.T. (local apparent time), whereas the other is longitude corrected. Thus the DeltaCAD program seemed to provide meaningful results.

Programs sometimes provide output that is less easily explained. Some call these bugs, others call them features. It depends on whether you are the buyer or the seller.

DeltaCAD, Excel/Open Office, and Shadows Expert software compared for example Latitude 33.5, longitude 112.2, legal meridian 105 , inclined $20^{\circ}$, declining S50W


Inclined Decliner dial. Check hour naming if inc/dec angles excessive.


The Excel/Open Office sheets agreed with the DeltaCAD macro, with an: $\mathrm{SD}=42.7^{\circ}, \mathrm{SH}=19.6^{\circ}$, and the hour line angles from the vertical also agree. Compared to Shadows Expert, the values were within $1 / 10$ of a degree to just over a degree or so. Of note, the Excel and Open Office sheets provide graphical guidance as well. It is guidance only due to aspect and other issues.

## CHAPTER TWENTY

## Cube dials

The cube dial is a hybrid of other dials, when one face loses its accuracy another face takes the lead. The disadvantage to them is that all dial faces need to be coherent and say the same things at the same time. Two common cube dials exist, one is not tilted and has the meridian, horizontal and vertical dials available. The other is tilted and has a polar and two equatorial dials as well as the two meridian dials. And in between the tilted and un-tilted dials there can be other variations.


There is little to add as far as design or construction goes. Small dials might use nails for gnomons, with a virtual style, and screws for the protective plexi-glass cover. If they are not of brass then be aware that they may cause a magnetic compass to read erroneously.

The pictures on this page as well as the entire body of this book should make details somewhat superfluous, however the next page will provide general clues to the titled dial's construction. The un-tilted dial should by now be self explanatory.

Screw placement and when to do what drilling are things to consider. In the tilted dial on the meridian face the plexi-glass retaining screws should be switched 90 degrees because the screw is too close to the gnomon. Plan in advance all screw holes before doing any drilling.

The drill holes for the gnomons should be drilled before the plexi-glass faces are applied, then the retaining screw holes should be drilled, and only then should the plexi-glass cover be married to the dial plate. If you screw the plexi-glass first and then try and drill the hole for the gnomon, Mr. Murphy will pay a visit and ensure that the gnomon hole is off center.

Also see: chapters which discuss pyramid
 dials that incline, decline.

Tilted cubes use one standard set of dial plates, level cubes usually have dial plates that are latitude specific.

## BUILDING THE TILTED CUBIC DIAL

A cube of wood cut from a 4 by 4 inch beam is the cube in this example. A template is in the appendix.


The template is cut and may be glued to the cube using Elmer's glue or PVC pipe glue. Remember that glue fumes may be toxic. A base is cut from a 1 inch thick 4 by 4 inch plank. A hole is drilled in the base, and a hole in the cube. The hole in the cube is on the cube face that faces the pole and faces the ground. If it has anything on it, it would be just data.


Pencil alignment marks on an east or west face and on the winter equatorial face will help when drilling the hole for the dowel support.

## CASE STUDY ~ A CLAY CUBE DIAL

This project was a cube dial, on an 8 by 8 by 8 cinder clock, with the faces or dial plates in clay. This dial will be for Silver City, NM, whose coordinates are:

```
location lat:
    32.75 N
    location long: 
```

Four dial plates were to be constructed, DeltaCAD produced. Chapter 31 discusses DeltaCAD programming in a BASIC like language.

The four dial plates.


Hour and hour line angle VERTICAL NON DECLINER

| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | -77 | -60 | -44 | -29 | -16 | -3 | 9 | 23 | 36 | 52 | 68 | 86 |
| Lat: |  | 32 |  |  | Long: | 108 |  |  |  |  |  |  |



Hour and hour line angle H-DIAL

| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 84 | -69 | -47 | -32 | -20 | -11 | -2 | 6 | 15 | 25 | 39 | 58 | 84 |
| Lat: | 32 | 32 |  |  | Long: | 108 |  |  |  |  |  |  |

While programming using spreadsheets such as Excel or DeltaCAD (using an adaptation of the BASIC programming language) is discussed in chapter 31, there are a number of different ways of designing the standard dials such as the meridian, vertical true south/north, and horizontal dials used in this case study. Some of those different ways are:-

- geometric
- trigonometric
- spreadsheet
- software
- CAD

The horizontal dial and vertical dial share a co-latitude relationship. Thus a horizontal dial for latitude 30 would have the same layout as a vertical dial for latitude 60 , with some exceptions. The exceptions are that calendar lines are reversed, and the direction of rotation of the hour lines for longitude correction is also reversed. And similarly the marking of the hours themselves.

The east and west meridian dial similarly are almost a mirror of each other. However longitude correction causes the hour lines to move up on one face, and down on the other. Thus only two dial plates need be reviewed, the horizontal and the meridian. And the diallist must then remember to handle longitude correction appropriately, name calendar lines appropriately, and mark the hour lines appropriately.

The horizontal and vertical dial hour lines are based only on the angle of the style with the substyle (latitude if true north or south). The linear height of the nodus is only used for calendar information. The two are independent, and when the tip of a style is used as the nodus, the benefit of a longer style is lost, and that benefit is accurate hour line reading.

A meridian dial that does not decline uses the linear height of the style from the sub-style as the entire basis for the hour lines, and any part of that style can be used to mark the calendar data

Assuming no longitude correction. Assuming hour lines only, and no calendar lines.

| GEOMETRIC |  | HORIZONTAL |
| :---: | :---: | :---: |
| TRIGONOMETRIC | linear distance from sub-style =glh * tan(hours) [hours is from 6 am or 6 pm ] [glh is gnomon linear height] | angular distance <br> from sub-style <br> $=\operatorname{atan}(\sin ($ lat $) ~ * \tan (\mathrm{lha}))$ <br> [Iha is $15^{*}$ hours from noon] |
| SPREADSHEET | tan uses (radians) | tan \& sin use (radians) |
| SOFTWARE | for $\mathrm{hr}=7$ to 11 step 1 ' or 1 pm to 5 pm etc $\mathrm{d}=\mathrm{glh} * \tan (\mathrm{rad}(\mathrm{hr}))$ <br> ' calc x1,y1 x2,y2 <br> ' draw line next hr | for $\mathrm{hr}=6$ to 18 step 1 <br> $a=\operatorname{deg}(\operatorname{atan}(\sin (l a t) * \tan (\mathrm{lha})))$ <br> calc end $x, y$ point <br> draw line <br> next hr |
| CAD | DeltaCAD use macro TurboCAD draw lines | DeltaCAD use macro TurboCAD draw lines |

The DeltaCAD dial plates when printed on $81 / 2$ by 11 inch paper were perfect for a dial plate for an $8 \times 8 \times 8$ concrete block. Holes were punched on the hour lines and other key places, and for the east and west meridian dials, the paper was cut along the calendar lines.

While the horizontal and the vertical dial use an angled gnomon, and in this case no calendar lines were present, this was not true for the meridian dials which had calendar lines, and in addition the gnomon style paralleled the dial plate and thus the critical dimension for the gnomon was its linear height above the sub-style. For this reason the DeltaCAD macro also shows the gnomon linear height, style to sub-style. This line would be inscribed on the meridian dial plate because clay shrinks, so this line should shrink similarly in the correct ratio. In fact the line was inscribed twice to show the linear contraction, about 11\% in the case of this terra-red clay.

USING CLAY AND MAKING SLIPS


Clay can be purchased at ceramics and pottery supply stores. In this case a clay designed for cone 6 (a measurement of heat) which is about 2230 degrees $F$ (a measurement of temperature). Heat and temperature are not the same thing.

Cone 6 clay was chosen because of the need for stoneware, in other words a need for something that could withstand the freeze thaw cycles of the seasonal extremes.

A section of clay was cut from the slab and pressed to the approximate size.


The noon line was offset because of longitude correction, and a cutout was made after the slip had been applied for the colored texture, but before firing. Bisque firing is a bit above 1800 degrees F and fixes the slip and clay. They are not waterproof, but they are fixed and glaze is used over the bisque dial plates to waterproof them and to bring the dial plate to life.

The clay surface was prepared by passing a wooden flat sheet over the clay surface, with a small amount of water. This removed scratch and other marks.

The paper printout from DeltaCAD was placed on the clay surface and the holes punched on key points was transcribed to the clay. The dial center, the tips of the hour lines, and the line that for a non longitude corrected dial would be 6 am to 6 pm was marked. This is critical for subsequent dial alignment.


The bisque firing is not done for several days because the moisture must leave the clay body and its slip. Do not move the clay pieces while drying as if you do the clay can remember and curve up. After the bisque firing and the subsequent cool down period, the glaze can be added.

The three pieces can easily be seen, along with the gnomon style cutout, and the alignment piece at the bottom of the two dial plate halves.

The preceding pictures show the horizontal dial plate, below is the vertical dial plate, again before the gnomon sub-style cutout was incised.


The gnomons were set in place with epoxy, and as it dried the dial plates were set in place temporarily to ensure the gnomon was placed correctly on the block, otherwise you could have a nicely oriented gnomon but a dial plate that didn't fit on the block.

When all plates were cured, then bisque fired, then cooled, then glazed, then fired at cone 6 or 2230 degrees $F$, the dial plates were next affixed to the $8 \times 8 \times 8$ concrete block. The medium used was Versabond, designed for exterior use.

When the gnomons were rigid, then the dial plate fixing medium (Versabond exterior) was used to affix the dial plates to the block. Protractors were used to ensure the gnomons were aligned at latitude, and that the meridian dial equinox line was correct. A pair of dividers was used to pick up the meridian dial linear height from the incised lines for just that purpose, and thus the gnomons were of the correct length.

When all was secure then a sand grout was applied of an adobe color, which is somewhat standard in New Mexico.


The grout was wiped clean off the
 dial plates with a damp sponge, and when the grout was almost firm, then the same sponge was used to texture the grout.

Above to the right is the finished dial taken from the southeast, and below to the left is the same dial design but in a slightly different motif.

These two dials were accurate, and certainly a welcome addition to the garden.

These dials were fired at the vitrification point, and sealed with a clear beeswax paste. After many freeze thaw cycles with intermittent moisture, they showed no signs of deterioration.

## VERTICAL DECLINING DIALS AND THE CUBE DIAL



The style of the declining dial's gnomon still parallels the polar axis as it is an hour angle dial.

However unlike the true north, south, east, and west facing vertical dials, the declining dial's plate itself is not aligned with the true cardinal points.

The dial plate is at an angle to the north/south meridian (longitude) line or the east/west latitude line.

This section deals with the cube dial that declines a little or a lot, however it is not tilted, it does not incline, recline, or procline.

- using available software programs
- using trigonometry - the formulae
- using spreadsheets which use the trigonometric formulae
- using geometry in step by step mode
- Actual construction of two final cube dials

NOTE: A design for South xx degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. The North facing decliner gnomons are inverted, and the vertical is midnight. If longitude correction is applied, care must be applied as hour lines shift.

- cube dial with dial plates almost east, south and west
- cube dial with dial plates almost southwest and southeast

NOTE: Gnomons for declining dials are usually rotated by an angle called the style distance (SD) and the sub-style to style angle is called the style height (SH). It is worth reviewing SD and SH.

Vertical almost south decliners can also use gnomons that are not rotated, they are in essence the gnomon that would exist if the dial were true south, only the hour angles would be different. This works well but loses the other benefits of using SD and SH, and is thus less common. Both methods are discussed extensively in this book.

## CASE STUDY ~ CUBE DIAL ON A COLUMN ~ ALMOST CARDINAL POINTS

Many years ago, I lived with a lovely cube dial on a pedestal, and the time had come to recreate it as best I could


In the background of the picture used as the cover for this book is the old dial. It stood towards the east of our lawn, its gnomons were missing however the remains of the hour lines were visible.

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ}$ | N |
| :--- | ---: | :--- |
| location long: | $108.2^{\circ}$ | W |
| magnetic declination: | $10.6^{\circ}$ | E |

Using TurboCAD, a model of the dial was built, only a south facing gnomon was shown, and the solar mesh used for detecting the sun's shadow at any date and time was added. The 3d stereogram shows the shadow at 10 o'clock for latitude 33, not corrected for longitude nor the equation of time.


A location was selected, and then a square base area was dug to just below the frost line, and the base made to parallel the road and wooden picket fence. This would mean that all dial plates would be decliners because the road was not aligned with true north.

The photo is taken from the east looking slightly north of west.

In these case studies, SD (style distance) and SH (style height) are the angular measures for vertical declining dials.

## PEDESTAL

DAY 1: The square base area is shown to the right, and was dug down deeper to the frost line, then a concrete base was poured. No rebar was used so if a magnetic compass was used for determining the cube's declination, magnetic disturbance would be minimized.

The alignment with the road was done by first verifying that the picket fence paralleled the road, and then that the base paralleled the picket fence.

Concrete was poured and a layer of blocks positioned and set level. Alignment was ensured by using a
 string paralleling the picket fence and touching the blocks. All in all four courses were added, and the top course had a mesh inserted in the four holes of those two upper blocks, and some concrete poured in filling the holes about half way up. Then all was allowed to set.


Those holes would be used when the rest of the column was added.

DAY 2: The column was built. This used 8 of the 8 by 8 by 16 inch concrete block, and a sack of mortar.

DAY 3: An 8 by 8 by 8 inch block was added along with some decorative mortar work.

And a 4 inch 12 by 12 inch capping block was then added.

This capping block would allow mortar or concrete to secure the 8 by 8 by 8 and the 4 inch tall 12 by 12 and be a firm base for the last 4 blocks that would form the cube that would rest on this column.

Because of the holes in the block and because of the layering, some wood was used to stop the mortar or concrete from seeping out, which in no way reducing the strength of the column.

Again, no rebar was used.
The decorative mortar work has radials on it, they are to make the column more interesting, and to deflect the eye from certain unevenness.

DAY 4: The cube itself was assembled on the column. The top four blocks were predrilled with a concrete bit, each hole being 1.5 inches from the top or bottom, and the side. These holes could be later used to affix the dial plates, each of which would be a little larger than one square foot.

Also, wood dowels were used in the concrete used to affix the top cube, this would add to lateral and torsional strength while avoiding metal rebar. Since those dowels would be completely encased, the chance for rot was substantially reduced.

The cube had four faces that were predominantly north, south, east, and west. The top could be used but giants are few and far between, so the more visible faces would only be used.

The street was not true north aligned nor was the pedestal, and the cube was off a bit as well.

So the next step was the determination of the south facing vertical surface's declination.

And then what medium would be used for the dial plate.

## ALIGNMENT

DAY 5: The alignment of the cube had to be measured. There are several techniques. One takes all day and measures the intersection of the sun's shadow on some
 circles. Another measures the sun's azimuth several times and uses an accurate clock. Another uses the astrocompass. And many people use a magnetic compass. Because there was no rebar, a Brunton surveyor's compass was used. The method used was to position oneself south of the cube, and aligned such that the east face was just no longer visible, then a bearing taken of the east edge of the south facing surface of the cube. It measured $355^{\circ}$. The process was repeated for the east and the west alignments, they were $265^{\circ}$ and $085^{\circ}$ so they agreed. This was not done for the north facing surface as a nearby parked car would have upset the magnetic field. Since three of the faces all agreed and the observer was a distance of about 20 feet from the dial, this reading would probably be accurate.

South facing surface: $355^{\circ}$ magnetic


| $0^{\circ}$ magnetic is | 10.6 true |
| :---: | :--- |
| $355^{\circ}$ magnetic is | 365.6 true |
| which is also | 5.6 true |


| location lat: | $108.2^{\circ} \mathrm{W}$ |
| :--- | ---: |
| location long: | $32.75^{\circ} \mathrm{N}$ |
| magnetic declination: | $10.6^{\circ} \mathrm{E}$ |

## SOUTH FACING DIAL PLATE

DAY 6: Having determined that the south facing surface was south $5.6^{\circ}$ west, this was a declining vertical dial, and the formulae gave us hour lines, a style distance and a style height. Style distance (SD) is the distance the gnomon is rotated from the vertical, and style height (SH) is the angle the style of the gnomon makes with the sub-style.


Drawing pictures of what is logical will help avoid mistakes and will make sense of the numbers you calculate for $\mathrm{SH}, \mathrm{SH}$, and hour line angles

NOTE: see chapter 16 for a south, east, and north declining dial on an almost square building about 4 or 5 degrees off the cardinal points, and using DeltaCAD macros for the dial plates and calendar lines.

The formulae for south decliners are:-
The hour line angles are based on: $\quad z=\operatorname{atan}(\cos (l a t) /(\cos (d e c) \cot (h a)+\sin (d e c) \sin (l a t))$
Gnomon rotation employs the following formula:-
Gnomon offset from vertical is: $\quad \mathbf{s d}=\operatorname{atan}(\sin (\mathrm{dec}) / \tan (\mathrm{lat})) \quad$ Style Distance
Style and sub style angle is: $\quad \mathbf{s h}=\operatorname{asin}(\cos (l a t) * \cos (d e c)$ ) Style Height


However, the illustratingShadows.xls file allows us to calculate these results.
The spreadsheet needs to consider longitude differences between the location and the legal meridian. While horizontal dials tend to be portable, a vertical declining dial is obviously tailored, thus the final dial plate should also be tailored.

Note that while the angles of hour lines change when the longitude is considered, the style distance (SD) does not.

The hour line angles can be readily checked against the SHADOWS software, and the style distance SD, similarly. The style height in SHADOWS requires a conversion from linear dimensions to trigonometric to angular. They were within less than a degree. When using SHADOWS software, remember to display the dial data with the longitude correction. This is because even if you enter a dial design location's longitude, the software does not make the longitude correction unless you specifically ask it to.

| Hour line angles from vertical. |  |
| ---: | ---: |
| TIME |  |
| hh.mm | DEC SxxW |
| 6.00 | 5.6 |
| 7.00 | 82.9 |
| 8.00 | -79.3 |
| 9.00 | -61.6 |
| 10.00 | -44.9 |
| 11.00 | -29.6 |
| 12.00 | -15.6 |
| 13.00 | -2.5 |
| 14.00 | 10.1 |
| 15.00 | 22.7 |
| 16.00 | 36.0 |
| 17.00 | 50.3 |
| 18.00 | 65.9 |
| STYLE:SD | 82.9 |
| STYLE:SH | 8.6 |



A dial plate of cardboard was first made, tacked to the column, and checked throughout the day.

Two sets of hour lines are seen. One is the hour lines assuming no longitude correction, the other took longitude into account.

An equinox line was drawn where it would look appropriate, and where it could be helpful securing the gnomon. The gnomon would be a copper sheet, the dial plate would be opalescent glass that would show a good shadow.

The paper dial proved to be accurate, so the final dial plate was then made from permanent materials, in this case, glass and copper.

This dial could have calendar lines added, and Italian lines to good effect, however, it was decided to have a larger gnomon, with a nodus notch for the equinox, and leave it at that. Other dials in the series might provide calendar information.

DAY 7: The actual glass plate is constructed.


An iridescent white glass was used for the dial plate, it showed a good shadow, and the shadow reflection was tested at many angles.

The noon line was the only hour with a marker.
The segments were assembled with $1 / 4$ inch copper foiled, soldered, and copper plated. The border was $1 / 4$ inch copper tube.

The dial plate was affixed using an adhesive suitable for tile and concrete block.

One piece of glass was not cut as well as I normally cut them. No excuse for it, but since this was a textured iridescent glass, a new piece would have upset the look. The technique was to add a brass dragonfly to distract the eye from the hour line being slightly offset. It can be seen on the 10 am hour line just above the equatorial line.

The lead lines (soldered copper foil) on the dial plate cannot be too tall as because they themselves can cause shadows at extreme solar angles.

Another distracter for the eye was added on the lower right corner, a glass jewel. Similarly a glass jewel was added where the hour lines converge. Care must be taken that such a jewel does not confuse the hour line shadow.

## DAY 8: EAST FACING DIAL PLATE

Just as the south facing plate was a decliner, so also was the east and the west facing plates, in this case it was a great decliner. Again, no calendar lines would be designed in since the object was to emulate a cube dial from ones youth.

Since the south dial was S 5.6 W, the west facing dial would be S 95.6 W , which is more than 90 degrees. It is also North 84.4 West. It uses the same spreadsheet as would be used for S 84.4 E.

Using S 84.4 E in the illustratingShadows.xls spreadsheet, and using the S...E column, the style distance (SD) was found to be -57.1 degrees.



The style height was 4.7 degrees, so the style would


And a reasonableness check for a great decliner such as this would be that this should be close to the dial's declination from south. And 4.7 degrees is close to the

The SD and SH reasonableness checks are for great decliners only just a bit off from the cardinal points.

When the SOUTH FACING DIAL shown above was designed (day 6), the orientation of all gnomons was drawn. This helped orient which gnomon went where, and, where the dial centers would be for the east and west dials. The dial center is where the extended style of the gnomon eventually meets the extended dial plate.

The next step was to derive the hour lines themselves. Since this was a great decliner, the hour lines would be angled with a convergence point some distance from the dial itself. Pure east and west dials have hour lines that are parallel and meet at infinity

TurboCAD was used to draw the hour lines, and also a square around the usable area, and even the style was drawn by projecting it, i.e. rotating it by 90 degrees so it was flat on the dial plate. The biggest problem was figuring out how to tell TurboCAD ~ OPTIONS ~ ANGLE to work. That tool is not intuitive and you may have to try several times before you get it to work for you.

## DAY 9: WEST FACING DIAL PLATE

The west facing dial used the S 84.4 E figures except for the lower column, covering noon to 6 pm.


Again, the spreadsheet hour angles become angled lines on a sheet of paper.

The style was easily drawn as a rotated projection of a style.


Having drafted the lines for the hours, and the style distance (SD) and style height (SH), they were printed out. While TurboCAD was used, any low cost drafting package would work.

After the lines were drafted, then a surrounding containing box was placed where the dial plate looked more satisfying.

Then the print out was placed in the middle of a cardboard mockup, and every line drawn to extend over the cardboard, remembering to use a different color for the SD and SH, Of course a leveling line was be included, visible at the top.

Then the cardboard dial plates were placed on the dial vertical surfaces.


While these dial plates were not made to display calendar lines, there is some benefit to an equinoctial line. If desired, the equinoctial line can be drawn easily. It is perpendicular to the substyle line, which was drawn using SD. Its distance from the selected nodus is determined by projection and geometry, or by simple trigonometry.

An exaggerated view of the gnomon is shown to the right.

The equinox range of movement of the sun is on a disk perpendicular to the style. A line is drawn perpendicular from the style until it intercepts the sub-style, see (1).

Then, perpendicular to the sub-style (2) from that point of intersection, is drawn the equinoctial line.


With glass and copper dial plates, an equinoctial line can be helpful as an additional anchor for the gnomon. When any calendar line exists, there must be a nodus. While for a predominantly east or west dial, a north or south edge of the gnomon can be used, for a south facing dial plate to have a long shadow, the style may need to be considerably longer than normal. Thus the style might need a nodus somewhere in the middle of the style. If an equinoctial line is drawn first at a place that is aesthetically pleasing or helpful in supporting the gnomon, reversing the above process will locate the required nodus.

## DAY 10: FINAL DIAL PLATE CONSTRUCTION

As before, an iridescent glass that shows the shadows clearly would be used, 1/4 inch copper foil, and the final plate would be affixed with a tile adhesive.


For the final dial plate, the print out was placed in the middle of the glass and every line drawn to extend over the surface. A different color was used for the SD and SH lines, and for the leveling line which was also included.

The glass was cut and foiled. Because some glass pieces were rather small, 7/32 and 3/16 foil was also used.

Always pick glass that will show the shadow from usable angles, some glasses may not be suitable.

Some rules of thumb: these gnomons are offset from a nearby hour line and are not rectangular. The way to tell if they are correctly aligned (not necessarily correctly set) is to rotate the dial plate and see if the style parallels each hour line. This ensures correct alignment, however as stated, it does not protect from lateral and vertical gnomon errors, only rotational errors.


The west dial had tape holding it in place until the adhesive had set.

The east facing dial shows 11 am standard time, or 12 noon summer time. The south facing dial showed the same time, which was a relief!

The east and west dial gnomons had a nodus notch in the center of their styles rather than using a gnomon edge.


The south dial gnomon had a nodus cut appropriately for the equinoctial line. Additionally, the south dial's gnomon was rounded to minimize injuries to people who got too close.

These dials were all calculated for the latitude of the town, their noon time lines were offset because the dials were also corrected for the longitude difference between the location and the legal time reference longitude.

The east and south dials have legal standard time marked at noon, the west facing dial is marked for noon and 1 pm standard time, as well as 1 pm and 2 pm summer time.

The equation of time is still required. To that end, a small plaque was made for passers by. Finally, the entire dial assembly was stuccoed with a sanded adobe colored grout.

This sundial has three faces, one for morning hours, one for middle of the day, and one for afternoon hours. It was designed for Silver City, NM latitude 32.75 N and longitude 108.2 W. Added to that the streets are 5.6 degrees east of true north. These dials are corrected to show standard time, so add one hour if daylight savings time is in effect. Also, pocket watches do a good job of showing 24 hours, but not of the daily rotation of the Earth. To force the universe to comply with artificial pocket watches, a table of corrections is used, called the equation of time or EOT, see the figure of 8 chart to the right.

For information about sundials please go to:
www.illustratingshadows.com


## CASE STUDY ~ A CUBE DIAL ON A COLUMN ~ mostly SWISE points

This dial was to be a similar design to the previous example, but to be primarily SE and SW as opposed to mostly South and mostly East and West. The design method was the spreadsheet. This dial would be for Silver City, NM, whose coordinates are:


| location lat: | $32.75^{\circ} \mathrm{N}$ |
| :--- | ---: |
| location long: | $108.2^{\circ} \mathrm{W}$ |
| magnetic declination: | $10.6^{\circ} \quad \mathrm{E}$ |

In these case studies, SD (style distance) and style height (SH) are the angular measures for vertical declining dials.

The column was built like the previous dial column using 12 of the $8 \times 8 \times 16$ concrete blocks, a sack of pre mix concrete and some sacks of mortar.

The pedestal was built to blend with surrounding construction angles.

When finished, the declinations were measured with two different compasses from several directions.


The southwest facing vertical surface faced magnetic 213 degrees, and the southeast vertical faced magnetic 123 degrees.

With an almost 11 degree magnetic declination to the east of true north this meant that the magnetic declination had to be added to the magnetic bearing to get the true bearing. Thus the faces were 224 degrees true and 134 degrees true respectively.

Now $224^{\circ}$ is southwest, subtracting 180 for south we get $\mathrm{S} 44^{\circ} \mathrm{W}$ for the southwest dial plate.
And $134^{\circ}$ is southeast which when subtracted from 180 gives $S 46^{\circ} \mathrm{E}$ for the southeast facing dial plate.


It was not the hoped for 45 degrees, but the exercise was worth that 2 degree difference.

## SOUTH FACING DIAL PLATES

Having determined that the south facing plates were $S 44^{\circ} \mathrm{W}$ and $\mathrm{S} 46^{\circ} \mathrm{E}$, the hour line angles must be deduced, as well as the sub-style, and style height.

Also, the longitude is $108.2^{\circ}$ which is $3.2^{\circ}$ west of the legal time $105^{\circ}$ meridian, that makes about a 12 minute difference.

The formulae for south decliners are:-
The hour line angles are based on: $\quad z=\operatorname{atan}(\cos (l a t) /(\cos (d e c) \cot (h a)+\sin (d e c) \sin (l a t)))$
Gnomon rotation employs the following formula:-
Gnomon offset from vertical is: $\quad \mathbf{s d}=\operatorname{atan}(\sin (\mathrm{dec}) / \tan (\mathrm{lat})) \quad$ Style Distance
Style and sub style angle is: $\quad \mathbf{s h}=\operatorname{asin}(\cos (l a t) * \cos (d e c)$ ) Style Height


However, the illustratingShadows.xls file allows us to calculate these results.
The spreadsheet needs to consider longitude differences between the location and the legal meridian. While horizontal dials tend to be portable, a vertical declining dial is obviously tailored, thus the final dial plate should also be tailored.

To make the tables simpler to follow, in the tables for the four faces that follow a correction of 12 minutes is used rather than 12.8.

The tables that follow use decimal hours (hh.hh) as opposed to hours and minutes (hh.mm). Always check to see whether decimals or normal time is being used.

Note that while the angles of hour lines change when the longitude is considered, the style distance (SD) does not.

The hour line angles can be readily checked against the SHADOWS software, and the style distance SD, similarly. The style height in SHADOWS requires a conversion from linear dimensions to trigonometric to angular. They were within less than a degree. When using SHADOWS software, remember to display the dial data with the longitude correction. This is because even if you enter a dial design location's longitude, the software does not make the longitude correction unless you specifically ask it to.

For the SOUTHWEST (noon to tea time) face we derived the following data.

| ENTER minutes correction to a normal dial |  |  | This shifts hour lines appropriately |
| :---: | :---: | :---: | :---: |
| 12 |  |  |  |
| ENTER A LATITUDE <br> 3275 | Legal $\longrightarrow$ | L.A.T. | Hour line angles from vertical. |
|  | TIME (decimal) | TIME (decimal) | DEC SxxW |
|  | hh.hh | hh.hh | 44 |
| ENTER A WALL DECLINATION | hours and decimals of hours |  |  |
| $44 \mathrm{l} \begin{aligned} & \text { SxxW is } \\ & \text { positive }\end{aligned}$ |  |  |  |
|  | $\rightarrow 12.00$ | $\longrightarrow 11.80$ | -3.6 |
|  | 12.5 | 12.30 | 5.1 |
|  | 13.00 | 12.80 | 12.6 |
|  | 13.50 | 13.30 | 19.3 |
|  | 14.00 | 13.80 | 25.2 |
|  | 14.50 | 14.30 | 30.6 |
|  | 15.00 | 14.80 | 35.6 |
|  | 15.50 | 15.30 | 40.3 |
|  | 16.00 | 15.80 | 44.9 |
|  | 16.50 | 16.30 | 49.5 |
|  | 17.00 | 16.80 | 54.1 |
|  | 17.50 | 17.30 | 58.8 |
|  | 18.00 | 17.80 | 63.8 |
|  |  | STYLE:SD | 47.2 |
|  |  | STYLE:SH | 37.2 |

Cross checking with other systems we found the style distance and style height were within tolerances. The 12:00 line (used 11.8 not 12.0 because of longitude correction) was - 3.6 and so on. The lines to be drawn, including SD, are copied below in order,

| $12: 00$ | $12: 30$ | $13: 00$ | $14: 00$ | $15: 00$ | 1600 | SD | $17: 00$ | $18: 00$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| -3.6 | 5.1 | 12.6 | 25.2 | 35.6 | 44.9 | 47.2 | 54.1 | 63.8 |



Similarly for the SOUTHEAST (breakfast to lunch time) face we derived the following data.


Cross checking with other systems we find the style distance and style height are within tolerances. The 12:00 line (used 11.8 not 12.0 because of longitude correction) was -3.5 and so on.

The schematic for the dial plate is to the right.
In this case the sub-style and 0800 line was within 0.3 degrees of each other and thus essentially co-located.


The next step was to decide what would be on the dial plates, and where. By drawing the substyle by using the style distance (SD), and the style height (SH), and thus the style, it was easy to decide what lines to have on the final dial plate, and what the gnomon would be like.


Two boxed areas, shaded above, looked appropriate for the dial plates. The next step was to make the cardboard mockups.


It turned out that there was a protractor drafting error on the hour lines for the SE morning mockup plate. Not a math error, only a drafting mistake. This emphasized the need for care. Better to receive a wake up call and get the final plate correct than have to remove and redo it.

A picture of the SE mockup is shown to the left, and it tested well, as did the SW mock plate, thus the final dial plates were constructed.

The final SE plate is shown to the right shortly before it was mounted.


The preceding page showed the southeast dial plate in both mockup and final glass form before mounting.


To the left is the south west dial plate as a glass sheet before cutting.

The hour lines have been sketched out as well as the sub-style line (style distance).

To the right is the final southwest plate cut and soldered.

The major lines were cut in the glass, however some hour lines and the roman numerals were crafted with awg 14 copper wire. That copper wire will expand and contract with temperature, so to ensure rigidity each was soldered along its length and the ends held in place with a wooden dowel as each part was soldered.

Before the gnomon was rigidly soldered to the dial plate, both the new dial plate and the paper mockup were affixed to a board so
 that each was parallel to the other.
This board was then rotated to observe the indicated hours. They matched. However if they had not then there was time to redo the gnomon or the hour lines made using wire. Foiled lines where glass was cut obviously could not easily be corrected.

The gnomon was marked on copper sheet, both the sub-style to the style, as well as another triangle which was the triangle made by the sub-style and the nearest hour line. That technique generates a rigid gnomon that is strong and less likely to bend over the years.

The calendar lines of which there were two, the winter solstice and the equinox, were drawn with a laser trigon and simple geometry respectively. The laser technique is discussed towards the end of chapter 23.

The NORTHEAST facing plate gets the sun also, so we may use the southwest plate's figures albeit mirrored.

| ENTER minutes correction to a normal dial |  |  |  | This shifts hours lines appropriately |
| :---: | :---: | :---: | :---: | :---: |
| 12 |  |  |  |  |
| ENTER A LATITUDE |  | LEGAL $\longrightarrow$ | L.A.T. | Hour line angles from vertical. |
| 32.75 |  | TIME (decimal hours) | TIME (decimal) | DEC SxxW |
|  |  | hh.hh | hh.hh | 44 |
| ENTER A WALL DECLINATION |  | 6.00 | 5.80 | 63.8 |
| 44 | SxxW is positive | 6.50 | 6.30 | 69.2 |
|  | SxxE is negative | 7.00 | 6.80 | 75.2 |
|  |  | 7.50 | 7.30 | 81.8 |
|  |  | 8.00 | 7.80 | 89.4 |
|  |  | 8.50 | 8.30 | -82.0 |
|  |  | 9.00 | 8.80 | -72.1 |
|  |  | 9.50 | 9.30 | -61.0 |
|  |  | 10.00 | 9.80 | -49.0 |
|  |  | 10.50 | 10.30 | -36.6 |
|  |  | 11.00 | 10.80 | -24.6 |
|  |  | 11.50 | 11.30 | -13.5 |
|  |  | 12.00 | 11.80 | -3.6 |
|  |  | 12.50 | 12.30 | 5.1 |
|  |  | 13.00 | 12.80 | 12.6 |
|  |  |  | STYLE:SD | 47.2 |
|  |  |  |  | 37.2 |

## SD 47.2 <br> SH 37.2

The hour line angle sign's change. What this means is that they passed through the horizontal. A depiction of the dial plate is shown to the right, and it may be tested, made, then installed on the northeast face.

The northwest facing plate process was the same.



The NORTHWEST facing plate got the sun also, so we used the southeast plate's figures albeit mirrored.


The hour line angle sign's change. What this means is that they passed through the horizontal. A depiction of the dial plate is shown to the right, and it was tested, made, then installed on the northwest face.


The first dial plate constructed was the NORTHEAST facing plate. This used an iridescent white glass that threw a good shadow. The dial plate was cut, each piece of glass foiled with $7 / 32$ inch foil, tacked, and the back then soldered, and again the front.

The gnomon was cut and applied, and roman numerals added. The gnomon was cut with the $90^{\circ}$ at the style, thus the equinox line was in this case at the gnomon's base, at right angles to the style distance line extended. With part of the gnomon construction on the rear side of the plate, that meant more places to solder to the gnomon, increasing its durability.

Copper foil can alter the relative distances on the dial plate, especially when soldered. So the final glass plate was attached to a board along with the original model. The two being secured, they were rotated at various angles to the sun, here they show about 9:30 am. All the hour lines matched the model providing assurance that so far the final dial plate was correct. The equinox line looks strange, it is in fact correct, and the semi-circle is not a calendar line, it merely marks half hours. Both of those features are conversation pieces.


That plate was affixed to the appropriate place on the cube on top of the column using a mastic, and along the top an epoxy. Mastic is not a perfect adhesive for glass but possible slightly better than thin set mortar in this instance.

Either way, the final cube would have a surround of grout or stucco enhancing adhesion by reducing water drainage behind the dial plate.

The equinox line for the north east face looks out of place especially since there were no solstice curve lines drawn. This particular face would only be seen by persons standing at the front door.

The final dial is shown to the right looking from the east towards the west.


## CHAPTER TWENTY ONE

## Altitude Dials

It was mentioned earlier in chapter 4 that there were three methods of telling the time from the sun, they were by measuring its:-

| altitude | (how high the sun is), or |
| :--- | :--- |
| azimuth | (how far east or west the sun is), or |
| hour angle | (the sun's angle around the north south polar axis) |

The altitude of the sun is its angular height above the horizon, it is latitude dependent, and is not corrected by tilting a dial as can be done with hour angle dials without considering other factors such as north south alignment which defeats the altitude dial's benefit of not needing a compass. The altitude of the sun at any time is determined by the date, time and the sun's declination.

ALTITUDE: $\quad$ The sun's altitude is its angle when looked at face on in degrees
alt $=$ degrees $(\operatorname{ASIN}(\operatorname{SIN}($ decl $) * \operatorname{SIN}($ lat $)+\operatorname{COS}($ decl $) ~ * ~$ COS(lat)*COS(lha) ) )

Iha local hour angle of the sun
lat angular direction north or south of the equator ~ do not confuse with LAT or L.A.T. (uppercase) meaning Local Apparent Time
decl declination of the sun which is date dependent
DECLINATION: $=(23.45 * \sin (r a d i a n s(0.9678(j d-80)))$
jd julian day of the year being 1 to 365

| Jan | Feb | Mar | Apr | May | Jun |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 31 | 59 | 90 | 120 | 151 |
| Jly | Aug | Sep | Oct | Nov | Dec |
| 181 | 212 | 243 | 273 | 304 | 334 |

alternatively, the declination can be found by
$=$ degrees $\left(0.006918-0.399912^{*} \cos (d a)+0.070257^{*} \sin (d a)\right.$
$-0.006758^{*} \cos \left(2^{*} d a\right)+0.000907 * \sin (2 * d a)$
$-0.002697^{*} \cos \left(3^{*} d a\right)+0.001480^{*} \sin \left(3^{*} d a\right)$
da day angle
$=2$ * $\mathrm{pi}^{*}(\mathrm{jd}-1) / 365$ (in radians, is an intermediate figure)
NOTE: The tables printed in this chapter are for illustration purposes, for actual dial design you should use the tables in the appendices, or print your own using the spreadsheets.

NOTE: Some pictorials have curves that do not seem to be smooth or perfect, and not up to the standards used in this book. The point was to portray the software mentioned as it actually works, and not to pretty it up. Thus such pictorials are true to the software used. Some pictorials are deliberately not to scale and a note states that fact. However, specific templates are of course accurate.

## THE CAPUCHIN DIAL (An altitude dial)

A Capuchin dial design begins with a horizontal line so that verticals can be drawn. A vertical is then drawn so that a semicircle may be drawn. The semicircle then has 15 degree arcs drawn and where those arcs hit the circumference of the semicircle, then verticals are drawn. These are the hour lines.


This is the first part of the design, and is nothing more than the hour line construction.

There are 5 added vertical lines, 6 if you include the very first vertical, and those 6 lines together with the place where the semicircle meets the horizontal on the left form the hour lines, 12 noon on the left and then 11 am and 1 pm next, then 10am and 2 pm , etc.

Where the top of the semicircle meets the horizontal line on the left, then is drawn a line up and to the right, at an angle equal to the dial's latitude. And, where that latitude line meets the original vertical, a line perpendicular to that latitude line is drawn, this is the calendar line.


Then lines are drawn for the solstices, their angle is always 23.5 degrees, which is the amount the sun appears to move below and above the equator. The lines are thus $23.5^{\circ}$ above the latitude line, and $23.5^{\circ}$ below it.

The top left line is the Winter solstice, December and January in the north hemisphere, the lower right line is the summer solstice, June July in the northern hemisphere.

June and July's angles with the sun are about the same, and December and January share similar angles also. These angles are called declination angles. One can be more precise and use the angles in the table below to make the calendar lines above and below the latitude line.

| Date | declination | Date | declination | Date | declination | Date | declination |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $1 / 1$ | -23.1 | $2 / 1$ | -17.3 | $3 / 1$ | -7.9 | $4 / 1$ | 4.2 |
| $5 / 1$ | 14.8 | $6 / 1$ | 21.9 | $7 / 1$ | 23.2 | $8 / 1$ | 18.2 |
| $9 / 1$ | 8.6 | $10 / 1$ | -2.9 | $11 / 1$ | -14.2 | $12 / 1$ | -21.7 |

One can also use the tables in the appendices, calculate them yourself using the formulas in the appendix, or build a spreadsheet using the techniques in the appendix on formulae. You can also measure the sun's angle with the horizon when it is at its highest point (solar noon) and subtract that from the co-latitude ( 90 degrees minus the latitude). You could also choose to use the middle of the month, whereas in the short table above, the first of the month was selected.

10. For each date on the calendar line, draw an arc from the 12 o'clock point towards the right.

The monthly lines are drawn above the latitude line for the minus declinations (October through February in the northern hemisphere), and below the latitude line for positive declinations (April through September in the northern hemisphere).

In the southern hemisphere the angles are switched.

One point per month causes one arc to be drawn per month, see pictorial on the left, where only the summer and winter solstice arc are drawn.

The dial is almost complete, holes are drilled in the calendar line, one hole per month point. And a wire or weighted thread is placed in the appropriate hole for the date, and the dial is ready for use. Its usage is described after the following notes on the use of computer aided design (CAD).

The following are the steps used with a computer aided design program (CAD), in fact, they were drawn with CAD (computer aided design) software.


This was drafted using CAD and the construction lines are also shown.

The left edge of the semicircle has drawn from it the latitude line and using the sun's declination from the tables or from a formula, a spreadsheet is shown for just this in the appendix, then date or calendar lines are drawn plus and minus from that latitude line. CAD makes this very easy to do.

To use the dial, a pin is placed in the center of the semicircle. The dial plate is then pointed towards the sun and rotated until the pin's shadow meets the horizontal. Thus the sun's altitude is measured.

A pendulum swings from the date in the slanted line and where it intersects its dated arc, then the time is read by the vertical hour lines. Longitude corrections, equation of time (EOT) corrections, and summer time corrections are applied. The dial is less accurate towards noon.

Of interest, the arcs associated for the dates show the sunrise and sunset times. The time of sunset or sunrise is where a vertical line drops from the date on the slanted line until it meets the date's associated arc. CAD systems can export the figure with construction lines, or a screen capture program can be used.


Studying the Capuchin dial provides understandings of the sun's northward journey in the European summer, its southward journey as Australia enters warmer times, and on how sunrise and its associated sunset may be deduced. The shepherd's dial discussed next provides clear evidence of how the sun's altitude limits are less in winter, more in summer. And for good measure shows how a dial can tell the time with no need for north south orientation.

In the wood dial to the right, the pendulum is in the November hole. So, the pendulum is read where it intersects the November arc, which is at 8 am or 4 pm . Care must be taken not to read from the top or bottom arc, but only from where the pendulum intersects its own date arc.

A capuchin dial is named after the Capuchin monks who wore hoods shaped like the arcs if inverted. The dial uses the sun's altitude only
 hence no compass is needed.

These dials are less accurate in the winter, less accurate as noon approaches, and have a tough time with hours near sunrise and sunset.

A DeltaCAD macro provided on the CD with this book will draft Capuchin dials, as will a FreeCAD Python macro.

## THE SHEPHERD'S DIAL (An altitude dial)

The shepherd's dial, like the capuchin, is an altitude dial and thus has no north orientation issues. It is simple to design yet may be more time consuming. The sun's altitudes must be derived. This can be done using a spreadsheet such as are in the appendices, and summarized below.


Some spreadsheets can produce charts, and the chart tempts us to use it directly for a shepherd's dial. However, the altitude lines are evenly spaced, whereas a protractor's angles, when extended to a line are not linear, they are the tangent of the angle. The solution is to modify the formula in the spreadsheet to be the tangent of the angle.



The chart above was produced by using a spreadsheet to produce a chart. The chart needs reversing to make it upside down, the axis needs marking, then it can be printed and used as-is. The dates need reversing also because of the chart rotation. The size of the gnomon is the distance from the top line (where the tan of the hour is 0 ) to the bottom of the noon line (where the tan of the hour is its largest), divided by the tan of largest noon altitude. The hour lines above are not smooth but some spreadsheets can draw smooth lines using CUSTOM charting, draw SMOOTH line options. This altitude dial is highly sensitive to the gnomon's length which must be accurate.

An alternative to using a spreadsheet is to use a CAD program. The CAD technique is merely the same technique that you would use if you were using a ruler and a protractor. But it allows the drawing to be saved and later modified.

The gnomon should probably be designed first. To do this, the dial plate would be as wide as is the circumference of the cylinder. Then an appropriate height is selected. Then the steepest solar altitude is identified from the chart, and for latitude 32 that is 81.4 degrees, then from the bottom of the chart a line is drawn of that angle, and a horizontal line is drawn on the top, which is the sunrise and sunset line, and where they meet is where a protractor is laid. Then the altitudes are drawn to the dial plate's side, and horizontal lines extended from that point across all the months. If you use lettering in some CAD systems, you may find that font size on the screen does not scale linearly with the pictorials as you zoom out, although options exist to correct this.

It is helpful to copy and paste from the spreadsheet into the CAD drawing, then a suitable zoom selected and then all drawing can be done without having to switch back and forth between spreadsheets and drawings. CAD files saved with pasted objects may take a lot of disk space.


In the above, a box of 6 by 6 inches was selected, the 81.4 degree line drawn from the bottom left of that box upwards until it intercepted a horizontal line extended along the top, and a protractor drawn at that point. Solar altitudes were measured from that protractor to the left vertical of the box, from whence horizontal lines were drawn to the month that had that solar altitude. One such angular line with its horizontal is shown for 10am or 2 pm for July.

The gnomon length is equal to the distance from the top left of the box to the center of the protractor, and when in use, the gnomon is exactly on the sunrise and sunset horizontal line, which obviously has 0 degrees from the protractor. The computer printed dial plate is protected with standard clear contact paper.

To use the dial, the dial is set level and the gnomon style is set over the date in question, and rotated until the shadow falls vertically below the gnomon at which time the time is read. Longitude corrections are shown for Los Angeles, Phoenix, and Silver City, and approximate equation of time (EOT) values shown. The EOT values were mid month. In a perfect world the hour lines would have used the mid month also, however that is a learning opportunity for the next time.

## IMPROVEMENT FOR WINTER MONTHS



The winter months have hour lines that are more tightly packed together than the summer months, and to increase accuracy of the dial, two gnomons can be used and the chart split into two halves. One half, "U" shaped, covers the winter months, and the other half is " $\cap$ " shaped covering the summer months.

To the right is a shepherd's dial using reclaimed waste water pipe material.

The CD and website for this book have DeltaCAD macros that can build a shepherd's dial using several line drawing techniques, can provide the dual gnomon dial plate, and provide animation of the hour curves over a latitude range. Similarly, the FreeCAD Python program can draft Shepherd's dials as well.



To the left is a DeltaCAD macro depiction, and above is that of the FreeCAD Python program.

## THE O-G DIAL WITH FIXED GNOMON (An altitude dial)



NOTE:
the hour curves look somewhat wobbly, this was a function of the software, and this book intends to show the real results of software, weak and strong. For a final dial, use a flexible curve, or use more points to smooth the curve.

This dial was constructed using CAD, and has little required structure. A gnomon at the top right casts a shadow on a horizontal line just like the capuchin dial, thus detecting the sun's altitude. For added benefit a protractor has been added.

Attached to that gnomon is a string or other cursor which tells the time by virtue of it intersecting some date lines.

The altitudes for each month came from the altitude tables for the latitude, and included on the dial plate is an EOT table as well as the longitude corrections for some cities.

This dial requires altitudes for every hour for each month, hence the need for tables, whereas the Capuchin dial was self designing provided the sun's declination was established by tables, trigonometry, or noon time observation during the year.

Accuracy is marginal in the noon hours, This dial is sometimes called an O-G dial for historical reasons, namely the curves likeness to the ogee form in architecture.

There are many variants of the altitude dial, some are somewhat self designing such as the Capuchin, some requiring calculations, such as the Shepherd, and the O-G dial here.

## THE HORIZONTAL ALTITUDE DIAL (An altitude dial)

A dial can be made that is horizontal yet is still based on altitude. It thus does not need compass alignment, it is rotated to align the shadow with the month in question.

## Altitude dial ~ yet horizontal



This process is simple, the altitude table for the latitude is taken, and the entire table subtracted from 90 degrees. This will make line drawing a lot easier.

A box is drawn, and 30 degree lines drawn from an off-center center, the center is somewhat to the left in this case. The 30 degree lines facilitate construction of the month hour lines. Months lines are drawn, the solstices are on the horizontal, and the other five months occupy the 30 degree lines. From the center, an arc is drawn above the box that will have as its radius the gnomon's height. Every month line, the two that make up the horizontal, and the other five, have a 90 degree perpendicular drawn, and it intersects the arc. The February October line has a perpendicular shown.

Where that perpendicular meets the arc, whose radius is the gnomon's height, lines are drawn whose angle equals the sun's altitude subtracted from 90 degrees. The table to the upper right is nothing more than a copy of the altitude table for the latitude, and each cell was subtracted from 90 degrees. The process is repeated for each calendar line. By using 30 degree separations, it is easier to construct the entire dial plate. CAD software was used for the process. The biggest problem was setting the CAD program's angle baseline. Of course a rule and protractor will work very well also.

CAD software's bezier line drawing was used to
 connect the hour points to the nautilus shaped line.

To use the dial, align the shadow of the gnomon with a month line, and read the time on the curved hour line. No compass alignment is needed. This dial just isn't practical for hours close to 6 or 7 am or 6 or 5 pm , and not very accurate around noon. In this dial, the gnomon's height is of course critical.

## THE PLANISPHERIC ASTROLABE (An altitude dial using the ecliptic)



Of old, the astrolabe was used for various astronomical and astrological calculations, In particular, it could also be used to determine the time from the sun's altitude, which when corrected for the equation of time and longitude, would provide the legal standard time.

Unlike the altitude devices discussed elsewhere, this device is directly based on the ecliptic, and since the Earth and the sun are always on the ecliptic, and since the ecliptic moves by the second, minute, and hour, the sun's altitude pinpoints an ecliptic point, and together with the rest of the magic of the astrolabe, the time is hence derived. Most astrolabes not only have the ecliptic, they also include the stars. The planispheric astrolabe, described here, uses a "projection" of the Earth and the celestial sphere to the equator. Another variant uses the Saphea projection onto a vertical plane.

The theory is that projecting onto a circular slice of a globe perpendicular to a projection focal point, results in circles on the surface being preserved on the projection plane, angles similarly.

A circle is placed on the globe at latitude $33^{\circ}$ for example,

latitude $33^{\circ}$ circles and lines drawn from the perimeter to the south pole in this case as a projection focal point. Those lines will intersect the equatorial plane and still form a circle. If several circles are projected, then several circles are the result, however, their centers on the equatorial plane may not be co-located, they may be offset.

If those circles were at latitude 90, it is obvious that the projection would be circles, and in that specific case they would also have a common center.

If the circles were on the equator, and projected to the equatorial plane, i.e. a 90 degree projection, they would still be circles.

The secret is to look at the projecting lines as a cone, and the projection is always the same angle of slice on that cone as are the original circles.

latitude $90^{\circ}$ circles

latitude $0^{\circ}$ circles

There are more rigorous proofs, mathematical and geometrical, however this explains the principle of stereographic projection as it is called.

Another key point is that the celestial sphere, stars and the ecliptic will exist on the surface of the globe. The third key point is that the ecliptic will be a plane slicing through the globe as a great circle whose angle of slice will be 23.5 degrees, being the Earth's polar axis offset from the plane of orbit, the ecliptic.

Astrolabes consist of pieces. The part that holds everything together is the mater, from the Latin for mother. Resting in or on the mater is a plate having projected circles depicting altitudes from the horizon, called almucanters. They are part of the time telling secret. And rotating on the almucanter plate, sometimes called a climate plate, is what is called the rete. The rete has the ecliptic circle on it, and sometimes the stars.

Using the astrolabe is simplicity itself. The sun's altitude is measured, usually by a simple protractor system on the back of the astrolabe. And then the rete which has dates on the ecliptic circle, is rotated until the date touches the sun's altitude circle (almucanter) and their intersection when extended to the outer circle on the mater from the center, depicts the local apparent time.

First, a climate plate must be constructed for the desired latitude.


A circle is drawn representing a vertical slice of the planet, a north south line and an equator line are drawn. Then a line for the desired latitude, $33^{\circ}$ in this case.

Then circles of altitude are drafted, depicted by two limiting lines, in this case they are 10 degrees apart. Two or five degree separation is more practical.

These are circles depicting altitude. The altitude they depict inversely matches their north/south distance from the observer. So $0^{\circ}$ from the observer is the $90^{\circ}$ altitude almucanter, $90^{\circ}$ south of the observer is the $0^{\circ}$ almucanter, etc.
To the right is the planet Earth with the equator, and part way up the globe is a cylinder depicting $80^{\circ}$ of altitude, or $10^{\circ}$ from the observer's vertical.

Those angles are altitudes seen by an observer on the surface of the planet at the center of all those circles. The picture to the right simplifies this by placing the observer in the Earth's center.

The planispheric astrolabe only uses two key concepts in its design.

First those cylinders of altitude which
 are circles on the surface of the planet, are reduced to projected circles on the equator, and they become the climates or the plate with the almucanters, or altitude circles. These are sometimes confused with latitudes and they are not, however, each almucanter of $n^{\circ}$ is $90-n^{\circ}$ from the observer on the observer's meridian or line of longitude.

Second, the ecliptic is reduced to scale using the same globe profile as for the climate or almucanter plate, on a movable plate called the rete. The next pages discuss this transformation.

The profile of the planet is drawn, as shown on the previous page. This has the altitude circles or cylinders, called almucanters. Then they are projected to the equator, as shown below.


When all altitude circles are completed, i.e. their lines projected to the equator, then those limits are used to draw the final circles for the climate plate. Above only the $10^{\circ}$ increments are shown, with $2^{\circ}$ being more common. This is done by drawing a circle, placing its left side on one of the projected circle's limits, and its right side similarly.

All the desired circles are drawn and then printed onto a dial plate, the climate plate, ready for use.

A point is drawn on the altitude circle or almucanter at the altitude equal to the latitude of the observer, or the dial plate or climate plate's design location. That point will be the center of the final climate plate,
 and around which the ecliptic rete plate will revolve. Hour lines of $15^{\circ}$ radiate from that center on the climate plate, and will be the final hour lines, they usually exist on the mater however.

To the right is the final layout for the climate plate holding the almucanters. Hours can then be marked on 15 degree radials from the almucanter matching the latitude of the observer, not from the zenith, or they can be added to the mater plate later.

The scale for the climate plate almucanters and the scale for the second and last plate, the rete which holds the ecliptic, must be the same.

The center of the 15 degree radials, which is the almucanter matching the observer's latitude, will also be the center of the rotating rete.

Next, draft the latitude independent rete.


The same circle used for initially projecting and drafting the altitude lines, is copied retaining the scale.

From the center of that new circle is drawn a line at $23.5^{\circ}$ line from the equator, being the ecliptic.

For the northern hemisphere, the upper part that intersects the circle is the June solstice. When projected, this will be the June 21st date on the rete, or projected ecliptic.

The lower intersection will be the December solstice. In the pictorial above, the ecliptic plane is projected to the equatorial line, a rete plate center located, and the limits of what will be the ecliptic circle are identified. Just as for the climate plate, and retaining the scale, the ecliptic circle is drawn. This is a circle and not an ellipse, the ecliptic being a collection of ecliptic intersections on the globe or planet, which forms a great circle, hence why a circle is used.

The final rete plate, whose scale is retained, is then printed onto a final plate. Note that its center of rotation is offset.


Just as hours were added to the 15 degree radials on the climate plate, dates are added to the rete. The June solstice is June 21, the December solstice is December 21st. Scale is critical, however some editorial resizing occurred in the above drawings.


Here the dates have been added in. Of course, the dates are symmetrical, thus March and September and their intervening dates can be swapped should you so wish.

Most astrolabes use the zodiac calendar on the rete plate, and the back side of the astrolabe has a conversion from the calendar in fashion at the time, to the zodiac. So take the "21" of the month, as shown, with a pinch of salt.

Similarly, the hours on the mater plate are traditionally shown clockwise, however they can be reversed. It all depends on how you make your rete rotate to intersect with a solar altitude curve or almucanter.

The final rete is cut and mostly consists of empty space so the almucanters can be seen.


Above are both the climate plate on the left and the rete on the right, scale retained. All that is needed is for the hours to be marked on the $15^{\circ}$ radials of the climate plate, they will be projected to the rim of the mater. From the rotational center of the climate and rete plates is usually attached a rule so the intersection of the sun's altitude with the date can be projected to the outer scale displaying the L.A.T..

The back of the astrolabe usually has an altitude measuring protractor, and often a conversion from the normal calendar to the zodiac. An equation of time table may be added, and a table of longitude differences for places on the design latitude similarly.

tangential lines for the rotational limits of the rete

The circle of the rete and its rotational point are copied to the climate plate because the rete's center is located on the almucanter associated with the climate plate's design latitude. Then, a circle is drawn from the rotational center to the limit of the rete, this last circle is the final climate limiting circle, and outside of that will be the mater. And hours are added as 15 degree radials from the rotational center. That mater's outer circle can be bigger than the rete's limits, not smaller. If bigger then it is easier to see where the rete is in relation to the almucanters.


The back of the astrolabe has date conversions, the EOT, and altitude protractor


The rete is latitude independent and has the dates


The outer periphery is the mater, enclosing the climate or almucanter plate of the sun's altitude lines

The above is the mater (outer area with the hours marked), within which is placed the climate or almucanter plate, which is fixed, and on that is the rete which rotates. Its date is set to the sun's altitude on an almucanter arc, and a line projected from the rotational center indicates the local apparent time on the mater. All that is needed is a rule to help point the time, and on the back, a protractor and alidade, the alidade is the name for the altitude measuring rule. The equation of time may be added, and other dial furniture.

The completed astrolabe in use is shown below. The altitude was found using the back, and the date and altitude made to intersect on the front. When that intersection was projected to the mater it indicated the time. After building an astrolabe, check some date/altitude intersections and their resulting time with the altitude tables or formula to verify the astrolabe's accuracy.


To the left is the alidade showing 35 degrees of sun altitude.

To the right, the
rete's date is rotated to the almucanter for the 35 degree sun altitude. The intersection when extended shows the L.A.T. and when longitude and EOT are considered, the legal standard
 time.

Sunset can be determined by moving the date on the rete to intersect with the 0 degree altitude line or almucanter. That intersection when extended to the mater from the rotational center will provide the L.A.T. for sunset, or sunrise. There is little written on astrolabe design. The appendices of this book take the preceding geometric figures and convert them to trigonometry suitable for a spreadsheet or a DeltaCAD macro. This book's website and CD have such a spreadsheet and DeltaCAD macro. There is a good article on stereographic projection for the climate plate at:-
http://www.math.ubc.ca/~cass/courses/m309-01a/montero/math309project.html
An astrolabe generator is available at::-
http://www.uwsp.edu/physastr/rislove/astrolabe/resource.htm or
on the CD and web site associated with this book as a DeltaCAD macro

Another set of excellent resources on astrolabes is at Mr. Morrison's web site:-
http://www.astrolabes.org/links.htm http://www.astrolabes.org/theastrolabe.htm [Mr. Morrison's definitive work on astrolabes]

Commercially available astrolabes can be acquired from:-
Norman Greene, 1215 4th St., Berkeley, CA, 94710
http://www.puzzlering.net/astrolabe.html
While the Norman Greene astrolabes are smaller and less detailed than some others, and while many commercial astrolabes suffer from a bright climate plate or plates with hard to read markings, the Norman Greene astrolabe is possibly one of the most practical commercial astrolabes to use overall. Until the TSA screeners at airports became suspicious of mine, I used to travel with it for many years.

## CHAPTER TWENTY TWO

## Azimuth Dials

It was mentioned earlier in chapter 4 that there were three methods of telling the time from the sun, they were by measuring its:-

| altitude | (how high the sun is), or |
| :--- | :--- |
| azimuth | (how far east or west the sun is), or <br> hour angle <br> (the sun's angle around the north south polar axis) |

The azimuth of the sun is its angular distance from true south, it is latitude dependent. The azimuth of the sun at any time is determined by the date, time and the sun's declination.

AZIMUTH:

$$
\begin{aligned}
\text { azi }= & \operatorname{ATAN}(\operatorname{SIN}(\text { lha }) /(\operatorname{(SIN}(\text { lat }) * \operatorname{COS}(\text { lha })) \\
& -(\operatorname{COS}(\text { lat }) * \operatorname{TAN}(\operatorname{decl})))
\end{aligned}
$$

DECLINATION:

$$
=(23.45 * \sin (\text { radians }(0.9678(\mathrm{jd}-80)))
$$

jd julian day of the year being 1 to 365

| Jan | Feb | Mar | Apr | May | Jun |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 31 | 59 | 90 | 120 | 151 |
| Jly | Aug | Sep | Oct | Nov | Dec |
| 181 | 212 | 243 | 273 | 304 | 334 |

alternatively, the declination can be found by

$$
\begin{array}{ll}
=\text { degrees }\left(0.006918-0.399912^{*} \cos (d a)+0.070257 * \sin (d a)\right. \\
& -0.006758^{*} \cos \left(2^{*} \mathrm{da}\right)+0.000907^{*} \sin \left(2^{*} \mathrm{da}\right) \\
& -0.002697^{*} \cos \left(3^{\star} \mathrm{da}\right)+0.001480^{*} \sin \left(3^{*} \mathrm{da}\right) \\
\text { da } \quad \begin{array}{l}
\text { day angle }
\end{array} \\
\quad=2 * \mathrm{pi}^{*}(\mathrm{jd}-1) / 365 \quad \text { (in radians, is an intermediate figure) }
\end{array}
$$

There are a number of common azimuth dials. Two of note are the Winged Azimuth dial, and the so called Analemmatic dial. So called "analemmatic" dials are "so called" since they have nothing to do with the analemma.

## SIMON'S AZIMUTH COLUMN

Selecting the azimuth table for a specified latitude, and selecting mid month and averaging the equation of time, a tailored azimuth table can be drawn.

AZIMUTH FOR MID MONTH FOR LATITUDE:

|  |  | prox |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Date | decl | EOT | 600 | 700 | 800 | 900 | 1000 | 1100 |
| Jan | -20.7 | +9 | 72.2 | 64.7 | 56.0 | 45.5 | 32.7 | 17.3 |
| Feb | -12.8 | +13 | 79.1 | 71.2 | 62.2 | 51.4 | 37.6 | 20.3 |
| Mar | -2.1 | +8 | 88.2 | 80.1 | 71.2 | 60.3 | 45.8 | 25.7 |
| Apr | 9.5 | =0 | 98.1 | 90.3 | 82.0 | 71.9 | 57.9 | 35.5 |
| May | 18.5 | -2 | 105.8 | 98.7 | 91.3 | 82.7 | 70.9 | 49.1 |
| Jun | 22.9 | =0 | 109.7 | 102.9 | 96.1 | 88.6 | 78.6 | 59.3 |
| Jly | 21.1 | +5 | 108.2 | 101.2 | 94.2 | 86.3 | 75.4 | 54.9 |
| Aug | 13.8 | +3 | 101.8 | 94.3 | 86.4 | 76.9 | 63.7 | 41.1 |
| Sep | 3.1 | -5 | 92.6 | 84.6 | 75.9 | 65.2 | 50.7 | 29.4 |
| Oct | -8.4 | -13 | 82.9 | 74.9 | 65.8 | 54.9 | 40.8 | 22.3 |
| Nov | -18.1 | -14 | 74.5 | 66.8 | 58.0 | 47.4 | 34.2 | 18.2 |
| Dec | -22.7 | -5 | 70.4 | 63.0 | 54.4 | 44.1 | 31.5 | 16.6 |
|  | decl |  | 6pm | 5 pm | 4pm | 3 pm | 2pm | 1pm |

Measure the diameter of a column or plastic plumbing pipe, and calculate the circumference by multiplying the diameter by 3.14.

The circumference being 3.14 times the diameter, and covering 360 degrees, each degree would use 0.0087 times the diameter as a linear distance on a sheet of paper (3.14 divided by 360). Or, 0.087 times the diameter for ten degrees.

A 1.9 inch diameter plastic irrigation pipe would thus consume $1.9^{*} 0.087$, or 0.165 inches per ten degrees. Mark off 0.165 inch columns across the top covering the azimuth range; for latitude 32 that might be + and -120 degrees, or two thirds of the circumference. Down the side mark the month. Place a dot on the month line for each hour. The dial is aligned true north/south, and the gnomon turned until the two shadows are a single vertical line, the time being read where that vertical shadow intersects the date line. Longitude and EOT corrections apply.



The gnomon has two parts so that their shadows merge and are vertical when the gnomon is aligned with the sun's current azimuth. Average declination is an approximation in the above spreadsheet.

## AZIMUTH DIALS - Table A4.2 cover latitudes 30 to 60

The azimuth of the sun varies hourly for any given date. Azimuth is the angle left or right of noon, as opposed to altitude which is the angle up from the horizon. And they are both different from the hour angle of the sun, which is measured around a polar axis aligned style.


One type of azimuth dial is shown above. Arcs of any convenient size are drawn, one for each month, and using an azimuth table, the azimuth angles are drawn for each hour on the monthly lines. June and December have their own lines, as they hold the solstices, other month lines are used for two months. The azimuth degrees are close enough.

This top left chart was drawn using DeltaCAD macros, and to the right is the same using FreeCAD Python programs, all provided with this book. The hour curves near the summer solstice seem to differ, this is because the programs use different methods to determine the radius for a given solar declination, and also slightly different declinations. Both are correct!

The dial above was read on December 10, and the displayed time is about 4:15 in the afternoon, to which 12 minutes is added for longitude correction, and 5 minutes subtracted for the equation of time, making it tell $4: 22 \mathrm{pm}$. It was fairly close. Lessons learned were to choose a thinner gnomon pole, and not to press too hard on the Plexiglas, otherwise it would crack.

It might seem obvious that the summer months should be closer to the gnomon, the winter months on the outer periphery. It may seem more logical, however this wise idea should be balanced with the need for better resolution of the hours in the May through September months.


The figure to the left shows the shapes when the summer months are drawn closest to the gnomon and the winter months further away. It just doesn't look impressive, in fact it looks depressed!

A common question is that the azimuth dial hour points do not form a straight line from anywhere, so it must be wrong.

The azimuth points are just directions from a gnomon and not a dial center they are not distances, not lines. Thus the azimuth points and the curve they live on are not intended to be points from which an hour line could be drawn from a dial center. The angles they represent are all they represent, and it is those angles that would exist on an hour line for an hour angle chart, and those angles would actually be calendar points for the hour in question!


## AZIMUTH DIAL ~ but in clay

Another dial that uses azimuth only has arcs for the sun's declination, and points on those separate arcs that indicate the hour in question.

To the right, drawn with DeltaCAD using a program on the CD accompanying this book, and also available on the web site, is a dial plate for an azimuth dial. Here the gnomon is fixed and doesn't move. An animated version is available showing how the hour lines vary with latitude.

This dial type was fully discussed elsewhere and is offered here to show it worked in clay.

For portability reasons, a paper azimuth dial was printed with no longitude correction. A slab of clay was rolled into a circle, and a drafting compass used to draw an outer circle and then the three declination arcs to scale. If portability is not a factor then the DeltaCAD program can produce such a dial with longitude correction.


The paper dial was cut at the 5 am (left) mark, following the curves to the 7 pm arc. The paper dial was placed on the dial plate after the slip had been applied and dried for a couple of hours.


Then a scribe was used to mark those two hour curves. The paper dial was removed and the next pair of hours was cut, then scribed, and so on until the entire dial was complete.

The equation of time (EOT) was marked on the outer circle with monthly average numbers, and the latitude inscribed. No longitude was marked as this dial was not longitude corrected.

After the picture was taken it was noticed that the solstice and equinox arcs were not marked, so they were added before the clay dried.

The clay dial plate was then left to dry, and in so doing the clay shrank about $10 \%$ or more as it dries.

The picture to the right shows the clay plate after it dried. The clay has nicely separated itself. Do not manually separate the clay yourself before it has dried, if you do so then the clay may curl up at the edges. Let it do its own thing.

The dial plate was then placed in a kiln and given a bisque firing, that is around 1800 degrees $F$. Kilns need clearance from combustible walls and floors, and should be vented. Venting is important because some materials used can be somewhat toxic during the firing process.


After the bisque firing, the clay was allowed to cool down, and then a glaze applied. The glaze looks somewhat opaque, as seen to the left. However when fired to cone 6 (heat) or to 2230 F (temperature), it then becomes transparent and melts, and fuses with the clay itself, becoming stoneware.

Clay being bisque fired can be stacked on top of other items in the kiln, see right. This is because the clay has dried. So has the slip, and slip is nothing more than clay, water, and pigment. The mixing of a coloring pigment with the clay being used makes a colored clay that expands or contracts with the clay material beneath it. Thus when dried, the slip coloring is in essence the same as the underlying clay. And it is not a glaze. That is why items being bisque fired can be stacked.


However, when glaze is added and fired, the glaze becomes in essence molten glass. Thus a glaze firing cannot be done by stacking as the glasses would merge. To the left is the plate after the glaze firing. Glaze was liberally applied, and the kiln held at cone 6 for some time in order to ensure the resulting stoneware would not be damaged by freezing moisture in the winter. When multiple pieces of clay are glaze fired, they are separated by kiln shelves or furniture. Those shelves should be coated with kiln wash. Kiln wash is a powder that creates a barrier between the item being fired and the firing shelves, should the glaze run.

The glaze firing being complete and cooled down, the dial plate was removed. A vertical gnomon such as a brass rod was located in the center of the circular concrete paver, secured with epoxy. Then an exterior tile adhesive was used to affix the clay plate pieces to a concrete paver which also reinforced the seal on the unglazed lower side and edges of the clay tile. Sanded grout was applied when the adhesives had dried. After a few days of curing, a second grout application may be needed to resolve residual cracks. When fully cured, the grout should be sealed, otherwise moisture can seep in and cause tile damage on a subsequent deep freeze. And... to the right is the finished dial.


## THE ANALEMMATIC DIAL

The analemmatic dial is an azimuth dial with no hour lines, only hour points, to which the extension of the shadow points, and the gnomon moves with date. The gnomon is not polar axis aligned, it is simply a vertical pole, sometimes a person in a garden area, however the ellipse of hour points is true north or south aligned..


The ellipse has a major axis left to right (west to east) and a minor axis up and down (north down to south).

The semi major axis $(M)$ is used in the formulae, being half of the east west major axis.
The semi minor axis ( $m$ ) is: $\quad m=M \sin$ ( lat )
semi minor axis is sin of latitude times major axis (at latitude 32, $\mathrm{m}=0.53$ times major axis M , or, if $M=6$ units wide then " $m$ " is 3.18 units deep)

Should you wish to draw an ellipse, set a drafting compass to match the semi-major radius M, i.e. C~ML. Move the drafting compass point to point "mt" and draw two short arcs at "a" and "b".


Insert a pin at "a", "b", and "mt", connect with a taught thread, place a pencil at "mt", then run the circumference to draw the ellipse. For a large dial such as in the following case study, a length of string may be used in place of the drafting compass and a taught thread.

If you do not wish to draw an ellipse because it is impractical, as for example for a dial on a lawn, then an hour points location is defined by:-
hour point's horizontal distance:- hour point's vertical distance:-

$$
\text { Ch' }=\mathrm{M} * \sin (15 * \text { hours }) \quad \mathrm{hh}=\mathrm{M} * \sin (\text { lat }) * \cos (15 * \text { hours })
$$

or to get an hour point by angle from C assuming an ellipse has been drawn

$$
\mathrm{x}=\arctan (\tan (\mathrm{t}) / \sin (\varnothing))
$$

| HOUR | HOUR | ANGLE | $\arctan (\mathrm{ta}$ |  | tan( hour from noon * 15) / sin ( latitude ) ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lat: | $>30$ | 32 | 34 | 36 | 38 | 40 | 45 | 50 | 55 |
| 12 | 12 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 11 | 13 | 28.2 | 26.8 | 25.6 | 24.5 | 23.5 | 22.6 | 20.8 | 19.3 | 18.1 |
| 10 | 14 | 49.1 | 47.5 | 45.9 | 44.5 | 43.2 | 41.9 | 39.2 | 37.0 | 35.2 |
| 9 | 15 | 63.4 | 62.1 | 60.8 | 59.6 | 58.4 | 57.3 | 54.7 | 52.5 | 50.7 |
| 8 | 16 | 73.9 | 73.0 | 72.1 | 71.3 | 70.4 | 69.6 | 67.8 | 66.1 | 64.7 |
| 7 | 17 | 82.4 | 81.9 | 81.5 | 81.0 | 80.6 | 80.2 | 79.3 | 78.4 | 77.6 |
| 6 | 18 | 90.0 | 90.0 | 90.0 | 90.0 | 90.0 | 90.0 | 90.0 | 90.0 | 90.0 |
| 5 | 19 | -82.4 | -81.9 | -81.5 | -81.0 | -80.6 | -80.2 | -79.3 | -78.4 | -77.6 |
| 4 | 20 | -73.9 | -73.0 | -72.1 | -71.3 | -70.4 | -69.6 | -67.8 | -66.1 | -64.7 |

Notice that the hour points are points and there are no hour lines. The next calculation is the gnomon, and this is north of the ellipse center in summer and south of it in winter.

The analemmatic points for the gnomon calendar are placed to put the gnomon as follows:-

| $\mathrm{z}=\mathrm{M}$ * $\tan (\mathrm{dec}) * \cos$ ( lat ) |  |  |  |  |  |  | Semi major axis "M" is 1.00 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| gnom | distan | p- | n | S |  | M * | ( la | tan |  |  |  |  |
| Date | Avg decl | 30 | 32 | 34 | 36 | 38 | 40 | 45 | 50 | 55 | 60 |  |
| Jan | -20.3 | -0.32 | -0.31 | -0.31 | -0.30 | -0.29 | -0.28 | -0.26 | -0.24 | -0.21 | -0.18 |  |
| Feb | -12.5 | -0.19 | -0.19 | -0.18 | -0.18 | -0.17 | -0.17 | -0.16 | -0.14 | -0.13 | -0.11 | 1.00 |
| Mar | -1.6 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.02 | -0.01 | 1.00 |
| Apr | 9.6 | 0.15 | 0.14 | 0.14 | 0.14 | 0.13 | 0.13 | 0.12 | 0.11 | 0.10 | 0.08 | 1.00 |
| May | 18.4 | 0.29 | 0.28 | 0.28 | 0.27 | 0.26 | 0.25 | 0.24 | 0.21 | 0.19 | 0.17 | 1.00 |
| Jun | 22.5 | 0.36 | 0.35 | 0.34 | 0.34 | 0.33 | 0.32 | 0.29 | 0.27 | 0.24 | 0.21 | 1.00 |
| Jly | 20.9 | 0.33 | 0.32 | 0.32 | 0.31 | 0.30 | 0.29 | 0.27 | 0.25 | 0.22 | 0.19 | 1.00 |
| Aug | 14.1 | 0.22 | 0.21 | 0.21 | 0.20 | 0.20 | 0.19 | 0.18 | 0.16 | 0.14 | 0.13 | 1.00 |
| Sep | 3.8 | 0.06 | 0.06 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.04 | 0.04 | 0.03 | 1.00 |
| Oct | -7.4 | -0.11 | -0.11 | -0.11 | -0.11 | -0.10 | -0.10 | -0.09 | -0.08 | -0.07 | -0.06 | 1.00 |
| Nov | -17 | -0.26 | -0.26 | -0.25 | -0.25 | -0.24 | -0.23 | -0.22 | -0.20 | -0.18 | -0.15 | 1.00 |
| Dec | -22.1 | -0.35 | -0.34 | -0.34 | -0.33 | -0.32 | -0.31 | -0.29 | -0.26 | -0.23 | -0.20 | 1.00 |

The dial may be constructed using a CAD system, and the angles may be proved by drawing a line from a calendar point to an hour point and checking the azimuth tables in the appendix for that latitude, date, and hour. Appendix 4.3 provides these values. Average declination is an approximation in the above spreadsheet.


An analemmatic dial drafted in CAD is shown, together with a real dial printed from CAD, with a gnomon inserted, displaying the correct time when longitude and EOT corrections are made.


## CASE STUDY ~ AZIMUTH DIAL ~ analemmatic dial for a garden

The local recreational center, where children on school breaks of vacations spend their time doing constructive things, is an ideal place for what is called the analemmatic dial. In essence it is an azimuth dial where there are no hour lines, only points on an ellipse, and there is no fixed gnomon, it moves north or south based on the month.

Constructing such a dial on the order of 20 feet
 across takes about an hour and in this example uses eleven $2 \times 12 \times 12$ pavers and 8 bricks.

The method is simple. A north south line is constructed with a compass after considering magnetic variation or declination.

Then the spreadsheet: illustratingShadows.xls from the main Illustrating Shadows web site or on the CD that comes with the book is opened, and the latitude entered along with the east to west dimension of the ellipse.

The result is a set of calendar points for the gnomon positioning which will have the winter solstice closest to the equator, and the summer solstice closest to the pole, on a north to south line.

Additional results are the horizontal and vertical distances for the hour points.

| ENTER |  | latitude | 32 |
| :---: | :---: | :---: | :---: |
| major ellipse east to west distance |  |  | 16.00 |
| gnomon distance up/down north/south based on ellipse center =$z=M * \cos (\text { lat }) * \tan (\mathrm{dec})$ |  |  |  |
|  |  |  |  |
|  | Avg | Latitude |  |
| Date | decl | 32 |  |
| Jan | -20.3 | -2.51 | -2.95 |
| Feb | -12.5 | -1.50 | total |
| Mar | -1.6 | -0.19 | north to |
| Apr | 9.6 | 1.15 | south distance |
| May | 18.4 | 2.26 | is |
| Jun | 22.5 | 2.81 | 5.90 |
| Jly | 20.9 | 2.59 | assuming |
| Aug | 14.1 | 1.70 | plus and |
| Sep | 3.8 | 0.45 | minus |
| Oct | -7.4 | -0.88 | 23.50 |
| Nov | -17 | -2.07 | extremes |
| Dec | -22.1 | -2.75 | 2.95 |

With the latitude and east west dimension entered, the spreadsheet provided the north and south distances for the base of the gnomon.

In this example of latitude 32 and a 16 foot wide ellipse, the January to June distances of the gnomon base range from minus 2.51 to plus 2.81, and for June to December the range is 2.81 to minus 2.75 .

The spreadsheet also shows the north south dimensions of the ellipse.


The above ellipse is not drawn to scale, the pictorial is simply to help orient the diallist. At this point an ellipse can be drawn and the angles from the ellipse center used to identify the hour points. Alternatively the east west and the north south coordinates of the points can be used to locate the hour points.
horizontal and vertical distances from ellipse center to the hour point
for a semi major radius of
8.00

| Latitude | 0 | Latitude |  | 32 |
| :---: | :---: | :---: | :---: | :---: |
| HOUR | HOUR | horizontal |  | vertical |
| am | pm | Ch' |  | hh' |
| 12 | 12 |  | 0.00 | 4.24 |
| 11 | 13 |  | 2.07 | 4.09 |
| 10 | 14 |  | 4.00 | 3.67 |
| 9 | 15 |  | 5.66 | 3.00 |
| 8 | 16 |  | 6.93 | 2.12 |
| 7 | 17 |  | 7.73 | 1.10 |
| 6 | 18 |  | 8.00 | 0.00 |
| 5 | 19 |  | 7.73 | -1.10 |
| 4 | 20 |  | 6.93 | -2.12 |
| am | pm |  |  |  |

to hour point on ellipse
hour point's vertical distance:-


Ch' $=M$ * $\sin$ ( 15 * hours from noon) hour point's horizontal distance:-

First, select a suitable location, and ensure ants and other wildlife will not be a factor. These ants, about six feet from the winter end of the gnomon track are normally friendly. However for some reason the day the dial was built they became more than inquisitive. These ants are about $1 / 2$ inch long.


Then the gnomon track was laid out, and this used a Brunton compass for alignment. The magnetic
 declination was 10.8 degrees easterly, thus a reading of 349 was required for correct alignment. Additionally, the east west axis of the eclipse was also measured both from the east and the west to minimize nearby magnetic disturbances from rebar or automobiles.

At about 9 pm the alignment was cross checked against the north star Polaris. This was done with a 6 foot wooden bar, used for the calendar information on the gnomon track. It was held vertical using its own weight and then sighted with the array track. Looking up at latitude, Polaris was sighted within close limits, remember, Polaris is not perfectly where the extended polar axis or the celestial polar axis, it is off, and precesses every 25,800 years.


Then for added measure, the astrocompass was used to check the other methods. Using the instructions in this book,
 a time of 1033 daylight savings, or 0933 mountain standard time was used. The longitude correction for a sundial is 12 minutes 48 seconds. The equation of time on July 2, 2006 when this was double checked was to add 4 minutes 12 seconds to a dial. The total correction to mst being thus exactly plus 17 minutes.

So, exactly 17 minutes is subtracted from the clock time to get the time used to calculate the hour angle. 0933 minus 17 minutes is 0916, which is 4 degrees ( 16 minutes of time) short of 45 degrees ( 3 hours) from noon. Or, 41 degrees from the noon line. The astrocompass was aligned and leveled, set to the latitude and approximate solar declination, and the local hour angle set to 41 minutes from noon. At the appointed time the shadow was found to well aligned.

Armed with this information the dial was laid out. An excellent project for the youth of the country.


There is a 6 foot wooden rod with dates marked on it, and a gnomon is placed on the north south line where the date indicates.


The gnomon casts a shadow and in this case the date rod is used to project the shadow to the hour point, it shows about ten minutes past 1 in the afternoon.

The equation of time must be factored in, 3 minutes to be added in this case, making it 1:13 pm.

And summer time must be added in making it 2:13 pm.

Longitude was not considered in this dial, although the spreadsheet can easily accommodate times on either side of the hour. In this case the dial was at longitude 108.2 and the legal time zone meridian was 105, making about 12 minutes to be added. Making the final correct time about 2:25 in the afternoon, which was quite close.

The formulae are straightforward:-

| lat | $=$ | latitude |
| :--- | :--- | :--- |
| M | $=$ | major axis (east to west) radius |
| dec | $=$ | declination for the calendar date |

gnomon distance up/down north/south based on
ellipse center $=\quad z=M * \cos ($ lat $) * \tan (d e c)$
north-south radius
$\mathrm{m}=\mathrm{M}$ * $\sin ($ lat $)$
angle from dial center to the hour point
$x=\arctan (\tan ($ hour from noon * 15) / sin (latitude ) )
hour point's vertical distance from dial center north/south)
hh' $=M^{*} \sin (l a t)^{*} \cos \left(15^{*}\right.$ hours from noon)
hour point's horizontal distance:-
Ch' $=$ M * $\sin$ (15 * hours from noon)

The sun's declination used for the calendar displacement can be found in tables or calculated. The simplest formula is possibly:

$$
\operatorname{dec}=(23.45 * \sin (\text { radians }(0.9678(j d-80)))) \quad \text { this formula agrees within half a degree }
$$ of a more accurate formula which is found in the appendices.

where jd = julian day of the year.
Not only does appendix 4 provide ready made tables for analemmatic dials, it also provides nomograms for their design.

The completed analemmatic dial with additional dials on columns at the $6 \mathrm{am}, 6 \mathrm{pm}$, and noon markings is shown below.


## CHAPTER TWENTY THREE

## dial furniture ~calendar or declination curves

Declination lines have been addressed several times. Sometimes for specific cases, sometimes in more general terms.

Declination lines appear as dial furniture on a number of dials, they indicate the approximate date, albeit with ambiguouity except at the solstices. This is because two dates will share the same declination.

To an observer, the sun orbits the polar axis of the planet, and moves towards the pole in the summer, the opposite in the winter. Another means of determining the date is to use the ecliptic dial, however, blending that concept which requires moving the dial, with a stationary dial plate is inconsistent, and thus not reviewed further here.

Below is a mesh showing the sun's travel as seven arcs in the sky. The extremes are June and December, the solstices. And in between are January through May, and July through November. As an aside, the 3d mesh displayed below is on the web site as well as the CD that comes with this book. It can be used in TurboCAD to generate accurate shadows for a date and time. Chapter 30 has details on how to snap the light and its target using snap to facet.


But are there perhaps more methods available to address sun dials such as the decliners, and are there other methods of locating the declination lines.

There are indications that the seasons were of more interest to early sun dial users, and that the calendar might have been more important than the time of day.

The spreadsheet associated with this book has a section on declination line data, and provides methods using both altitude as well as azimuth.

The DeltaCAD macros accompanying this book can also provide a graphical depiction of calendar curves with many options, and these can then be transposed to any hour angle type of dial plate.

NOTE: For declination or calendar lines on a horizontal or vertical dial, a template is available in appendix 9 to facilitate this process whose use is described in chapter 12.

NOTE: The term "gnomon linear height" will be used, it is the shortest distance from the style's nodus directly to the dial plate.

There are indications that the seasons were of more interest to early sun dial users, and that the calendar might have been more important than the time of day.

The sun appears to oscillate annually from minus 23.5 degrees when it is over the southern tropics, to plus 23.5 degrees when it is over the northern hemisphere tropics.

This angle made with the equator is called the declination. Many of the tables show the declination for various days in the year. In particular appendix 2 shows the sun's declination for each day of the average year. This book shows two formulae, and one is:-

```
DEGREES(0.006918-0.399912*COS(((2*3.1416*(jd-1)) / 365)) + 0.070257*SIN(((2*3.1416*
    (jd-1)) / 365)) - 0.006758*COS(2*((2*3.1416*(jd-1)) / 365)) + 0.000907*SIN(2*((2*
    3.1416*(jd-1)) / 365)) - 0.002697*COS(3*((2*3.1416*(jd-1)) / 365)) + 0.00148*SIN(3*
    ((2*3.1416*(jd-1)) / 365))) where "jd" is the day in the year 1 to 365
```

The declination plus the co-latitude provides the sun's altitude at noon.

There are a number of methods for drafting calendar or declination lines. Many dials have on their dial furniture just three lines or curves. They are the winter solstice (shortest day), summer solstice (longest day), and the two equinoxes, when day and night time are equal. Some dials have seven such lines, of which two are the solstices, leaving five, and those five account for the ten remaining months since all but the solstice months share a declination. However any calendar line may be drawn.

This chapter may repeat in summary form information in other preceding chapters so that the information is all in one place. Additional information not expressed in preceding chapters is explained more fully.

## ARMILLARY DIALS

The equinox line of an armillary dial runs as a straight line in the center of the dial plate. Solstice lines parallel the equinox line. The equinox line is perpendicular to the style and dropped from the nodus. In other words a notch or blob or other nodus is needed to show the calendar information. The solstice lines are above and below the equinox line by 23.5 degrees, and simple trigonometry can be used to calculate the linear distance above and below the equinox line. Since the tangent of 23.5 degrees is 0.43 , the linear distance above and below the equinox line is 0.43 times the radius of the armillary dial plate.


The above shows the circular dial plate, albeit it depicted as flat.

## EQUATORIAL DIALS

The equatorial dial has its dial plate paralleling the equator, perpendicular to its polar axis aligned style. The calendar lines are circles surrounding the gnomon. Their linear distance from the base of the gnomon is the co-tangent of the declination times the gnomon's linear height.

the radius of the calendar line, circle in this case, is the cotangent of the declination times the nodus linear height, however some spreadsheets may not support the cotan function so it is also:-
radius of calendar circle = nodus linear height / tan(declination)
and for the solstices, the cotan of 23.5 is the reciprocal of the tan, namely $1 / 0.43$, or 2.3 times the nodus height. The winter calendar lines are on the equatorial plate's lower surface, the summer lines on the upper. There is no equinox line since at equinox, the declination is zero, placing the equinox circle at infinity. If the gnomon is used to support the dial plate, then there needs to be a clear nodus on the lower part of the gnomon in order to cast a calendar indicating shadow. An equatorial dial can show the times of sunrise and sunset, appendix 6 addresses this. Appendix 3 provides all the calendar data for equatorial dials including sunrise/set data.


A horizon line is drawn from the nodus as shown above, and it crosses the 15 degree hour lines and the declination circles. For any date, the declination circle is found, and sunrise or set is the intersection of that declination circle with the hour line. Winter would use the lower nodus.

## POLAR DIALS

The polar dial uses a simple geometric construction, or a simple trigonometric formula.


The distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style linear height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines, then the points joined to form a hyperbolic curve for the solstices, or a straight line for the equinox. Appendix 3 has tables to facilitate this activity.

## MERIDIAN DIALS

The meridian dial faces true east or true west. The method is exactly the same as for the polar dial, see above, however, in this book the baseline for meridian dials is 6 o'clock (am or pm), whereas the polar dial uses 12 o'clock noon as the basis for hour lines. Some books use the 6 o'clock basis for meridian dials, some use noon, the result is formulae that look quite different due to the different reference hour, however the end results are exactly the same.

## HORIZONTAL DIALS

This chapter provides a geometric method as well as a trigonometric method for calendar lines, a short preview is covered below. The trigonometric method is the basis for the spreadsheets and procedural code (DeltaCAD, and FreeCAD) described later in this chapter.


The distance from the foot of the style, point T , and not from the dial center C , to a given hour line's declination point (TD) is determined by:-
distance TD = style or gnomon linear height / tan (sun's altitude)

$$
\text { sun's altitude }=\arcsin (\sin (d e c) * \sin (l a t) \quad+\cos (d e c) * \cos (\text { lat }) * \cos (\text { time }))
$$

The declination is found in several tables in the appendices. Appendix 2 has the declination tabulated by the day, the altitude spreadsheets on the CD or web site may be used for each hour, or their formula can be used directly.

Alternatively, the azimuth may be used from T , however this does not work for noon.

## VERTICAL SOUTH DIALS (true south or north)

The vertical dial is drawn exactly the same as a horizontal dial, but the dial is designed for the colatitude. Thus a vertical dial for latitude 40 degrees would be the same as a horizontal dial for latitude 50 degrees. Note that a vertical dial's summer solstice is the horizontal dial's winter solstice, and the vertical dial's winter solstice is the horizontal dial's summer solstice.

## VERTICAL DECLINERS (not true north, south, east, or west)

Vertical decliners have an equivalent dial that is not declining and is horizontal somewhere else on this planet. Find that location and the virtual horizontal dial with its declinations lines may be drafted, and then used by matching that horizontal dial's noon line with the vertical decliner's extended sub style line.

The term style height, SH , an angular and not a linear distance is used in this book, and more defined in chapters 16, and 17.

SH is the latitude to use for a "horizontal" dial to have calendar lines, and the noon of that calendar "horizontal" dial will align with the SD, style distance, of the actual dial, see next page.

While this is an insight, some other insights exist that facilitate vertical decliner calendar lines. A key insight is that at the equinox, day time equals night time, thus 6 am and 6 pm solar local apparent time is when the sun rises and sets. Thus the intersection of a horizontal line from the base of the sub-style towards a reachable 6 o'clock hour line defines one point of the equinox line. From that point, moving perpendicular to the sub-style line (of a rotated gnomon), the equinox line is thus drawn.

This only works for gnomons rotated using the SD technique because the sub-style line when extended provides the angle of the equinox line because that equinox line is perpendicular to SD, the style distance line extended.

For the derivation and use of SD and SH see chapter 16 as


Thus the equinox line is simply drafted as long as a 6 o'clock line is in place.
Another technique can be used for a vertical decliner's calendar lines. Again, the extended substyle line is used, along with a horizontal auxiliary dial. This technique assumes the gnomon has been rotated using the Style Distance techniques discussed in chapters 16 and 17.


A horizontal dial is built whose latitude is the vertical dial's style height and not the actual latitude, and the associated horizontal dial's gnomon dimensions equal those of the vertical dial's gnomon. The entire set of lines from the auxiliary dial is slid up such that its equinox line overlays the equinox line for the vertical decliner drawn using the methods of the previous page, or the horizontal dial is slid up so its gnomon blends with the rotated vertical dial's gnomon, same end result. The vertical dial's winter solstice is the horizontal dial's summer solstice, and the vertical dial's summer solstice is the horizontal dial's winter solstice.

Thus a vertical decliner at latitude 32, where the wall is South 10 degrees West would provide the data shown to the right. SD is 15.5 degrees, and the noon line of the auxiliary sundial will merge with the extended sub-style line. SH is

| 15.5 | SD |
| :--- | :--- |
| 56.6 | SH | 56.6 degrees, thus the horizontal dial is designed for latitude 56.6 degrees. The physical dimensions of the vertical decliner's rotated gnomon are exactly used as-is on the auxiliary horizontal dial. Consider double checking your design using software such as the DeltaCAD macros that Illustrating Shadows.

ANOTHER TECHNIQUE for the vertical decliner projects the azimuth and altitude (for each solar hour for a given declination) from the nodus, using the point on the sub-style "below" (i.e. horizontal to) the nodus as a base point. The FreeCAD vertical declining dial program does this.

Many methods and techniques exist. Whichever method you select, double check it with other software or methods, and consider making a mockup cardboard dial to verify it in practice.

## Declination lines for the horizontal (or vertical) dial using geometry.

This section shows how to draw calendar lines on a horizontal or vertical dial, step by step, and a template is available in appendix 9 to facilitate this process whose use is described in chapter 12.

A horizontal dial with three hour lines shown.


First, draw a gnomon for the dial center "C". From the nodus draw the equinox line ( 90 degrees to the style), and from that the solstice lines (approx 23.5 degrees on either side). The three lines (equinox and the solstices) intersect the gnomon's base line extended, or the noon line, at points $\mathrm{a}, \mathrm{b}$, and c . These three points whose distances from the dial center are $\mathrm{Ca}, \mathrm{Cb}$, and Cc are then transcribed to the dial plate (right pictorial to left pictorial).


The equinox line is then drawn perpendicular to the noon line, and it produces equinox intercepts for those additional hour lines, d and e. Distances Cd and Ce are then located from the left dial plate to the right hand picture on its equinox line. This produces two more hour lines on the right hand side picture, Cd and Ce . Those hour lines on the right hand side do not have angles that match their hour lines on the dial plate, and this is because this is a projection.

Now that there are two more hour lines, or as many as you choose, this produces intercept points for the solstice lines, namely points $f, g, h$ and $i$.


Points $f, h, g$, and $i$ are now transferred back to the dial plate, from the right projection pictorial to the left picture.


When this process is completed for as many hour lines as desired, the dots are connected and then the declination lines drawn.

NOTE: The horizontal and vertical dials have


## Declination lines using trigonometry.

Horizontal dial ~ calendar or declination curve logic
The simplest approach for a horizontal dial is to select a solar declination such as $23.44,-23.44$, or 0 , and run the hours from morning to afternoon. Running the hours means using LAT (local apparent time), and for each time increment, derive the sun's azimuth and altitude. That part of the process is the same for the vertical, horizontal, and vertical declining dial. Assume "glh" is the gnomon linear height:-

For the horizontal dial, distance " $d$ " is:thus


The very first calculation is the first point of a declination or calendar curve, and thus no line or curve segment is drawn, but the " $x, y$ " coordinates are saved. The hour is bumped up to the next time and a new " $x, y$ " coordinate calculated, and this time a line segment is used because we have the previous " $x, y$ " pair. This is repeated until the end of the time frame. This process can be repeated for all desired declinations. The " $x, y$ " coordinates are relocated by the coordinates of the base of the nodus, "NODUSY, NODUSY", and not from dial center.

```
/*\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
' \# MAIN LOOP \# the solar declinations in this example is -23.44 \#
" \# makeLine is a function to draw a line segment, see "functions.js" \#
' \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
*/
/* this uses NODUSX,Y from the gnomon drawing earlier */
wx = \(9999 \quad / *\) these, when 0, tell the calendar */
wy \(=9999 \quad /^{*}\) line draw to draw nothing */
for (hr=7; \(h r<17 ; h r=h r+0.25)\) \{
    /* get an altitude and azimuth for this one hour at this SOLAR declination */
    al = altitude( hr, lat, -23.44 ) /* degrees altitude */
    \(\mathrm{d}=\) glh / Math.tan(al*2*3.1416/360) /* radial distance: nodus base to cal curve */
    az = azimuth( hr, lat, -23.44 )
    /* degrees azimuth */
    \(x x x=\) d * Math.sin(2*3.1416*az/360)
    \(y y y=d\) * Math.cos(2*3.1416*az/360)
    if ( wx<9999 \&\& wy<9999 ) \{
            if ( Math.abs(nodusx+wx)<150 \&\& Math.abs(nodusx+xxx)<150 ) \{
                if ( Math.abs(nodus \(x+w x\) ) \(<130\) \&\& Math.abs(nodusx+xxx)<130 ) \{
                        // above code keeps the curve within reasonable bounds
                        makeLine( nodusx + wx, nodusy + wy , nodusx + xxx, nodusy + yyy )
                \}
            \}
    \}
    wX = XXX
    \(w y=y y y\)
\}
```

The logic is shown above, this is written in Java Script as used in the free NanoCAD program. The azimuth and altitude formula are the standard formulae, see the appendices. Some of the Illustrating Shadows CAD programs may use a different algorithm.

Vertical dial ~ calendar or declination curve logic


As for the horizontal dial, the approach for a vertical dial is to select a solar declination and run the hours from morning to afternoon. Running the hours means using LAT (local apparent time), and for each time increment, derive the sun's azimuth and altitude. That part of the process is the same for the vertical, horizontal, and vertical declining dial. Assume " glh " is the gnomon linear height:-

For the vertical dial:- $\quad x=$ glh * tan(azimuth)
and, distance "d" is:- $\quad d=g l h / \cos (a z i m u t h)$
which is needed for the " $y$ " ordinate:-

$$
\begin{aligned}
y & =d^{*} \tan (\text { altitude }) \\
& =\tan (\text { altitude })^{*} g l h / \cos (\text { azimuth })
\end{aligned}
$$

```
' #######################################################################################
' # MAIN LOOP # the solar declinations in this example is -23.44
' #########################################################################################
'# this uses NODUSX,Y from the gnomon drawing earlier
wx = 0 '# these, when 0, tell the calendar
wy = 0 '# line draw to draw nothing
hr = 7 '# start at 0700 am (1st seg not drawn)
while hr < 17 '# end at 1700
    '# get an altitude and azimuth for this one hour at this SOLAR declination
    az = azi( hr, latr, -23.44 ) ' degrees azimuth
    xxx= glh * tan(2*3.1416*(az)/360)
    al = alt( hr, latr, -23.44 ) ' degrees altitude
    yyy= glh * tan(2*3.1416*al/360) / cos((2*3.1416*(az)/360))
    if wx<>0 and wy<>0 then
        xl=nodusx+wx
        yl=nodusy-wy
        xn=nodusx+xxx
        yn=nodusy - yyy
        ptb = CStr(xl)+","+CStr(yl)+","+CStr(0)
        pte = CStr(xn)+","+CStr(yn)+","+CStr(0)
        if abs(xl) < 130 and abs(xn)<130 then ' all x values in range?
            if yl > -130 and yl<0 then ' last Y in range ?
                if yn > -130 and yn<0 then ' last Y in range ?
                    Set lineObj = ThisDrawing.ModelSpace.AddLine(ptb,pte)
                end if
            end if
        end if
    end if
    wx = xxx
    wy = yyy
    hr = hr + 0.25
wend
```

The process is similar to the horizontal dial process, and uses the base of the nodus and not dial center. The above code is written in Visual Basic Script as used in the free NanoCAD program. Some of the Illustrating Shadows CAD programs may use a different algorithm.

Vertical Declining dial ~ calendar or declination curve logic
As for the horizontal dial, the approach for a vertical declining dial is to select a solar declination and run the hours from morning to afternoon. Running the hours means using LAT (local apparent time), and for each time increment, derive the sun's azimuth and altitude. That part of the process is the same for the vertical, horizontal, and vertical declining dial. Assume "glh" is the gnomon linear height. For the vertical declining dial, the process is similar to the vertical dial, except the vertical dial plate, or wall, has a declination which is factored in. As before, the nodus base point is located at "nodusx, nodusy"

So: $\quad x=$ glh * tan(azimuth + wall declination $)$


And:- $\quad d=g l h / \cos ($ azimuth $\quad$ which is needed for the " $y$ " ordinate:-
so: $\quad \mathrm{y}=\mathrm{d} * \tan ($ altitude $) \quad=\mathrm{glh} * \tan ($ altitude $) / \cos ($ azimuth+wall declination $)$

```
########################################################################################
# MAIN LOOP # the solar declinations in this example is -23.44
########################################################################################
# this uses NODUSX,Y from the gnomon drawing earlier
# set the summer and winter x,y coordinates to 0 to begin with
wx = 0 # these, when 0, tell the calendar
wy = 0 # line draw to draw nothing
hr = 7 # start at 0700 am (1st seg not drawn)
# keep "decl" useful for azimuth on the dial plate
decl = dec
if dec >0:
    decl = -1 * dec
while hr < 19 : # end at 1700 or 5pm
    # get an altitude and azimuth for this one hour at this winter SOLAR declination
    al = alt( hr, lat, -23.44 )
    az = azi( hr, lat, -23.44 )
    xxx = glh * math.tan(math.radians(az+decl))
    yyy = glh * math.tan(math.radians(al)) / math.cos(math.radians(az+decl))
    if dec > 0:
                xxx = -1 * xxx
        if abs(xxx)< 3 and abs(yyy)<2 :
        if wx<>0 and wy<>>0:
                c = Part.makeLine((nodusx+xxx, nodusy-yyy,0),(nodusx+wx, nodusy-wy, 0))
                Part.show(c)
        wx = xxx
        wy = yyy
#
```

The process is similar to the vertical dial process, and uses the base of the nodus and not dial center. The above code is written in Python as used in the free FreeCAD program. Some of the Illustrating Shadows CAD programs may use a different algorithm. Python uses indentation rather than an approach using braces $\{\ldots\}$ as in Java Script, or some other code block terminator such as IF...END IF, or WHILE...WEND as used in Visual Basic.

Illustrating Shadows provides:- functions.py, functions.js, functions.vbs to aid programming.

The Java Script produced horizontal dial with calendar curves created in NanoCAD:-


The Visual BASIC Script (vbs) produced vertical dial created in NanoCAD:-


The vertical declining dial using Python as in FreeCAD:-


FreeCAD and NanoCAD are CAD programs available at no cost. FreeCAD is open source, and NanoCAD while free, requires a serial number (immediately available by email), and registration (immediately provided online). Both install easily, work first time, and do not need extra software to be downloaded. They have been tested on Windows 8 and earlier editions. Being free, documentation is limited, so Programming Shadows on the Illustrating Shadows web site or CD is essential for anyone programming in those systems. Supplemental Shadows has more information on declination curve methods.

## Some key insights about flat dial calendar lines

The following two notes are critical to understanding the relationship between the "virtual dial" used to draft calendar or declination lines, and the "real" dial that will show the time.

## You do not have to use "the" hour lines

You can use the actual hour lines, although it is an extra step if the dial is longitude corrected. However, you can use any hour lines you like when drafting the calendar or declination lines. You could use one hour and three or four hours on either side of "noon". While it is tempting to use 10 degrees and 30 or 40 degrees on either side of the extended sub-style, it is not that simple. This is because you would have to work back from the angle with the sub-style, from whence the azimuth or altitude are readily found, back to the time that would produce those azimuth and altitude figures.

## The "noon" and the hour lines on declining dials are NOT real noon and hour lines

When designing a declining dial, or even an inclining declining dial, the calendar lines use "noon" for the dial that is appropriate for a horizontal dial whose latitude is the style height. In other words, "noon" for calendar line design on declining dials is the sub-style line extended, and not noon for the real dial's location. So, for those dials, the calendar lines will obviously use "hour lines" that are not those that will finally be displayed on the dial. The "noon" for calendar lines is the sub-style extended, the "noon" for the real dial will be a vertical line dropped from dial center (for vertical dials), or a true north-south line passing through dial center (for horizontal dials). And those real noon lines may be displaced if the dial is longitude corrected.

The following two notes are critical to understanding the relationship between date and hour.

The nodus is for solar declination (calendar date information), the style is for the hour.
There is a recent tendency to use a rod as the gnomon, the tip being the nodus, and that tip serving a dual purpose. One purpose is to indicate the solar declination, in other words the date. And, where the nodus shadow falls on an hour line is also the time.

Anything that is dual purpose usually has somewhere in it a compromise. The compromise is the ease or accuracy of reading the hour. With a style, there is a long shadow, and the eye can parallel that style shadow with an hour line and thus increase readability, and thus accuracy. With a rod as the dual purpose shadow caster, the ability to read the time depends on one shadow tip, and on a more perfectly aligned gnomon.

The corollary is that sometimes ridiculously small gnomons exist on dial plates, because the style tip is used for marking the calendar declination lines, and thus the dial plate has hour lines contained within the area bounded by the winter solstice and the dial center. What on Earth, or in the solar system, is wrong with having a long style and using all the dial plate area for those hour lines, and treating them separately from the nodus, which would be part way up the style, and have the nodus for just the calendar lines or curves?

Be bold. Use all the dial plate for the hour lines. Use the maximum effective area for the calendar lines and curves, and mark the nodus at an appropriate place somewhere along the style.

## The practical laser trigon for calendar lines

Just because we live in a scientific age with things to the nano second, the micro nano millimeter, and digital this and that, doesn't mean that other methods do not have a place. In days of old, a device called a trigon was often employed with a protractor and a thread.


To the left is a more modern trigon made from copper plate and a conference presenter's laser pointer. The laser is held in this case by one of two plates angled at plus and minus 23.5 degrees, secured to a rigid copper wire of awg 10 size, inserted into a copper tube that is itself secured to a vice of two copper plates which is placed on the gnomon's style.

Such a device can be made in a few minutes, and the vice can be a pair of plates such as is shown for placing on a gnomon of copper plate, or it can be two copper plates at right angles such as would be used for a thick gnomon. The variations are endless.


Of course, the laser angles need calibrating, and to the right is a pager cutout of a gnomon with the winter solstice, the equinoxes, and the summer solstice shown.

The laser is depressed and the angle of the light verified against the solstice line, as shown. While the declination angle is thus measured, it is important that the rotational plane of the laser be correct. This can be tested by marking three points on the extended sub-style line which result from three angles from the nodus, $-23.5,0$, and +23.5 . If the laser strikes both at the correct predicted distance then the rotational plane is probably correct. Also, the axis around which the laser rotates must parallel the style.


To the left is an actual dial plate with the paper gnomon placed showing a solstice line and the equinox line.

This simple device is useful for dial plates that are not flat, such as an S-shaped roofing tile, or any other irregular surface for that matter.

Good declinations to use, considering month ambiguity, might be:-


## CHAPTER TWENTY FOUR

## dial furniture ~ Italian, Babylonian, and day of length dial furniture

## LINES THAT ARE NOT HOUR LINES AS SUCH

There are some lines or curves that appear as dial furniture that are not time of day hours, and yet they indicate a time of sorts. Yet, they appear as furniture which places them in common with calendar lines. For that reason, these lines and curves are discussed here.

## LENGTH OF DAY LINES (DAYLIGHT HOURS):

The declination can be used not only for certain traditional dates, but also to indicate the length of a day. Looking at sunrise and sunset tables will show days of say 8,9 , or any other number of hours. Then for the dates associated with those day lengths, the sun's declination may be found.

Using the techniques for declination lines in this book, curves can be drawn such that when the nodus shadow is on that curve, then the day-length is indicated.

If $L$ is the difference between the hours in the day and 12 , multiplied by 15 (degrees in an hour) then:-

$$
\begin{aligned}
& \text { declination }=\arctan (\sin (L / 2) * \cot (\text { latitude })) \text { or } \\
& \text { declination }=\arctan (\sin (L / 2) / \tan (\text { latitude }))
\end{aligned}
$$

Thus, work out for your latitude the declination for an $8,10,12,14$ hour day, for example, and since you now have the sun's declination, you may now use it with the empirical, geometrical or trigonometric methods for the drawing declination lines. Instead of labeling them with a date, label them with the day's duration.

For example, consider a 14 hour day at latitude 57. the difference between 14 and 12 is 2 , so...
$L$ is $2 * 15$, so $L / 2=15$, and $\cot (57)=0.6494$,
thus declination $=\arctan ((\sin (15) * \cot (57)))$
or $\arctan (0.2588 * 0.6494)=\arctan (0.1681)=9.5$ degrees
This provides four dates, two for 14 hours ( $14-12$ is 2 ), and two for 10 hours ( $12-10$ is 2 ), and using tables, they are Feb 24, Oct 19, and Apr 15 and Aug 29 for the specified latitude.

A number of tables in the appendices have the declination for a given day. Using this declination, declination lines can be drawn using the same methods used for equinox and solstice lines.

## UNEQUAL, TEMPORARY, or BIBLICAL HOURS

The unequal hours, temporary hours, or biblical hours are simply a given day's daylight (see above) divided into 12 equal parts. Those parts vary from day to day, shorter in winter, longer in summer, one hour during the equinoxes. They are drawn by deriving the length of day at the winter solstice (for vertical dials) or summer solstice (for horizontal dials), dividing that duration by 12 , and marking those times from noon on the solstice curve. At the equinox mark the times in 1 hour increments from noon. Connect the two points for each of the 12 "parts" and extend the lines. The result is a set of "hour" lines indicated by the nodus. Not much use these days, but of historical interest, and as such they can be found on some European dials.

## LENGTH OF DAY TRANSITIONING TO SUNRISE AND SET TIMES

To determine the L.A.T. (local apparent time) of sunrise or sunset for any day of the year, the hour angle when the sun is on the horizon is used. The formula for that hour angle, with no longitude or EOT correction is:-

$$
\text { hsr }=\arccos (\tan (\text { latitude }) * \tan (\text { declination }))
$$

The declination is determined by:-

$$
\begin{aligned}
& \text { declination }=\text { degrees }\left(0.006918-0.399912^{*} \cos (d a)+0.070257^{*} \sin (d a)\right. \\
& \quad-0.006758^{*} \cos \left(2^{*} d a\right)+0.000907^{*} \sin \left(2^{*} d a\right) \\
& -0.002697^{*} \cos \left(3^{\star} d a\right)+0.001480^{\star} \sin \left(3^{\star} d a\right)
\end{aligned}
$$

Where "da" the "day angle" (in radians, an intermediate figure), is determined by [ $\mathrm{j}=$ Julian day ]

$$
\text { da }=2 \text { * } 3.1416 \text { * }(\text { Julian day of the year }-1 \text { ) / } 365
$$

For the winter and summer solstice, the sunrise and sunset times from the equator to latitude 65 are depicted below, and generated using a DeltaCAD program on the CD and web site. The appendices also have a nomogram to determine sunrise and set times built with DeltaCAD.


A solstice sunset time, together with the equinox sunset time of 6 pm (longitude not considered, nor EOT), provide a means of determining the number of hours to sunset, or of course the number of hours since sunrise, discussed on the next page as Italian and Babylonian hours.

## BABYLONIAN AND ITALIAN LINES

Once a dial is built, and solstice and equinox lines drawn, then all sorts of options exist for additional clutter (dial furniture) on the dial. Calendar information was often helpful, when to plant, sow, and reap in a then unhurried pace. In particular, some other interesting "hours" were used. Babylonian hours showed the time from sunrise. The Italian hours showed the time from sunset. In practice the Italian hours were massaged to show the number of hours until sunset, a figure helpful to the gardener or worker toiling in the fields.

The technique for drawing Italian hour lines is simplicity itself. The time of sunset is noted for the solstices and the equinoxes. The appendix has approximate formulae for this calculation, and many online almanacs have readily available accurate times. Some free software available consists of spreadsheets which are very accurate. Appendix 6 may be helpful.

NOTE: If the dial is neither longitude nor EOT corrected, then use sunset times with no longitude or EOT correction. If the dial has longitude corrections built in, then use sunset times with a longitude but no EOT correction. If you use the spreadsheets on the web site, you can effect this by setting the reference longitude to the location's longitude, and the EOT can be set to zero by changing the EOT multiplier from 1 (normal) to 0 (no correction desired). Appendix 6 provides true (L.A.T.) sunrise, sunset, and day-length for many latitudes.

For a vertical dial, mark on the winter solstice calendar line the time when sun sets, and one hour before that, and two, and three, and so on. Then for the equinox perform the same operation (sunset is 6 pm L.A.T.), and similarly for the summer solstice if shown. Some dials may not have room for both solstice curves, so use just one solstice and the equinox. Simply connect the sunset time's dots on the solstice and equinox lines and draw a straight line. Mark that line as "sunset". Then connect the dots for one hour before sunset, and mark the resulting line "1 hour to sunset", and proceed for all the desired Italian hours. Take care with summer time adjustments when marking the hour points. Babylonian hours, while less useful, follow a similar process. The process is similar for horizontal or other dials.

CAUTION: Italian and Babylonian hours are latitude specific. Tilting an hour angle dial for a new latitude will correct the normal hours but not the Italian or Babylonian lines. While hour lines use the hour angle around the style, Italian and Babylonian lines are latitude and geometry based, detecting when and where the sun crosses the horizon. That geometry is latitude dependent as it uses the curvature of the Earth for a specific latitude and is thus not correctable by tilting.

A VERTICAL DIAL WITH SOME ITALIAN HOUR LINES


## CHAPTER TWENTY FIVE

## dial furniture ~ the Analemma, and the use of "DL"

The analemma is a depiction of the equation of time and the day of the year. Shown on globes as a distorted figure of eight, it is also shown as a wavy graph.


Since the date is less practical for the purpose of trigonometry, the sun's declination is used.

The sun's declination (for a given date), and the equation of time (eot) are used to derive $X$ and $Y$ values for an analemma on a dial plate, or on a gnomon. Sometimes the altitude and azimuth are needed as well.

The analemma on a dial plate has the benefit of being simpler, and as shall be seen, more accurate. However the dial plate gets cluttered, especially when half and quarter hours are used, as well as calendar curves, and Italian lines.

The benefit of having the analemma on a gnomon's style is that the dial plate is less cluttered. However there are drawbacks. If the style has a 3d bobbin form or bowling pin kind of analemma, inaccuracies are introduced. This is because it is either symmetrical or it is not. If it is symmetrical, then for a given solar declination which may have two different eot values, an average is used, and that average generates an error.


If it is not symmetrical then there are still inaccuracies because the same point on the bobbin gnomon style will displace the shadow on the dial plate differently for different hours on the same day, as shown above. In essence, 3d bobbins or bowling pins must be symmetrical because you must be consistent about the part of the shadow you are using, i.e. the left part, or the right part. So for 3d analemma gnomons, the 3d analemma is symmetrical and thus always provides an average of the EOTs for any given declination.

In summary, dial plate analemmas are more accurate than gnomon based 3d analemmas. And gnomon based analemmas need a fixed dial plate line or curve to read the analemma shadow.

The solution is to have a flat and not a 3d gnomon analemma. While this does allow an asymmetrical analemma, it also requires that the analemma be rotated to be perpendicular to the suns rays. The conclusion about analemma depiction on sundials is thus:

An analemma on the dial plate
is easy to make accurate
works directly only for the hours it is associated with requires interpretation for other hours
clutters the dial plate when other furniture is there


A 3d bobbin on the gnomon's style has to be symmetrical is thus an average of EOTs for the same solar declination is harder to make than an analemma on a dial plate is harder to make than a 3d analemma is easy to read for any hour needs a fixed dial plate line or curve to read the analemma shadow

A 2d analemma on the gnomon's style can be asymmetrical thus has accurate EOT depiction is easier to make than a 3d bobbin but it requires you to rotate it needs a fixed dial plate line or curve to read the analemma shadow

A table or chart of EOTs
is easy to use
requires only one simple addition or subtraction
does not clutter the dial plate or gnomon
but requires it to be placed somewhere in easy reach
and it can incorporate longitude corrections
Illustrating Shadows provides Excel spreadsheets with $X$ and $Y$ values facilitating graph paper depiction of an analemma. This is more accurate then polar coordinates (angle and distance) since the angles are rather small thus errors can easily creep in.
analemma.xls
Also, DeltaCAD programs provide a pictorial of the analemma, as well as $X$ and $Y$ coordinates. And the standard v-dial and h-dial DeltaCAD macros depict the analemma, and in addition a single DeltaCAD macro:
analemma.bas

The diallist must choose whether to place an analemma on a dial plate, or use the gnomon's style with a 3d bobbin, or a 2d template, or whether to use a table or graph of EOTs.

NOTE: Always double check the EOT analemma template and make sure the analemma is oriented properly.

NOTES ON TECHNIQUE FOR THE VARIOUS DIAL TYPES: The armillary and equatorial dials use the declination and EOT directly. The armillary is a special case as the gnomon and dial plate analemma formula are symmetrical. The equatorial dial full plate uses polar coordinates whereby the EOT is converted to degrees with the hour and longitude difference added which then becomes the angle, and the declination is used as the radius, and these two are then converted to $X: Y$ coordinates. The polar and meridian dials use declination and EOT directly. The horizontal, vertical, and vertical decliner dials require solar altitude and azimuth to be derived.

## ARMILLARY DIAL

The analemma is on the gnomon style.
There are some assumptions and approximations.



VIEWED SIDEWAYS FROM EAST OR WEST


VIEWED FROM THE EQUATOR LOOKING TO THE POLE

The formula for Y (north or south along the style of the gnomon) and Y (east or west parallel to the earth) for an analemma is derived as shown above, and is:-

```
eot = 7.5* *in(RAD(jd-5))-10.2*Sin(RAD(1.93*(jd-80)))+0.5*Sin(RAD(1.5*(jd-62)))
decl = 23.45* Sin(rad(0.9678*(jd-80)))
y = r*Tan(rad(decl) )
x = V*Tan(rad(eot/4))
    r *Tan(rad(eot/4) / Cos(rad(decl) )
```

There is an approximation even still, in the above. It assumes the plane of the analemma is perpendicular to the line from the hour line up to the analemma's bobbin center. Of course, instead of a bobbin in 3d, a 2d plate can be used which is rotated to parallel the dial plate surface where the hour line is located.

Excel .spreadsheets exist:-
And a DeltaCAD macro is also available:-
analemma.xls
analemma.bas

Scaling is important if you use DeltaCAD, and aspect ratio as well as scaling if you use Excel or Open Office. For DeltaCAD the OPTIONS, DRAWING SCALE, and VIEW SC, must be 1 and in addition, FILE, SET PRINT REGION, PRINT SCALE must be 1. For spreadsheets, the data is more useful than the pictorial display since spreadsheets make little pretence at retaining aspect ratio when graphics are displayed. After printing the DeltaCAD analemma gnomon, verify the north and south X and Y distances and compare it to the Excel range.

The DeltaCAD macro supports gnomon based analemmas for armillary dials.

## ARMILLARY DIAL

The analemma is on the dial plate.

There are some assumptions and approximations.


Gosulb gmonn neareght


As with the bobbin (3d shaped gnomon) or 2d rotatable analemma, all on the gnomon style, the formula for the analemma on the dial plate turns out to be substantially the same.

```
eot = 7.5 * Sin(RAD(jd-5))-10.2* Sin(RAD(1.93*(jd-80)))+0.5* Sin(RAD(1.5*(jd-62)))
decl = 23.45 * Sin(rad(0.9678*(jd-80)))
```

```
y = r*Tan(rad(decl) )
```

y = r*Tan(rad(decl) )
x = V * Tan(rad(eot/4))
x = V * Tan(rad(eot/4))
= r*Tan(rad(eot/4) / Cos(rad(decl) )

```
    = r*Tan(rad(eot/4) / Cos(rad(decl) )
```

Actually, almost the same. Whereas the larger EOT swing is to the south on an analemma built in to the gnomon, it is to the north when the analemma is on the dial plate itself. This is shown pictorially below. Also, not only is the analemma rotated about the east west axis as mentioned above, the picture on the right also shows that it must be rotated about the north south axis as well.

That rotation about the east west axis is critical because the end of the year EOT swing is some 16 minutes whereas the mid year swing is about half that, there is no symmetry.


Most 3d analemma gnomons assume there is symmetry of the + and - swings, but even that is not true. The end of year swing is 14.15 to -16.29 , a difference of some two minutes, the mid year swing is 6.32 to -3.40 , a difference of almost three minutes. The lobe location relative to south is valid for both hemispheres, that is why the term end of year and mid year were used, not summer and winter.

| EOT 2009 | mm.ss |  |
| :--- | :--- | :--- | :--- |
| end of year  mid year  <br> MAJOR PEAKS  MINOR PEAKS  <br> MAX MIN MAX MIN <br> 14.15 -16.29 6.32 -3.40 |  |  |

The DeltaCAD macro supports a single detailed dial plate analemma, and a full dial plate for armillary dials to be used as is.

## EQUATORIAL DIAL

The analemma is on the gnomon style,

There are some assumptions and approximations.


In this case, "d" is a fixed radius from the base of the gnomon rod, and falls on an hour line or some other time division that will be corrected by the gnomon. As usual, we need $X$ and $Y$, and we also have the EOT and Declination. Assuming that the distance from "p" to the base of the gnomon, known as " $x$ " is substantially the same as from " $p$ " to point " t ", which is not correct, but is almost the same, then:-

$$
\tan (\mathrm{decl}) \quad=\quad \mathrm{y} / \mathrm{d}
$$

thus $y \quad=\quad d * \tan (d e c l)$
and assuming that EOT as shown is the same angle on the slope, as it would be between line "d" and the line "pt", which is not correct, but is almost the same, then:-
thus

$$
\tan (e o t) \quad=\quad x / d
$$

thus $x \quad=\quad d * \tan ($ eot $)$
and again, these are assumptions that are close but not perfect. The rule of a $2 d$ rotatable analemma on the gnomon still applies, it is more accurate and easier to make than a 3d bobbin.


The DeltaCAD macro supports gnomon based analemmas.

## EQUATORIAL DIAL

The analemma is on the dial plate.
There are some assumptions and approximations.


For a point on top of the gnomon, the nodus, a shadow will be drawn whose tip is based on the sun's declination, and whose displacement from a given hour line will be based on the equation of time, EOT. If the linear height of the gnomon rod is "R" inches then the following applies.

|  | $\tan (\mathrm{decl})$ | $=$ | $\mathrm{R} / \mathrm{y}$ |
| :--- | :--- | :--- | :--- |
| thus | y | $=$ | $\mathrm{R} / \tan (\mathrm{decl})$ |
| and | $\sin (\mathrm{decl})$ | $=$ | $\mathrm{R} / \mathrm{V}$ |
| thus | V | $=$ | $\mathrm{R} / \sin (\mathrm{decl})$ |
| and | $\tan (\mathrm{EOTm} / 4)$ | $=$ | $\mathrm{x} / \mathrm{V}$ |
| thus | x | $=$ | $\mathrm{V} * \tan (\mathrm{EOTm} / 4)$ |
|  |  | $=$ | $\tan ($ EOTm$/ 4) * \mathrm{R} / \sin (\mathrm{decl})$ |

For the northern hemisphere, the summer or mid year surface uses late March through late September, and the winter surface uses late September to late March 20. And the same dates but the opposite surface are of course used for the southern hemisphere.

However, at the equinoxes the shadow tip is at an infinite distance, thus a more practical range for analemmas is when the declination is not from $0^{\circ}$ but say from $10^{\circ}$, to 23.44 degrees.

$$
\begin{array}{llll}
\text { end of year } & \text { Oct } 22 \text { - Feb } 23 & \text { JD } & 295-365, \text { and } 1-54 \\
\text { mid year } & \text { Apr } 15 \text { - August 28 } & \text { JD } & 105-240
\end{array}
$$

The best solution in fact would be where the value of $Y$ is less than equal the radius of the dial plate, DR.

NOTE: Northern hemisphere: mid year (summer) is facing sky, end year (winter) face ground.
NOTE: Southern hemisphere: mid year (winter) is facing ground, end year (summer) faces sky.
NOTE: If the sun is slow, i.e. the EOT is positive, the analemma point is to the west so it gets hit earlier. If the sun is fast, i.e. the EOT is negative, the analemma point is to the east so it gets hit later.

The DeltaCAD macro supports a single detailed dial plate analemma, and a full dial plate for summer and winter equatorial dials to be used as is.

## HORIZONTAL AND VERTICAL DIALS - key notes

The armillary and equatorial dials were special cases in so far as the sun's declination was usable directly. The armillary dial formulae were symmetrical for an analemma on the gnomon or the dial plate, not so for the equatorial dial. For horizontal and vertical dials, a simple declination based formula is only usable at local apparent noon where the altitude of the sun is the colatitude plus the declination. For hours other than local apparent noon, this is not true, and altitude and azimuth become the factors that must be used.

Hour lines match a clock when the longitude correction and the equation of time are factored in. Dial furniture can accommodate this information relieving the observer from this simple adjustment. The longitude correction is a fixed adjustment to the sun's hour angle, but the equation of time varies by the day.

The analemma figure of eight attempts to show the correct time for the latitude, longitude and date by where the nodus falls. These legal standard time curves approximate figures of eight, and as such are ambiguous for the most part. In other words, the same solar declination may have associated with it two different EOTs, this is because except for the solstices, two dates share the same declination.

The derivation of the analemma is simple. From the base of the nodus, a line is drawn at an angle and for a distance. Then for a selected time which the analemma figure of eight will represent, the shadow tip is calculated for the yearly cycle. That angle is the sun's azimuth at the dial's location for a desired time. And that distance is the nodus linear perpendicular height above the dial plate divided by the tangent of the sun's altitude. Of course, any time can be used. Complications arise for latitudes below 23.5 as the altitude "goes over the top", and for some hours when the azimuth moves from north to south. When programming an analemma, complications exist for all latitudes when the hour is such that the sign of the $X$ or $Y$ coordinate reverses, and this is fixed with simple programming code, which is used in the DeltaCAD analemma macros.

The polar coordinate system is simple, it is the azimuth for direction, and the nodus linear height divided by the tangent of the altitude, for the appointed time. And that time is the legal time modified by the longitude correction (fixed) and the EOT (varies by the day). Polar coordinates are simple, but suffer from the need for accurate angles, thus, $x, y$ coordinates may be more accurate in practical drafting. Conversion from azimuth/distance to $x, y$ cartesian coordinates is also simple, for some latitudes and times, some extra steps are needed because of altitudes passing through 90 degrees, or azimuths moving from north to south or vice versa..

```
x = polar distance * 部(azimuth)
y = polar distance * cos(azimuth)
```

The altitude and azimuth are simple calculations, and their basic formulae are in the appendix. The illustratingShadows.xls spreadsheet has these calculations programmed and all that is needed is a latitude, longitude, time and a gnomon linear height. For example, consider the following data:

| FOR THIS LATITUDE: | 32.75 |
| :--- | ---: |
| ENTER A TIME FOR THE ANALEMMA DATA hh.hh: | 12.00 |
| ENTER A GNOMON HEIGHT IN ANY UNITS: | 100 |

The spreadsheet provides a summary table for about every ten days with both the polar coordinates (azimuth and distance) as well as the Cartesian coordinates ( $x$ and $y$ distances from the base of the gnomon). Note that this is the gnomon base and not dial center. The spreadsheet is designed for latitudes between 0 and 60 approximately, and the noon hour plus or minus a few hours.



NOTE These figures are in close agrement with other systems. However, each system uses different EOT formulae as well as declination formulae, so this data may be slightly different from other programs.

NOTE: design latitude is from 0 degrees to 66 degrees

The graph to the right is the graph of the $x$ and $y$ coordinates of the point that the nodus tip will impact a horizontal dial for a given legal time at the dial's latitude and longitude.

While Italian hour lines convey useful information, the analemma figure of eight tends to clutter the dial, and still requires the ambiguous month for it to work correctly, so it does not save much observer time for the added cluttering of the dial plate. Some ingenious diallists have made gnomons whose shape is a double pear drop or 3d bobbin like figure of eight where the approximation is that the figure of eight is symmetrical, which it is not. And other ingenious variations on that theme exist. A number of dials use these figures of eight, however the majority do not, either from ignorance, or from the provision of an EOT and longitude correction chart. Most of the author's dials are longitude corrected so only an EOT chart is needed.

Since many diallists use DeltaCAD, Illustrating Shadows provides integrated analemmas on vertical and horizontal dials complete with calendar (declination) curves, for most latitudes and longitude corrections. That same DeltaCAD macro also provides equatorial and armillary analemma support.

## Horizontal Dial - analemma on dial plate - DeltaCAD program

Program:
analemma.bas
which is also a choice of the standard: MAIN-h-dials.bas

NOTE: regardless of hemisphere, the summer part of the analemma is closer to dial center, the winter part of the year farther from it.
winter in the northern hemisphere is end of and beginning of the year.

h -dial and calendar using gnomon linear height
Lat: 32.8 d.Long: 03.2
$\begin{array}{ccccccccccccc}6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\ 84.1 & -68.9 & -47.0 & -31.2 & -19.5 & -10.1 & -01.7 & 06.4 & 15.3 & 25.8 & 39.6 & 58.7 & 84.1\end{array}$

## Vertical Dial - analemma on dial plate - DeltaCAD program

Program:
which is also a choice of the standard:
analemma.bas
MAIN-v-dials.bas

NOTE: regardless of hemisphere, the winter part of the analemma is closer to dial center, the summer farther from it.
winter in the northern hemisphere is end of and beginning of the year.


NOTE: Chapter 23 covers the geometric and trigonometric methods of deriving calendar or declination points, and thus the basis for the horizontal, vertical, and vertical decliner dial analemmas. These dials use the sun's altitude and azimuth. The formulae can be seen by looking at the above named DeltaCAD macros or the Illustrating Shadows spreadsheets. For the southern hemisphere, the analemma is reversed top to bottom.

The DeltaCAD macro supports a full dial plate for horizontal, vertical, and vertical decliner dials. The DeltaCAD SELECT and COPY followed by a PASTE into a WORD file sometimes has issues with the vertical dial due to the DeltaCAD object rotation feature used.

## Vertical Declining Dial - analemma on dial plate

For vertical decliners, the normal dial can be created as a decliner, then a horizontal dial with a latitude equal to the vertical decliner dial's SH can be slid over the vertical decliner's dial plate such that 12 noon local apparent time on the pseudo horizontal dial plate matches the SD on the real vertical decliner.

The process above is discussed in chapter 23 covering calendar or declination curves.
For the analemmas however, problems arise. The hour lines on the real vertical decliner, and the hour lines on the pseudo horizontal surrogate dial will not normally match. And thus nor will the analemma figures.

At this point a philosophical point comes into play.
Either you have one master analemma, and put that on the noon local apparent time of the pseudo surrogate horizontal dial, which results in a single analemma on the SD line of the real vertical decliner, or, generate pure analemmas for the final dial plate.

The first option is simple and easy, and reduces clutter on the dial plate.
The second option requires adjusting the hour lines on the surrogate plate to match those on the vertical decliner. This is done by using a longitude adjustment. That adjustment has little to do with the real vertical decliner's design longitude. It is a value that will simulate a longitude that will enable the surrogate dial's hour lines, and thus analemmas, to match the real dial. The surrogate dial's latitude is of course SH (style height).

This magical longitude difference is called "DL" or difference in longitude, and is derived from a simple formula from the vertical decliner.

The " $v$-dec mostly facing the equator" choice of the vertical dial DeltaCAD macro provides that DL, as does a " $v$-dec dial calculator" choice of the analemma DeltaCAD macro, as does the master spreadsheet:
illustratingShadows.xls
the formula is very simple, and its proof is in chapter 16 along with a short discussion:-

$$
\text { DL }=\operatorname{Atn}(\operatorname{Tan}(\mathrm{dec}) \quad / \operatorname{Sin}(\mathrm{lat}) \quad)+\mathrm{v} \text {-dec longitude }- \text { legal meridian longitude }
$$

Use this DL as the surrogate dial's longitude but reduce its magnitude by 15 degrees if it was larger than 15 , until the result is less than 15 degrees. Then use a legal meridian of 0 in the horizontal dial plate generator. The resulting hours will produce hour lines that match the real vertical decliner, and thus the analemma figures will also align. But while the analemmas will align, as discussed on the next page, they need to be rotated left to right and top to bottom.

KEY POINT: One analemma anywhere on a dial is just as useful as well aligned analemmas on many hour lines, as they are only directly usable for their hour line.

MORAL: Only place one analemma on a dial plate, it is uncluttered and still gets the message across. Or place a chart or table on the dial. Keep it simple.

However, for the perfectionist, the next two pages show a full set of analemmas for a vertical decliner. The DeltaCAD macro supports a full dial plate for vertical decliner dials with no extra work. The DeltaCAD SELECT and COPY followed by a PASTE into a WORD file sometimes has issues with the vertical decliner due to the DeltaCAD object rotation feature used.

Consider a vertical decliner for latitude 32.75 , facing $\mathrm{S} 45^{\circ} \mathrm{W}$, or in this DeltaCAD program, $-45^{\circ}$. The vertical dial DeltaCAD macro creates the vertical decliner dial plate shown below to the left.


Hour and hour line angle VERTICAL DECLINER DL= -64.8


DL, or difference in longitude, is a value that in essence says how much of a longitude shift would be needed in order for a surrogate horizontal dial to be designed, and have its hour lines match those of the vertical decliner, when the horizontal dial is superimposed on the vertical decliner's sub-style. Chapter 16 discusses DL, and chapter 23 shows the calendar curve process for vertical decliners. The process that works for calendar curves also works for the analemma.


The DL (effective difference in longitude) is $-64.8^{\circ}$ because the raw DL is actually $-61.6^{\circ}$ (not shown in the above dial plate since the math is done for you) minus the $v$-dec dial's longitude from the meridian of $3.2^{\circ}$. It is minus the "vertical decliner's longitude from the legal meridian difference" because the signs reverse when going from a vertical to a horizontal dial design, see chapter 13. DL will be the surrogate horizontal dial's effective longitude difference from a legal meridian of 0 . But as shown on the next page, there is a problem with the analemma of a horizontal dial when used in a vertical plane, however that problem is also easily solved.

A longitude difference exceeding 15 degrees can be unwieldy, so one approach is to keep reducing the 64.8 degrees magnitude by 15 degrees while adjusting the hour label by one for each $15^{\circ}$ since 15 degrees is one hour. So 64.8 minus 4 hours would be a new DL of 4.8 degrees, and we have to remember the shift of 4 hours. This is reviewed on the next page.

The SH (style height) is 36.5 degrees which is used as the latitude for the surrogate horizontal dial with calendar data. And note that the SD is where the surrogate horizontal dial's true noon (not longitude corrected) will be positioned in the next step.

If only calendar or declination curves were needed, then a horizontal dial for a latitude equal to 36.5 would be used as described in chapter 23. The next page shows the horizontal dial superimposed on the real vertical decliner.


Since analemmas are required, the horizontal dial with analemmas choice of the analemma DeltaCAD macro would be used but there is a problem as mentioned on the last page. The analemma for a horizontal dial needs to be (1) rotated top to bottom and (2) left to right because the (1) solstice curves for the two types are reversed, and (2) the hour lines rotate in opposite directions. A special choice of the DeltaCAD analemma macro derives the correct DL and also invokes the surrogate horizontal dial while specially triggering those analemma reversals. That choice derives a latitude of 36.5, (SH from vertical decliner) and a longitude of -64.8 (DL from the vertical decliner) with a legal reference meridian of 0 . As mentioned on the previous page, a longitude 64.8 presents problems for the program since the span exceeds normal time zone references, so reducing that by 15 until the result is 15 or less works better. So -4.8 is the result. And the reduction was 60 degrees, or at 15 degrees per hour, the resulting shift is 4 hours.


To cross check, from the previous page, using the vertical decliner's SD of 47.7 minus the 1400 ( 2 pm ) hour angle of 25.3 , we get 22.4 , and the 1400 hour is 3 hour lines from SD; 2pm was chosen randomly.

Looking at its matching horizontal dial on the previous page, 3 hour lines from local noon, (coincidentally 1400), it is 22.4 degrees. Those figures are circled.

Thus this is the correct surrogate dial for a vertical decliner and can be slid along the decliner's SD line as discussed in chapter 22. In fact, as long as the surrogate is rotated to align with the real dial's SD, it can be used as-is as long as the hour numbers are adjusted to match.

To the right is shown another option of the DeltaCAD macros for the analemma and the vertical dial. It matches the results of the steps above, which it fully automates.

The intent of this and the previous page is to show the process used to design calendar curves and analemmas for a vertical decliner. And also the tweaking a horizontal dial needs when used in the vertical plane.

The Illustrating Shadows DeltaCAD macros provide full automation for those dials while still retaining the knowledge of those steps involved.


## Polar Dial ~ and by definition a Meridian Dial also ~ analemma on the dial plate

Whereas the horizontal, vertical, and vertical decliner dials used the sun's altitude and azimuth to calculate the nodus shadow for a given date (see chapter 23), the polar dial can use a different set of formulae for drawing the analemma. The method used here still derives the equation of time (EOT) and declination, however in the Polar and Meridian dial's case the altitude and azimuth are not used at all since the declination and EOT can be used directly.

The reason this simple method can be used is that the hour line is located at a fixed distance from the sub-style, a distance " $X$ " on the polar or meridian dial, whereas for the horizontal, vertical, and vertical decliner, the " $X$ " value varies with declination necessitating the altitude and azimuth approach.

The logic is simply to locate the hour line, longitude corrected (not EOT corrected), which is:-

$$
X=\text { sub-style to hour line } \quad=\quad \text { style height * tan( Iha from noon })
$$

And then for the selected hour, the EOT is derived for each day of the year, along with the declination. Then we vary the " $X$ " coordinate (which is the hour) by the EOT, and then the " $Y$ " coordinate is derived using:-

> Y = distance up an hour line which is longitude and EOT corrected, to its matching declination point $\quad=\quad$ style height*tan(declination) / cos(lha from noon)

In this fashion, the altitude and azimuth are not required, and the figure of eight distortions due to " $X$ " increasing with distance are also managed. $X$ and $Y$ are used progressively for drawing a series of short lines that make up the complete analemma.


This same dial plate is usable as a meridian dial, a vertical dial facing true east or true west. For the northern hemisphere the dial plate is rotated counter clockwise to the co-latitude angle for a west facing dial, and clockwise to the co-latitude angle for an east facing dial so that the gnomon and sub-style are both aligned with latitude. Of course the hours need renumbering. Then the afternoon hours are dropped for a morning or east facing dial, and the morning hours are dropped for an afternoon or west facing dial. For the southern hemisphere, the analemma is reversed top to bottom (in essence, the declination is reversed) but no adjustment is made to the EOT factor.

The DeltaCAD macro supports a full dial plate for polar, and east and west meridian dials. The DeltaCAD SELECT and COPY followed by a PASTE into a WORD file sometimes has issues with the meridian dial due to the DeltaCAD object rotation feature used.

## NORTH AND SOUTH HEMISPHERE RELATIONSHIPS

Just as chapter 6 discussed north and south hemispheres in the context of hour lines, and chapter 13 showed the similarities and differences of the horizontal and vertical dial, this pictorial shows the simplest way to consider analemma drafting for the two hemispheres.

A latitude $-30^{\circ}$ horizontal dial in the southern hemisphere matches a $+60^{\circ}$ vertical dial in the northern hemisphere exactly.

$$
\begin{aligned}
& \text { the sun } \\
& \text { orizontal } \\
& \text { southern } \\
& \text { ches a } \\
& \text { in the } \\
& \text { nisphere }
\end{aligned}
$$

A latitude $-30^{\circ}$ vertical dial in the southern hemisphere matches a $+60^{\circ}$ horizontal dial in the
So the analemma for a latitude $-30^{\circ}$ horizontal dial is the same as a latitude $+60^{\circ}$ vertical dial. This is a latitude:co-latitude relationship. northern hemisphere exactly.

So the analemma for a latitude $-30^{\circ}$ vertical dial is the same as a latitude $+60^{\circ}$ horizontal dial. This is a latitude:co-latitude relationship.

The spreadsheets for the analemma: and:
illustratingShadows.xls
analemma.xls
were designed for the northern hemisphere. Using the about notes and concepts, they can be used for the southern hemisphere as well.

Another concept to consider. When the sun is slow, it is more to the east because it lags, so shadows are more to the west. Whether that is more to the left or more to the right, depends on which direction the viewer is facing.

## Other Interesting Hour Angle Dials

## GLOBE DIALS

The globe dial is as simple as the armillary or equatorial dial in design.
Globes can be found in masonry supply facilities as globe finials. A globe dial needs an axis that matches the polar axis of our planet, namely set at the latitude. The latitude angle was marked on the globe using a laser on a tripod, the laser was tilted to latitude, and raised until the laser line bisected the globe. This was marked with a pencil after verifying the red laser light again for latitude. The process was repeated on the other half, and the two laser guided half circles then joined up. Later a 1/4 inch copper tube will circle around those pencil lines.

Two holes can now be drilled in the globe at its poles and copper rods inserted. Now semicircles of copper tubing are affixed to those two rods to provide a full circle - pole to east to pole to west and back to pole. A copper plate then spans the equator of the globe and should be as thin a copper sheet as possible.

Then, using a protractor, lines are marked every 15 degrees on the equatorial band, and adjustments made for longitude if so desired, however that would limit the globe dial to this location. In this example the 15 degrees were marked by measuring the band where the hours would be displayed, and dividing that length into twelve equal segments, this was in place of using a protractor.


When using the laser to aid in drawing the latitude line which will be the 6 am and 6 pm hour line also, care must be taken to ensure that the lines thus marked are truly bisecting the globe. Take care not to look into the laser light.

A movable gnomon, a copper fin, is finally added that can rotate around the polar axis.


Time is read by rotating the thin plate until the shadow reaches minimum size. The figure above to the left is not correct as the shadow is large, the movable gnomon needs to be turned away from the shadow. The figure to the right, is correct as it has a minimum shadow. The equation of time is applied along with the longitude factor, and summer time also if appropriate. In this case the L.A.T. shows $2: 20 \mathrm{pm}$, the longitude correction was plus 12 minutes, the EOT was plus 2 minutes (June 22) which means plus 1 hour for summer time also. The legal standard time was thus $3: 34 \mathrm{pm}$, which was within a few minutes of the time on the author's atomically set clock.

## Alternatives to the globe dial

Concrete globes can be found in many masonry supply shops, however, there is an alternative. A pipe may be used instead.


In the picture on the left, simple PVC piping was used. The entire contraption is sloped at latitude. The gnomon at the top can be almost any shape, since it is turned until there is no shadow on either side. The gnomon pointer then indicates the time on the time scale, to which the EOT must be applied. The longitude correction is also required, and the middle pipe has at its lower end another pointer that points to a scale of 1 degree increments. This allows the longitude correction to be applied.

As with the other dials made of PVC, the scales and other information were printed on normal printer paper, applied with normal PVC cement, and then protected with clear contact paper.

Another variation is shown to the right, it has thirteen gnomons. Each gnomon is the same length, equidistant, and radiates outwards paralleling the equator, the pipe being set at latitude and north/south. The gnomon height is such that when the sun's shadow is 15 degrees from it, the gnomon's shadow just touches the base of the adjacent gnomon. The hour marks are correctly 15 degrees apart. The quarter hours are shown as equidistant however this is not strictly correct because the curved surface is not concave, it is convex.

Time is read in one of two ways. If the sun is on a gnomon, i.e. the time is on the hour, then the null shadow indicates the time as in the preceding dial. For other times, it will be seen that there will be a place where the shadows diverge, as is emphasized in the picture on the right. Both happen to be on the half hour point, indicating the time is about half an hour after the gnomon on the right, and half an hour before the gnomon on the left. If one was close to the 15 minutes after point, the other would be near the 45 minutes
 before point, and so on. For somewhat improved accuracy, the shadow closest to a gnomon base is used. This reading was taken on the fall side of the equinox, hence the shadows fall north of the equinoctial center line. The time strip is wide enough to hold the shadow tip at the solstices, namely a 47 degree (twice 23.5) spread from the nodus to the adjacent gnomon's base line.

This dial suffers from the use of linear 15 minute assumptions. In addition it suffers from the requirement to accurately set a number of gnomons, thirteen in the above example. In spite of those complications, it is a dial with instructive value. Care should be taken to avoid injury from the nails. Copper fins can also be used which would be safer and less painful.

## BIFILAR SUNDIAL

In 1923, a German named Hugo Michnik developed a bifilar hour-angle sundial that was horizontal with two cross wires of differing heights. The two cross wires replaced the gnomon, one ran north-south, the other east-west, and they were parallel to the dial plate. The secret was in the ratio of the heights of those two wires. At a certain ratio, the hour lines are evenly spaced at 15 degrees just like the armillary and equatorial sundials, and unlike other hour angle dials such as the horizontal and vertical dials. The design was simplicity itself.

The north south wire can be any height, the east west wire height is equal to:-
east west wire height = height of the north south wire * sin(latitude)

While the north south wire is placed over the noon line, the east west wire is placed at a distance from the dial center that is equal to:-

```
dist from dial center = height of the north south wire * cos(latitude)
```

 wire is height of the north south wire * cos(latitude)

The problem with this design is that the shadow of the cross-hair can be off the scale, so it is possible to mistake the shadow of the south most rod supporting the wire with the shadow from which time is read. Similarly, the south support may at times shadow the cross-hair shadow. The moral is do not make the wires too high off the plate, and keep the south, east and west supports clear of the dial plate.


## POLARIZED LIGHT AND THE SUN DIAL

Clearly the sundial provides a measure of the time when the sun shines on the gnomon. A lunar dial can provide the time at night for a couple of weeks in a month, and for when the moon shines not, a nocturnal can provide the time from a few select stars on moonless nights. But what about determining the time when the sun is below the horizon and the moon or stars are insufficient for use? What about when a cloud limits the light to a gnomon?

Around 1848, Sir Charles Wheatstone used the polarizing effect of light coming from the celestial pole. Light from the sun reflected by the sky becomes polarized. Light coming through clouds tends to de-polarize. The alignment of this polarized light varies with the sun's position, the observer's position, and the part of the sky being observed. While the most polarizing effect does not always come from the celestial pole or polar axis extended, measuring the polarization from thence does provide a linear hour angle measurement.

Sir Charles used a polarizer which he rotated, the result was the observed sky became brighter or darker. The observer seeks the darkest light from the sky as it is a narrower region. As the sun orbits, so does the place where the polarizer provides the darkest light. And by using the celestial pole, that angle varies linearly and directly with the sun.

In true Illustrating Shadows form, four pieces of PVC piping were used, a mirror, and a lens from a cheap pair of polarized sun glasses.

A long tube points to the polar axis extended to infinity, the celestial pole. At its end is a rotating collar in which has been placed a circular cutout of one lens from a pair of polarized sunglasses. The lens was removed and cut using a Dremel rotating saw. At the
 base of the long tube is a 90 degree union, in which is placed part of a mirror angled at 45 degrees. This enables the light to be viewed from the polar axis easily. Without that 90 degree bend it would be hard to see the light from the sky because your eye would have to be below dial level.

The perimeter of the long tube has a paper collar divided evenly into 24 hours, and time is indicated by a pointer attached to the rotating collar containing the polarizer.

The pointer is set by seeking the darkest light and affixing a pointer to indicate the L.A.T. on the paper collar of hours, backed off by the equation of time (EOT). For fine tuning, the paper collar can be adjusted and then set rigid with a piece of masking tape. This technique means longitude corrections are automatically applied, and of course EOT must be considered.

From then on, whenever the time is required the long tube is pointed to the celestial pole and the polarizing collar rotated back and forth around the darkest light, until the darkest orientation is found. The EOT is applied and the time then read. Other polarizers can be used, however camera polarizers seem to be less satisfactory. This dial can read the time as long as the sun is illuminating the extended pole and as long as that extended pole is blue sky. Thus, before sunrise until after sunset, the time may be read, similarly if the observer's location has no well formed shadows.

This is not an all inclusive guide on polarizing dials, rather a starting point from which to work, and that starting point is a working polarizing dial.

## REFLECTING OR CEILING DIALS

Ceiling dials are not common, Sir Isaac Newton and Christopher Wren both built them, and having an unused ceiling in my barn, I decided to follow suit.


The dial has hour lines, a meridian line depicted by sun declination markers, and the equinox and solstice curves. The pictures above show the time approaching 1 pm .


Having a habit of forgetting key information, a data plate shows the latitude and longitude of design, the declination (why the meridian is offset), and the distance from the ceiling to the mirror (CTM).

Care must be taken with ceiling dials, stepping backwards off the step ladder while marking the ceiling is a real danger. When working with ladders it is easy to become disoriented, especially when wearing glasses.

With a reflecting dial, or ceiling dial, a mirror is placed on a surface such as a window sill, of course it is set level. The sun is reflected from the mirror at point " m ", and for this example, the equinox is assumed. The reflected ray displays a reflection of the sun on the ceiling, a spot, and this is at point " n ". Because the following uses the equinox, angle " Cmq " is a right angle.


The sun comes through " $q$ " to " $m$ " and is reflected to " $n$ ". If this were a true horizontal dial, the gnomon would be "BmC", where "Bm" is the style linear height, and this matches the height of the ceiling above the little mirror, "mb". (NOTE: lat $=l a t_{2}=l a t_{3}$, subscripts are order of derivation).

Angle " BCm " is the latitude, so also is "bcm" and "nmb".
$\begin{array}{llll}\text { Since } \mathrm{nmb}=\text { latitude, } & \tan (l a t)=\mathrm{nb} / \mathrm{bm} & \text { thus } & \mathrm{nb}=\mathrm{mb} * \tan \text { (lat) } \\ \text { Since } \mathrm{bcm}=\text { latitude, } & \tan (\text { lat })=\mathrm{mb} / \mathrm{bc} & \text { thus } & \mathrm{bc}=\mathrm{mb} / \tan \text { (lat) }\end{array}$
thus, distance from equinox point on meridian
to the dial canter " c " is $\mathrm{nc}=\mathrm{nb}+\mathrm{bc}$

$$
\begin{align*}
& \text { hence } \mathrm{nc}=\mathrm{mb} * \tan (\mathrm{lat})+\mathrm{mb} / \tan (\mathrm{lat}) \\
& =m b \text { * }(\tan (l a t)+1 / \tan (\text { lat })) \\
& \text { [The conventional formula is }=2^{\star} \mathrm{mb} / \sin \left(2^{\star} \mid a t\right) \text { ] }
\end{align*}
$$

Thus we have a dial plate, with a meridian line (L.A.T. noon) and an equinox point on it, thus the equinox line can be drawn.

Similarly, we have the dial center, it is located from the equinox point " n " at a distance:-

$$
\begin{equation*}
=m b *(\tan (\text { lat })+1 /(\tan (\text { lat })) \quad \text { derived formula } \tag{2}
\end{equation*}
$$



The dial plate on the ceiling is reversed, otherwise it is a normal horizontal dial. The morning hours are to our right as we look up at the ceiling, whereas looking down on a normal horizontal dial, they are to the left. With this in mid, the dial layout is a normal horizontal dial for the design latitude and longitude, except we reverse the sense of the angles.

Ceiling dials have a dial center that is not in existence, but it is used to locate the hour line angles. For example, 2 pm , the " $h 14$ " line will have an hour line angle of some value "x". Since the dial center "c" is not in physical existence, we use the distance from point " n ", hour point on equinox line h14 from "n"

$$
\begin{aligned}
\mathrm{nh}_{14} & =\tan (\text { hour line angle }) * \mathrm{nc} \\
& =\tan (\text { hour line angle }) * \mathrm{mb} *(\tan (\text { lat })+1 /(\tan (\text { lat }))
\end{aligned}
$$

and at point h14, a line is drawn of 90 -hour line angle. Hence the hour lines can be drawn. Calendar curves can also be drawn using the same techniques covered elsewhere. A key point to remember is that these dials can get rather large, reducing the mirror's height from the ceiling will make the dial smaller.

REFLECTING OR CEILING DIALS

| 1 | $\mathrm{nb}=\mathrm{mb} * \tan (\mathrm{lat})$ [1] | $\mathrm{nc}=\mathrm{mb}^{*}(\tan (\mathrm{lat})+1 / \tan (\mathrm{lat})$ ) [2] |
| :---: | :---: | :---: |
| Latitude | Distance from point above mirror to equinox point on the meridian line | Distance from meridian equinox point to dial center |
| 30 | 0.5774 | 2.3094 |
| 31 | 0.6009 | 2.2651 |
| 32 | 0.6249 | 2.2252 |
| 33 | 0.6494 | 2.1893 |
| 34 | 0.6745 | 2.1571 |
| 35 | 0.7002 | 2.1284 |
| 36 | 0.7265 | 2.1029 |
| 37 | 0.7536 | 2.0806 |
| 38 | 0.7813 | 2.0612 |
| 39 | 0.8098 | 2.0447 |
| 40 | 0.8391 | 2.0309 |
| 41 | 0.8693 | 2.0197 |
| 42 | 0.9004 | 2.0110 |
| 43 | 0.9325 | 2.0049 |
| 44 | 0.9657 | 2.0012 |
| 45 | 1.0000 | 2.0000 |

Distance from mirror to ceiling (style linear height)

| Latitude | Distance from <br> point above <br> mirror to <br> equinox point <br> on the meridian <br> line | Distance from <br> meridian equinox <br> point to dial center |
| :---: | :--- | :--- |
| 46 | 1.0355 | 2.0012 |
| 47 | 1.0724 | 2.0049 |
| 48 | 1.1106 | 2.0110 |
| 49 | 1.1504 | 2.0197 |
| 50 | 1.1918 | 2.0309 |
| 51 | 1.2349 | 2.0447 |
| 52 | 1.2799 | 2.0612 |
| 53 | 1.3270 | 2.0806 |
| 54 | 1.3764 | 2.1029 |
| 55 | 1.4281 | 2.1284 |
| 56 | 1.4826 | 2.1571 |
| 57 | 1.5399 | 2.1893 |
| 58 | 1.6003 | 2.2252 |
| 59 | 1.6643 | 2.2651 |
| 60 | 1.7321 | 2.3094 |
| 61 | 1.8040 | 2.3584 |



First, a place was estimated about where to place a mirror. Then in this case a cross piece was affixed to both sides of a window frame. This was level in all exes.

Then a point on the ceiling was selected, and a small hook located there, and a plumb line dropped.


A small piece of wood was placed on the cross piece, and a reflective surface added. Again, it was leveled in all axes.

The distance from the reflective surface to the ceiling was measured and found to be 48.5 inches.

The illustratingShadows.xls spreadsheet was run with the design latitude of 32.75 and longitude of 108.2, and the vertical distance of 48.75 entered.

The date was November 30, 2008, the sun was fast so the EOT was negative, being - 10:59 minutes : seconds

Also, the sun would be slow due to the longitude separation from the legal meridian, and $108.2-105.0$ is some 3.2 degrees, or $+12: 48$ minutes : seconds

The total correction being: $\quad+1: 48$ minutes $/$ seconds
So at 12:00, plus the final correction of $+1: 48$ minutes/secs, the light spot is finally marked on the ceiling at 1201:48 and then a line was drawn from that spot light to the point directly above the mirror, this was the noon meridian line.

The spreadsheet estimated a distance from the point above the mirror, to the light spot for the $11 / 30 / 08$ date using the calculated sun's declination of -21.69 degrees as 67.85 inches. The actual distance was measured at 67.0" thus the calculations were matching the observations within less than an inch.

NOTE: The correction for longitude and EOT used the astronomical EOT. A "noon transit" of the sun table can be used, such as in the appendices or in chapter 6. Additionally, the spreadsheet on the CD that comes with this book as well as on the web site has a noon transit table corrected for the dial's longitude, this makes locating the meridian even easier.

The main spreadsheet has all the data on the first section, see immediately below.

| 48.5 | Distance from mirror to ceiling taken from main spreadsheet gnomon linear height |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Todays EOT/Diff.Long correction: |  | 2.39 |
|  | $\mathrm{nb}=\mathrm{mb} * \tan (\text { lat }) \quad \mathrm{nc}=\mathrm{mb}^{*}(\tan (\text { lat })+1 / \tan (\text { lat }))$ |  |  |  |  |
| Latitude | Distance from point above mirror to equinox point on the meridian line | Distance from meridian equinox point to dial center | Distance from point above mirror to summer solstice point on the meridian line | Distance from point above mirror to today's decl point on the meridian line | Distance from point above mirror to winter solstice point on the meridian line |
| 32.75 | 31.1962 | 106.5979 | 7.951 | 67.847 | 72.421 |
|  |  |  |  | Today's decl |  |
| DESIGN | ONGITUDE | 108.20 |  | -21.6913 |  |
| DESIGN | EGAL MERIDIAN | 105.00 |  | 11/30/08 |  |

Then the summer solstice, equinox, and winter solstice points were marked, they were derived from the spreadsheet as: 8 inches (7.951), 31.2 inches (311962), and 72.4 inches (72.421) respectively.

A line perpendicular to the meridian line was drawn at the equinox point on the meridian line, being at 31.2 inches.

The distance from the meridian line, along the equinoctial line for hour points and, repeated, are the inverse hour line angles at those hour points,

| Latitude | hour line angle at the hour point on the equinoctial line on the meridian ~ taken from above inverse hour line angles | Distance from meridian equinox point to dial center | Distance along equinoctial line from meridian line |
| :---: | :---: | :---: | :---: |
| 32.75 |  | 106.5979 |  |
| am to pm | angle |  |  |
| 6.0 | 5.90 |  | 1031.446 |
| 7.0 | 21.12 |  | 276.035 |
| 8.0 | 43.04 |  | 114.161 |
| 9.0 | 58.82 |  | 64.497 |
| 10.0 | 70.51 |  | 37.736 |
| 11.0 | 79.91 |  | 18.960 |
| 12.0 | 88.27 |  | 3.224 |
| 13.0 | 83.55 |  | 12.047 |
| 14.0 | 74.72 |  | 29.130 |
| 15.0 | 64.19 |  | 51.560 |
| 16.0 | 50.42 |  | 88.124 |
| 17.0 | 31.29 |  | 175.394 |
| 18.0 | 5.90 |  | 1031.446 |

The last section of the spreadsheet identifies hour line distances along the equinoctial line from the meridian. It also copies the inverse hour line angles at those hour points. The hour points along the equinox were marked. At those hour line points, inverse hour line angles were drawn.

Then the hour lines themselves can be completed, they would meet at the dial center which is of course outside the building. The equinox line is used since it is straight. Any other declination other than 0 produces a hyperbolic line. This dial used empirical methods to draw the north south meridian line, albeit using calculated time shifts for the EOT and longitude differences. Calculations were used for the equinox point, and the day's spot was cross checked with its calculated position on the meridian (north south) line.

Hour lines were drawn using calculated hour point distances along the equinoctial line, and angles using inverse hour line angles, those all being calculated distances and angles.


All required information is provided by:

The main spreadsheet:
The PDA spreadsheet:
And a tabular spreadsheet:
And a DeltaCAD program, choice 7:

## illustratingShadows.xIs

a small spreadsheet exists for PDAs a general tabular spreadsheet exists MAIN-h-dials.bas

The main spreadsheet also has within it an almanac and a sun's transit table. The transit table only needs the longitude correction to be converted to minutes and added (if west of the meridian) or subtracted (if east of the meridian). The longitude difference is multiplied by 4 to convert from degrees to minutes.

Thus using the November 30th example, the noon transit time for this location (longitude 108.2) and for this time zone (105) is 3.2 degrees, or 12.8 minutes. The 1148.46 and 12.8 thus added become 1201.6 hhmm.m or 1201:36 as hhmm:ss which is within a few seconds of the calculation for the meridian earlier, the above table for transit time is a middle of the four year span using an astronomically correct EOT, thus the 12 seconds difference is surprisingly accurate.

To the right is a DeltaCAD plate for a ceiling dial.


## CALENDAR CURVES AND THE CEILING DIAL (or large horizontal)

The CAD program can be used to measure distances from the point on the ceiling above the mirror to calendar points on the hour lines.

To the left are CAD measurements of the ceiling point above the mirror to the summer and winter solstices, and the equinox, on the north south meridian (not the noon line).

The measurements are less than an inch off which is fair given the size of the dial plate.


Measurements for each calendar curve are made, in the case to the left, the winter solstice was used.

These distances allow a point to marked on each hour line for the several calendar curves, thus allowing the calendar curves to be drawn.


## FINAL DIAL COMPLETION

Wood molding was used to mark the significant points, wood trim was used for the hour lines and for calendar curve segments. And a stucco used to fill in the areas.

The mirror was enhanced, this was not simple because of the need for perfect alignment with the prior mirror so that the hour lines and calendar curves would still be accurate.

The plan was simple. A laser was set up to send a dot of light off the original mirror and mark that point on the ceiling. Then the old mirror would be removed and replaced with the one to the right. The new mirror would be fine tuned so that the spot would be at the same place as with the original mirror. This works because of the law of reflection.

The laser was set up, and work resumed on the wood for the dial plate borders, and trim for the hour lines and calendar curves.

At this point, some wood fell down and that was the end of the original mirror. What to do? The benefits of that
 original meridian line sprang to mind. Not only does the meridian line indicate the declination of the walls of the building housing the ceiling dial, not only does the meridian line indicate the central point for the calendar curves, not only does it provide the line from which the equinoctial line stems perpendicular to it, but it also provides the means of calibrating a mirror.

If the sun's noon transit is measured, using the noon transit data from the spreadsheet, or from the time of noon corrected for EOT and longitude, then at noon the sun must be reflected onto that north south meridian line.

And since the distance from the mirror to the ceiling is known, and the day's solar declination, then the distance from the ceiling point above that mirror, to the center of the sun's reflected spot on the ceiling can be deduced.

The ceiling dial spreadsheet has this table in one degree steps. And for a distance of 48.5, and a solar declination of -23.13 shows the spot to be 71.58 inches from the reference point. Thus tragedy was averted. The dial works well, however at lower latitudes the sun's altitude is such that no spot is reflected up, and of course morning and afternoon hours are inhibited due to the suns azimuth being excessive.

DECLINATION POINTS ALONG THE MERIDIAN AS MEASURED FROM THE CEILING POINT ABOVE THE MIRROR
FOR A LINEAR DISTANCE OF:

| -23.44 | 72.42 |
| ---: | ---: |
| -23 | 71.23 |
| -22 | 68.63 |
| -21 | 66.15 |
| -20 | 63.78 |
| -19 | 61.52 |
| -18 | 59.36 |

25-Dec

| today's decl: |
| :---: |
| -23.13 |
| today's spot |
| 71.58 |
|  |

## Night time dials

## A LUNAR DIAL

There are several techniques that allow the time to be read from the moon. This also brings into light the definition of being a purist; is a purist someone who in this context seeks the most accurate timepiece, or is a purist one who uses just the moon, albeit empirically, and excludes references to the sun. Three techniques spring to mind, they are:-

- A lunar dial using a sundial and a correction table
- A lunar dial using the moon but with solar associations in its mathematics
- A lunar dial using just the moon


## A lunar dial using a sundial and a correction table - some old dials may have this table

A sundial itself can be used, however some adjustments are required. First, an Earthling's hour is marked by about $14.5^{\circ}$ of lunar orbit around the Earth's polar axis, compared to $15^{\circ}$ for the sun. Thus solar hours would be larger by half a degree, in other words 2 minutes. In 24 hours, that accounts for 12 degrees, or in Earthling terms, 48 minutes. Thus a table can be drafted such that the moon's phase indicates the time to be added to or subtracted from the sundial's 12 o'clock midnight baseline time. Of course those 2 minutes per hour must also be considered for the hours on either side of 12 o'clock.

In the mid 1700's such a table was used that assumed a 30 day lunar cycle, however there are other variables that make this method somewhat impractical. At 15 days after the new moon, i.e. a full moon, 12 hours is added to the dial, thus 12 noon means 12 midnight. Again, this was an approximation.

Before and after full moon one must remember to subtract or add as appropriate. This system works with a little effort, and requires the date of the new or full moon to be known. Of course, it still needs some fine tuning since full moon happens at a specific date, time, and place and that may not be over your dial. Lunar dials are not known for their accuracy.

| 30 day |  |  |
| :--- | :--- | :--- |
| Days <br> before <br> full <br> moon | Days <br> after <br> full <br> moon | Hours to <br> adjust <br> dial <br> L.A.T. |
| 7 | 7 | 6.4 |
| 6 | 6 | 7.2 |
| 5 | 5 | 8.0 |
| 4 | 4 | 8.8 |
| 3 | 3 | 9.6 |
| 2 | 2 | 10.4 |
| 1 | 1 | 11.2 |
| 0 | 0 | 12.0 |

## A lunar dial using the moon but with solar associations in its mathematics

The most accurate method is to use the moon's orbital information in conjunction with that of the sun's apparent Earthly orbit, and from thence, derive an accurate system.

Here is a method of determining new moon, first quarter, full moon, and last quarter for any date. The following formulae are approximations of approximations, and are drawn from chapter 32 p159 of Astronomical Formulae for Calculators, Jean Meeus. Some of the formulae in that book have been simplified elsewhere in this book. The formulae use an epoch base of 1900 which is somewhat academic, however it emphasizes the need for formulae not to be mixed from author to author, or article to article. Many books use the year 2000 as the epoch base.

Additionally, these figures are for the mean lunar cycle. That is 29 days, 12 hours, 44 minutes, 3 seconds, or 29.5305891203704 days, however interaction with the sun can vary this by about 6 hours on either side.

Intermediate figures

| M23 | 40 = | mm/12 |
| :---: | :---: | :---: |
| N23 | = | (dd/30)/12 |
| O23 | = | $(\mathrm{mm}+\mathrm{N} 23) / 12$ |
| P23 | = | O23+yyy |
| k | = | (P23-1900)*12.3685 |
| int $\sim$ of $\sim$ | - | INT(k) |
| T | 40 | k/1236.85 |

Julian date for new moon, first quarter, full moon, third quarter, nearest to the specified date.

```
=2415020.75933 + 29.53058868 * (int~of~k + 0.00) + 0.0001178*T*T - 0.000000155*T*Т*Т
=2415020.75933+29.53058868* (int~of~k + 0.25) + 0.0001178*T*T - 0.000000155*T*T*T
=2415020.75933+29.53058868* (int~of~k + 0.50) + 0.0001178*T*T - 0.000000155*丁*丁*T
=2415020.75933+29.53058868* (int~of~k + 0.75) + 0.0001178*T*T - 0.000000155*T*T*\top
```

Julian date you can use as a base for the above dates

```
=INT(365.25 * (4716 + (IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))) +
    INT(30.6001*((IF((IF(mm>2,1,0))=0, mm+12,mm))+1))) + dd-1524.5 +
    (2-INT((IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))/100) +
    INT(INT((IF((IF(mm>2,1,0))=0, yyyy-1, yyyy))/100)/4))
```

It is emphasized that these formulae are approximations, and should be treated in that light. These formulae are in the generalized spreadsheet on this book's web site. The same spreadsheet also has the formulae to convert mean lunar phase data to true lunar phase data.

## A lunar dial using just the moon

The lunar dial discussed here does not require the above formulae, it is simply set on the first quarter of the moon, and rotated each day by 12.2 degrees eastward. This lunar dial uses just the moon itself, and is simpler to use and more practical than a sundial with a table of corrections,

The sun's maximum declination is 23.5 degrees and the moon has a 5 degree orbital offset from the ecliptic, thus its declination range is 28.5 degrees.

A dial plate designed for 14.5 degree hour angles and an adjustment for 12.2 degrees per day can be used based on the observation of the phase of the moon. An armillary dial can also be used, as can an equatorial dial, all of these will require a daily rotational adjustment around the polar axis, be that mechanical or mathematical.

Because the moon's apparent orbit around the Earth is a bit slower than the sun's, an hour uses less lunar degrees around the Earth's polar axis than does the sun. Since the moon retards about 2 minutes per Earth hour, that is in polar axis hour angles $1 / 2$ a degree since 1 degree represents 4 minutes. Thus a lunar hour is indicated by about $14.5^{\circ}$ and not 15 which is used for the sun.

Because a mean lunar month is about 29.5 days (a true lunar month may vary from the mean by 6 hours), and because this must account for 360 degrees of rotation before the moon is back in synchronization with itself, this means a daily rotation backwards of about 12.2 degrees is required. An analysis of almanacs over many years shows that the actual daily shift can be as little as 9.5 or as much as 14.6 degrees, and the daily variation may be as much as 1 degree. There are a couple of waves of this variation throughout a lunar month. This dial needs to be set on the first usable day (first quarter) and may need fine tuning over the next two weeks which is the life of the usable shadow. Two weeks of lunar data from March 2003 follows showing the moon's GHA (Greenwich Hour Angle) at 0000 GMT from the first to the last quarter. The daily shift range is $-12.42^{\circ}$ to $-14.22^{\circ}$ (compared to $12.2^{\circ}$ ), the daily difference in that shift varies from $-0.58^{\circ}$ to $+0.32^{\circ}$.

For that entire lunar cycle those figures were $-9.77^{\circ}$ to $-14.22^{\circ}$, and $-0.62^{\circ}$ to $+0.86^{\circ}$.



Because of the need for a daily dial adjustment, and the baseline to start the two weeks of good lunar use varies, this dial is easy to use.

To the left is a picture of the dial. The top rotating section is a moving gnomon, using a null shadow. That null shadow causes the extension of the gnomon to show the time on the next rotating section down.

The 12 o'clock hour line on the second rotating section has an arrow pointing down, which is set to the day from the first quarter, the days are numbered $0-14$, or as $0-9$, and A-E. For people who go to bed early, days 0 through 9 work well, for late night people, all 14 days are usable.

That two week lunar span section is also rotated around the lowest or first section to calibrate the time against a standard time watch. This corrects for lunar variations up to the first quarter, and the dial's longitude. And the hour section is rotated one detent each day.

Because of the many and less easily predictable variations in the lunar orbit, by the day, month, and year, these dials are not excessively accurate, and probably need recalibration every few days, maybe set once on the first quarter, and again on the full moon.

Appendix 9 has a template for this dial, it was designed for a 2.25 outer diameter PVC pipe, however you can enlarge or shrink it to suit your purposes.

This is not an all inclusive guide to lunar dials, rather a starting point from which to work, and that starting point is a working lunar dial.

## THE NOCTURNAL OR STAR CLOCK

The bigger picture of the solar system and its planets shows that the planets move close to the ecliptic. Hence the planets and the moon generally are in the same line in the night sky. And that line is roughly on the ecliptic, or within about 10 degrees of it.

The planets move on their own orbits, thus their locations appear to move compared to the stars which are considered to be fixed. The first four planets from the sun are:Mercury, Venus, Earth, then Mars.

The stars are everywhere and appear to rotate around the Earth's poles. As the Earth orbits the sun, different stars close to the ecliptic become visible in different seasons because they are seen on the night side of the Earth away from the sun.

Polaris ~ the north star, within a degree of the Earth's extended north polar axis, visible all year in the northern hemisphere, and is nowhere near the ecliptic, it is perpendicular to and north of it.


The stars in the northern hemisphere seem to rotate around the north star, Polaris. They revolve about 360 degrees in a day. And for any given time of night, they rotate 360 degrees in a year.

This all means that a 360 degree map of the stars can be drawn, and throughout the year some stars come into view, others leave. And similarly, in the evening, the stars for the season rotate 360 degrees in a day. Approximately. This means that should one know the month, then the time can be approximated, and this is the basis for a nocturnal dial. In fact, chapter 6 covering true north direction determination showed such a star map. This section is not intended to be anything more than an overview of how a nocturnal time piece may be designed. Of course, the southern hemisphere has similar rules, however there is no Polaris, only an empty place which is however pointed to by some constellations. Some nocturnal dials use Ursa major and Ursa minor, some use Cassiopeia, this section uses Ursa Minor and Cassiopeia for good annual coverage.

All the stars visible for this date, or season ... rotate daily and completion one rotation annually


Stars rotate east to west above Polaris as the night passes - they rotate 360 degrees approximately in a 24 hour time frame.

Stars rotate west to east below Polaris as the night passes.

LOOKING NORTH

The June view is rotated 180 degrees from the December view.


Cassiopeia - an upside down "W" in this pictorial is a marker for Polaris, the north star.


From any star map, such as in the book "Stars and Planets" by lan Ridpath which has a monthly sky map with times for three differing dates, you select the stars you wish as markers. In this case the middle of the 5 stars in Cassiopeia and the last of the 7 stars in Ursa minor were chosen. They have an almost 155 degree relationship when connected to Polaris. Cassiopeia center is $40^{\circ}$ from Polaris and Ursa Minor outer is $165^{\circ}$ at 2100 on December 30, approximately.

For Cassiopeia's central star being offset $40^{\circ}$ from vertical at 2100 on December 30, an index is placed $40^{\circ}$ back from 2100 , which at $15^{\circ} /$ hour is 2 hours 40 minutes, or at 1820 . Similarly, for the last star in Ursa Minor, it is offset $165^{\circ}$ from Polaris, which is 11 hours from 2100 but the other side, 0800. Thus a date plate can be drawn with an hour plate with 24 hours, and there will be an index at 6 pm for that Cassiopeia star and one at 8 am for that Ursa Minor star. The Cassiopeia 6 pm index was chosen because when 6 pm is placed on the date in question (December 30 e.g.), the cursor bar will indicate the correct time (if longitude corrections are applied) when using Cassiopeia. In other words the index is backed off as many hours as the reference star is ahead of the cursor, and vice versa. A star atlas is invaluable here. Some nocturnals use different hours for these same stars, this is not inconsistent, they are using a different date-wheel alignment, this uses December at the top, some use March. Similarly, some use different stars in Cassiopeia, and so on. Set the desired star's cursor to the date. The dial center is held where Polaris is, some dials have a hole in the center attachment screw, relaxing the eyes to see double also works. Then tilt the dial to match the co-latitude because the stars revolve in a circle around Polaris, and Polaris is at latitude. This avoids distortion that would otherwise occur, especially at northern latitudes. The cursor is moved to the pointer star, and the local time is read. Longitude corrections apply.


THE NORTHERN HEMISPHERE START CHART AND A NOCTURNAL DIAL

| Time read from cursor that points <br> to the index or pointer star. |
| :--- |

It would be educational to validate the hour points for the two indices for Cassiopeia and Ursa Minor. Locate a star map such as above, and prove they are correct. Also, develop another star's cursor from scratch. Note: different star maps may not agree as angles may differ by some degrees.

The nocturnal dial to the right is set for Cassiopeia, December 20th and shows a time of $8: 30 \mathrm{pm}$ standard time, to which the longitude correction is applied. The dial must be held vertical but tilted at co-latitude, because this is an hour angle dial.

The template for this dial is in appendix 9, and was designed with a computer CD in mind for the larger dial plate.


## CHAPTER TWENTY EIGHT

## The use of stained glass for a sun dial and other media of interest such as popup and paper dials

This chapter covers the following topics...
THE USE OF OPAQUE GLASS as in reflected light dials

THE USE OF TRANSPARENT GLASS
where light and shadow are transmitted through the glass

THE USE OF PAPER MODELS FOR DIALS transparent (as in the meridian dials) opaque as in the horizontal, polar, and armillary (and part time equatorial) 3d cutout and fold popup dials

## AND BRASS ENGRAVED DIAL PLATES.

This is not intended to be a treatise on stained glass or any other medium, but rather a few pointers to consider when constructing a dial using glass.

Glass may provide for a matte surface such that the shadow is seen by reflection, as with any common sundial made of other opaque materials. While horizontal dials produce few challenges, a vertical reflective glass dial indoors requires consideration in placement,
 and may prove impractical since the dial must be on the other side of the observer compared to the sun.

Glass allows transparency and thus may allow a shadow to be seen through the glass, such as is seen in stained glass windows. This would tend not to be practical for horizontal dials but produces splendid results for the vertical dial which would be between the observer and the sun. Doing so, however, introduces some things to think about.

First, the dial plate would be drawn using a mirror image template. Second, the gnomon needs rigidity which may take extra thought for declining dials. Third, the techniques for marking a vertical dial plate may use more advanced skills for the glazier. Fourth, the nodus linear height is to the frosted surface upon which it typically resides, and not to the front or non weather side of the glass dial.

## GLASS DIALS

Sun dials may be made in almost any media, and glass has been one of them used down through the centuries. Historically, glass sun dials were of the painted and stained variety, both of which are processes requiring kiln firing. And they used the property of the light coming through the glass as opposed to being reflected by it.

However, the non transparent and more opaque of the glasses may be used on a dial plate where the observer looks at the gnomon's shadows on that glass which are then reflected towards the observer as on the left side of the pictorial below.


A translucent dial where light and shadow are seen through the glass, it would reside at the window itself.

A reflective dial where light and shadow are reflected by the dial plate, it would reside away from the window or in this example outside.

Reflecting dial plates need little explanation since most dials are of that type. The question becomes what is different with glass dials where the glass dial is transparent, as on the right side of the pictorial above.

A transparent dial such as a vertical meridian dial pictured above would reside in a window, and receive the sun some of the morning (if facing east) or some of the afternoon (if facing west). If a glass reflecting meridian dial were used, then it would have to reflect light which means it could not be in the window itself, but rather on the wall receiving light from the window with the observer in between. And that lighted area will move dramatically making the indoor reflecting dial less practical. Thus for indoor use of vertical dials, transparent glass is much more effective than a reflective dial of an opaque glass. The same would apply to a vertical south facing dial.

For indoor use transparent glass serves best. But how does one see a shadow on transparent glass? The shadow is hard to see on all but the opaque glasses and in a weak light no shadow would be seen. The solution uses a clear glass with a frosted surface, either by an etching cream or a sand blasted process both of which have safety considerations.

The markings may be painted, and there are modern methods not requiring a kiln, however the author prefers a tracing black and brown matte such as were used in the 13th centuries, and fired around 750 degrees Celsius. The pigments are mixed with a flux, each of which has differing benefits. While water may be used, the paint or matte can flake off. Oil of cloves, anise, and other mediums make a very firm bonding with the glass.

The tracing black is used for the hour lines, calendar lines, and numerals indicating the time. Lettering is often done best by the negative technique: a matte covers the area and the lettering done by removing matte material. Traditionally they are drawn on the non weather side of the glass which makes writing much simpler. For true meridian and true south dials the process is simple, but for a declining dial care must be used in making the template which will be held below the glass, and above which the dial plate will be drawn in. Care is needed because the hour lines are no longer symmetrical. The template used should thus be a mirror image print of the declining dial because we will be looking south through the glass and not be looking north from a southerly position. Some printing software can print mirror images.

The firing is done and the glass cooled, and for texture a matte may be added with texturing from a brush. That would be a second firing. If the fluxes used for the first firing were effective, then the matte could be applied before that second firing, and thus both would be fired in one step.

The lines and indications need to stand out, and this may use the same technique that is used to make the gnomon's shadow stand out, or they may be marked on a dark matte as in a negative. The weather side of the glass traditionally holds stains and sometimes frosting. The tradition goes back to the fact that silver stains (producing the wonderful golden yellow textures of the 14th century, weather well and last hundreds of years, whereas the paints and mattes have a much shorter life when exposed to the elements.

If an etching cream is used then all safety precautions must be observed. While the author has had some success with this, more success has come from a silicon carbide grit in a sand blasting box under the pressure of a compressor. Here again safety must be paramount.

At this point we have lines on the room side of the glass, and a frosted texture on the weather side. In between the paint firing and the frosting may come a stain firing, the wonderful golden yellow stain is derived from silver compounds, fired around 1250 Fahrenheit for a longer period of time. This stain process is on the weather side of the glass, and by doing it before the frosting process, any stain found in unwanted areas may be rectified by sand blasting.

The gnomon must be rigid, and to ensure that, it is best if it lies on a lead or foil line. Simple for the meridian dial and the south facing dial, but some creative thinking may be needed for a decliner.

The author uses $1 / 4$ inch copper pipe commonly used for refrigerator ice making lines as the frame. If the dial is to hang, then stress must be considered to avoid glass separation from the frame, as in any hanging window or panel.

If the panel is to stand on a surface such as shown on the prior page, then a stand must be able to sustain jarring forces. The author uses pyramids with large glass marbles touching the surface. Three marbles create stability. The glass pyramid has three panels of 45 or more degrees at the bases. By not beveling the edges, a very sturdy solder line is formed that is structurally sound, and for decoration, solder buttons are applied.

The entire panel is cleaned, then its border brushed with copper sulfate, a caustic material with safety considerations. Then washed and dried.

## SUMMARY

The templates used are mirror image of a template that would be used for a reflective dial, especially important for declining dials. Chemical and other safety considerations exist. However the end result is a wonderful delight to behold. A resource for stained glass dials is: www.stainedglasssundials.com

## POPUP AND PAPER DIALS

Sun dials may be made in almost any media, and paper is certainly no exception, though less practical in humid climates or when a storm is overhead, or quantities of termites thrive.

Paper dials fulfill a valuable role. For example, they can be used to validate a design. To that end, some paper templates are included in appendix 9. They are instructive. They are fun.

The templates in appendix 9 cover:-

- a horizontal dial template with all the protractor data included
- a vertical dial template with all the protractor data included
- a template for generating declination lines
- a protractor and graph paper to facilitate declining dial design
additionally, appendix 9 includes some paper cutout dials such as:-
- an armillary dial which is also an equatorial one in the summer months
- a meridian dial for east and west facing situations, using paper transparency
- a polar dial
- a horizontal dial
however, paper sun dials may be designed using popup methods popular in the late 19th century.
- a latitude 32 popup 3d paper dial with both horizontal and vertical dial plates

Designed for latitude 32, this dial only needs the equinox line and the hour lines to be altered to match a new latitude. To adjust to another latitude for the horizontal part of the dial, on paper rotate the gnomon 90 degrees, find the new dial center by running a line at co-latitude down to intercept the 12 noon hour line at latitude. That is the new dial center. Another line at 90 degrees to the new style from the nodus goes up to intercept the noon line, and that is where the new equinox line goes. A similar process would be used for the vertical dial.
 the horizontal dial.

A good book discussing these techniques is "The Pop-Up Book" by Paul Jackson.

## A PORTABLE ENGRAVED BRASS HORIZONTAL DIAL

Since the engraver was lying around, and there was spare engraver's brass, this project was an engraved dial, horizontal, longitude adjusted dial, designed for placing on a window sill.

This dial will be for Silver City, NM, whose coordinates are:

| location lat: | $32.75^{\circ} \mathrm{N}$ | location long: | $108.2^{\circ} \mathrm{W}$ |
| :--- | :--- | :--- | :--- |
| magnetic declination: | $10.6^{\circ} \mathrm{E}$ |  |  |

The spreadsheet: illustratingShadows.xls which is on the www.illustratingshadows.com web site was used to provide the longitude corrected hour line angles.

The engraving system used was a Roland EGX-20 which is the bottom of the line, attached by a parallel port to a lap top using Windows XP. As this was indoor dial, engraver's brass was used.

Using a CAD system, a dial plate was drawn up. The CAD image is shown to the right. Dr Engrave software was then used, and the CAD picture was pasted in as a model. It can be moved, and with shift plus the mouse, it can also be resized. Then a dial plate with gnomon was designed over the CAD image, even though images as such can be imported into Dr Engrave. When the final dial plate looked appropriate, the original CAD image was discarded.


The Dr Engrave image is shown to the left. A piece of engraver's brass was placed in the EGX-20 printer, and then the dial plate engraved.

After engraving the dial plate looks anemic. Engraver's brass has a coating which the engraving process cuts through as it cuts into the metal. The rotating bit doesn't just scrape, it has a diamond bit which cuts or burnishes into the brass. This exposes freshly ground brass.

A brass blackening solution is used resulting in the lines and letters standing out. The solution has no affect on the coated parts of the brass, but those where the cutter has traveled become oxidized and blackened.

Holes were drilled for the extremes of the substyle, and on the back of the dial plate, a straight line was cut with a Dremel rotating blade. The gnomon was cut, bent at the bottom, then with some epoxy on the bent base, the gnomon was pushed through the dial plate cut, and thus the gnomon was fastened. For added measure a compass was added, which was unaffected by the brass. The final dial is shown
 to the right, taken from the north looking south.

The author is in no way adept at engraving, this was a by product of the author's desire to have engraved data plates and engraved EOT charts for use with his garden dials. Dial plates can be engraved by other means, such as the photo resist method using an acid bath. Some truly beautiful dial plates result.

## CHAPTER TWENTY NINE

## Buying dials and making them work

## WHAT TO DO BEFORE BUYING ~ WHAT NOT TO BUY

There has been an increase in the number of portable sundials for sale on web based auction sites, and buyers have asked questions as to why their portable sundials do not work.

Sundials can be as accurate as a second or two, true the location must be exact and the predictable equation of time must be applied somehow. The equation of time enables watches to be corrected to reflect the sun's time as indicated on Earth, although nowadays it is used to correct the sundial to the watch. Many simple sundials can be built in a matter of an hour or two that are accurate to within 2 minutes.

Remembering that web sites come and go, these urls or ones like them should be searched for keywords such as "buying sundials" and other variations. The following web sites have useful information for purchasers.
http://www.wiz.to/sundials/
This website has a few simple rules about what to look for before buying any sundial. Web site managed by Mike Shaw.
http://www.mysundial.ca/sdu/sdu_check_a_dial.html
This website has a few simple rules about what to look for before buying any sundial. Web site managed by Carl Sabanski.
http://www.illustratingshadows.com
This chapter has a section on reverse engineering store bought sundials, and the generic spreadsheet on the CD with this book or this book's web site has a sheet for simplifying that operation, allowing the best design latitude to be deduced, from whence those notes explain how to set the sundial. Those notes are condensed in this book and follow in a few pages.

## WHAT NOT TO BUY

There are some fine dials on the auction sites, however there are a number that cannot work.

## LATITUDE ADJUSTABLE ARMILLARY/EQUATORIAL

The armillary dial to the right looks nice, however, the latitude index is not accurate. It looks nice, works if you use an external protractor and plumb line, but it is not functional out of the box. This dial was set to latitude 35 using an external protractor. The latitude guide has two problems. First it shows about 8 to 10 degrees, secondly the line of the armillary dial plate isn't even parallel to the lines on the protractor of the dial.



The dial to the left comes with a level, it is a plumb line and it can only move in one axis. A dial plate when leveled should be level in all axes, which is why the plumb line should be a thread, or why two spirit levels used.

## LATITUDE ADJUSTABLE HORIZONTAL

The dial to the right is fairly well made, however in this case the latitude index is so far off that it is useless. In one case the dial plate it tilted in the wrong direction. This dial is interesting, it is hard to tell whether it is a horizontal dial or an equatorial dial. The gnomon suggests a horizontal dial made portable by a tilting mechanism. With the style of the
 gnomon set level the
 protractor should show 0 degrees, it shows somewhere from 10 to 20 degrees. And even more interesting, its dial plate to the left has hour lines about 17 degrees apart, between 16 and 18 degrees when the angles are measured from the center of the circular dial plate. This suggests a poorly designed equatorial dial, yet the sloped gnomon contradicts that. If it were a horizontal dial as its gnomon suggests, then the noon and 6 am and 6 pm hours should meet at the base of the complete slope of the gnomon which they do not. In fact, measuring the hours from the gnomon's base, or dial center, suggests no real dial design.

## TRIANGULAR LATITUDE ADJUSTABLE HORIZONTAL

To the right is a dial whose latitude index has no connection with real life. At latitude 14 on the protractor, the gnomon is at 55 degrees. At latitude 8 the gnomon's style is at 90 degrees. The center of the protractor is not the tilting dial plate's center of rotation. What is the direction of, say, the 6 o'clock lines? Where the arrows point to, or, from the VI to the dial center? At latitude 32 the gnomon is too small to cast any shadow at all in the summer. A nice dial for the coffee table, but unable to offer the time of day. As they say, this dial plus
 a dollar gets you that dollar cup of coffee.

## A CIRCULAR LATITUDE ADJUSTABLE DIAL



Ignoring the rather painful patina, the dial on the left has a dial plate that might work. However at protractor latitude 10 the style is at latitude zero. While the center of the protractor and the center of the dial plate rotation do coincide, the gnomon size is such that no shadow is visible due to the cutouts in the dial plate.

These are reproductions of dials, bearing the names of manufacturers that seem legitimate. Some look nice on the coffee table as long as the room is not well lit. Telling the time is not their forte.

One portable multi faceted sundial sold on the web has no gnomons, and its text even says that it is turned so all shadows say the same time, when it is now aligned correctly. Not true except in the rare cases where different modes are used to determine the time, and that dial "ain't one of them". Multi faceted dials will always show the same time on all faces regardless of how they are oriented.


I asked this dealer why 6 am and pm were not perpendicular to noon, the response was he never had a complaint, he had these items made for over 25 years. He just sells and does not understand much about any of the items he sells.

LILY PAD DIALS, AND THE LIKE FOR GARDENS
Many dials can be made to work by calculating their design latitude and then being tilted. The next section explains this, as does chapter 15 on inclined surfaces. However, some dials have the noon mark correct but the 6 am and 6 pm below the dial center, so they do not work.

Some beautiful armillary dials for gardens have no latitude adjustment, and the dial plate where the time is read is so polished that no shadow may be seen.

Web based auction sites dealers are as a rule an honest group, but they are not diallists. Be very wary of dials for sale on auction sites. Some are excellent, but you get what you pay for. Educate yourself.


- Study the links to buyer's guides on this book's web site as well as the NASS and BSS.
- Study closely the dial you want and ask what guarantee exists for its accuracy.
- When you get the dial, check and verify your dial, and return it asap if it cannot work.
- Check the NASS and the BSS web sites for ideas and reputable diallists, this book's web site has links to NASS and BSS web pages.


## REVERSE ENGINEERING STORE-BOUGHT GENERIC DIALS

Have you ever purchased a delightful sun dial for your garden, but found it does not work. No problem. Take a photograph of the plan view and the profile, and on the plan view mark the noon and 6 o'clock lines, thus providing the dial center. Then measure the angles of the hour lines, and average the pairs, thus 11 am and 1 pm are averaged, 10 am and 2 pm , and so on. Then select the
 hour angle tables in the appendix for horizontal dials, and see if any latitude column comes close to those hour line angles.

Measuring the gnomon's style angle may confirm the latitude for which the dial was designed. Assuming a dial design latitude was found, then adjust the gnomon to that latitude.

Then place the dial on a surface with a wedge so that while the gnomon points true north, the style is adjusted by that wedge to match your location's latitude.

A few artistic dials are simply incorrect. The 6 am to 6 pm hour line may not be straight, not 90 degrees to noon, there may be no apparent dial center, the gnomon latitude may be incorrect, and where it meets the dial may not be at dial center. Sadly, success is not readily to hand. Keep it out of the sun and enjoy its art work. Time telling is not its forte!


The above dial appears to be a good fit for a design latitude of 46 or 47 degrees. The gnomon showed a design latitude of about 40 degrees.

With a dial plate for latitude 46 degrees, and a style set at 40 degrees, it may be assumed this dial was not designed as a true sun dial, or, that the gnomon was damaged in transit. However, if you can adjust the style to be 46 degrees then the dial would work for latitude 46 degrees. This
 assumes the style will still meet at the dial center.

A wedge may be used to tilt the entire dial such that the final style is tilted at your latitude. Living at latitude 32 degrees, I needed a wedge of 14 degrees.

A temporary wedge was built, and tested in the real sun, and when fully satisfied, a permanent wedge was designed, thus providing life to an otherwise useless sundial.

The end result was a dial designed for latitude 46 with a wedge to thus emulate latitude 32, and the sun dial that was being neglected due to its inability to indicate the time consistently, now becomes not only a delightful garden ornament, but also a functional time piece.

Why commercial sundial vendors do not provide a simple sheet of paper showing how to do this amazes many purchasers.

Of course the EOT needs to be considered, and the longitude correction may be included in a tailored EOT chart, or shown as a separate figure to adjust the apparent time. Or, using Atkinson's Theorem, the dial could be rotated about the polar axis by an amount matching the longitude difference, although the end result might look strange, and the wedge might become more complex.


Alternatively, a spread sheet can be used that reverses the hour line angle formula

| REVERSE ENGINEERING | $\mathrm{H}=\operatorname{atan}(\sin (l a t) * \tan (\mathrm{lha}))$ |
| :---: | :--- |
| thus | $\mathrm{lat}=\operatorname{asin}(\tan (\mathrm{hour} \operatorname{line} \operatorname{angle}) / \tan ($ hour * 15) $)$ |
|  | lat $=\operatorname{DEGREES}(\operatorname{ASIN}(\operatorname{TAN}(\operatorname{RADIANS}(C 7)) /$ |
| or... | TAN(RADIANS $(15 * C 5))))$ |


| angle | $\begin{aligned} & 12 \\ & 12 \end{aligned}$ | 12 |  | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 11 | 10 | 9 | 8 | 7 | 6 |
|  | 0 | 11.90 | 22.66 | 37.28 | 51.82 | 68.93 | 90.00 |
|  | 0 | 11.73 | 22.38 | 35.34 | 51.81 | 70.68 | 90.00 |
| lat a | xxx | 51.9 | 46.3 | 49.6 | 47.2 | 44.1 | xxx |
| lat b | xxx | 50.8 | 45.5 | 45.2 | 47.2 | 49.8 | xxx |
| avg a \& b |  | 51.3 | 45.9 | 47.4 | 47.2 | 47.0 |  |

Final average
47.8

| hla | 0 | 11.22 | 23.14 | 36.51 | 52.05 | 70.10 | 90 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| delta a | xxx | 0.68 | 0.48 | 0.77 | 0.23 | 1.17 | xxx |
| delta b | xxx | 0.51 | 0.76 | 1.17 | 0.24 | 0.58 | xxx |

Max error
1.17 degrees

And when the hours are plugged in for times other than 6 o'clock and 12 o'clock (which for a dial not corrected for longitude) are 0 degrees and 90 degrees regardless of latitude, we see the average latitude for the hour line angles for each hour, and the average latitude for all the hours.

In this case, latitude 47.8 is suggested. This agrees with the use of the tables above, however, the gnomon clearly was designed for latitude 40, and those 7 degrees of difference between the gnomon design latitude and the hour line angle design latitude is significant. The gnomon needs to be adjusted to reflect 7 more degrees while keeping the extension of the style's line such that it intercepts dial center. And the dial is then tilted to cause the resulting gnomon angle to match your latitude.

The above spreadsheet is on the website as: illustratingShadows.xls

From chapter 15 it may be recalled that reclining dials were discussed. The example here is of a latitude 50 dial reclined by 18 degrees producing a style with a resulting angle of 32 degrees, and the picture to the right shows the two. The upper dial was designed for latitude 32, the lower for latitude 50 . They both agree on the time.

The latitude 50 paper dial is reclined by virtue of two protractors that have been cut to recline the dial by 18 degrees. Appendix 9 has cutout paper dials.


## CHAPTER THIRTY

## programming or use of $\sim$ 3D-CAD, vrml, and spreadsheets

THE USE OF CAD, OR COMPUTER AIDED DESIGN, AND VRML ON THE WEB
3D object modeling. Light manipulation in a CAD 3D dial
Generating VRML files from 3D CAD entities. Also see Appendix 10 for viewing VRML
Also see chapters 15 and 18 for 3D CAD techniques on declined and reclined surfaces
Also see chapter 31 for 2D drawing and programming of DeltaCAD
Also see chapter 31 for 2D VBS and Parametric Script programming of TurboCAD


THE USE OF SPREADSHEET PROGRAMS (Open Office is free and very similar to Excel)
Converting formulae used in math to formulae usable in a spreadsheet
Using graph functions for altitude curves and hour lines

Spreadsheets are software programs that allow data to be stored in individual cells, or in columns, or in rows, and for manipulation of that data to take place. This seems simple enough, however there a few traps along the way.

NOTE: The "illustratingShadows.xls" spreadsheet is in Microsoft Office Excel format. Open Office is a free package that is compatible with Microsoft Office, and it opens the Illustrating Shadows spreadsheets with no trouble. Similarly, Kingsoft has a free and a purchased spreadsheet program that works well with Microsoft Excel spreadsheets.

## THE USE OF CAD, OR COMPUTER AIDED DESIGN

CAD comes in a couple of flavors, one is 2 d drafting, the other is 3d modeling. There are some freeware products available, and the program products for which a fee is charged, vary from $\$ 100$ to many hundreds of dollars. Free CAD programs may not provide the angular resolution required. The author elected to use TurboCAD by IMSI. This section glosses over a few techniques that may vary from package to package, and offers ideas based on the author's learning curve.

DRAFTING:
In 2d mode, drafting is simple with many options for lines and orthogonal lines. The first learning curve was selecting the starting point for an angle [OPTIONS, ANGLE, BASE ANGLE] for drawing or for measurement. Second, setting grid snaps could cause problems, as it does with any such program. A third problem was that the default tool was LINE DRAW and not SELECT which caused and still causes much consternation. Replication, linear or angular, requires caution when multiple copies are made. Otherwise 2d drafting is simple.

## MODELING:

In 2d mode, modeling a 3d object may cause confusion. For example drawing a cube may result in a message saying that you can't have a length of zero. However if one shifts from the normal view to say a south-west view (Isometric SW) then "no worries". When modeling an object, the color etc must be set before drawing the item. It can be set afterwards however the object must be selected first and then the attributes changed.

Positioning an object in 3d such as a column takes patience. The concept of a WORKPLANE and how to set it is critical, and affects all sorts of things such as subsequent objects, as well as lights. Failing to use the work-plane will result in frustration if you attempt to position objects since a movement on one dimension may cause an unexpected shift in another.

Objects are modified in wire-frame mode, not in the other display modes. Saving files regularly is critical. This book used a screen capture program to capture final rendered figures as JPG files.


In 2d drawing mode, tables from a spreadsheet or pictures such as JPG can be pasted into the drawing which keeps the data to hand facilitating line and curve drawing. However the pasted item can dramatically increase the final file size. Also, such objects do not appear in 3d mode.

Text can be made to appear in 3d mode. Enter the text, then select it, then FORMAT, EXPLODE it twice, right click for properties, then enter a thickness in 3d.

Rotating a 3d object around the $X$ axis is done by grabbing the $Y$ axis of the model, rotating a 3d object around the $Y$ axis is done by grabbing the $Z$ axis of the model, and rotating a 3d object around the $Z$ axis is done by grabbing the $X$ axis of the model, The work-plane axis can be displayed if desired by OPTIONS, PREFERENCE, SHOW WORLD CS, it shows three little arrows in the bottom left of the screen showing where the $\mathrm{X}, \mathrm{Y}$, and Z axes are, which is critical when rotating an object, especially given the possibility of confusion when rotating an object.

Direction, point, and spot lighting are treated as objects, thus when a light is created, it goes on the work-plane, and its direction is adjusted using the same method as any other 3d object. Using four tiled windows with top, left, right, and southwest views facilitates object manipulation.

Different TuboCAD releases may not read their .TCW files. Use .DXF formats for compatibility, however colors may not be adjustable once saved in that format.

Should you use the free CAD program mentioned in this book for dial design, ensure you set SNAP to none, other wise line angles will not be suitable for dial plates.

## MANAGING A CAD FILE AS AN INTERNET VIEWABLE VRML OBJECT

A CAD file may be saved for viewing on the web in very simple steps. There are three steps depending on how your computer system is established. For people who only wish to view 3d, not generate it, steps 2 and 3 would apply. For people with a VRML browser plug-in already installed, step 3 only would apply. For designers, all three steps would apply.

1. Building a VRML (WRL) file for others to see (TurboCAD or Parallel Graphics ISB)
2. Installing the VRML browser plug-in if you do not have one
3. Viewing any 3d VRML file on the internet (WRL file suffix).

## BUILDING A 3D WEB PICTURE FOR ANYONE ON THE WEB FROM TURBOCAD

In TurboCAD save as WRL: The SETUP option in the SAVE panel should be version 2.0, do not save as WRZ, it may have browser problems on the internet. In ISB, use PUBLISH.

STEP 1: SETTING UP TO VIEW 3D PICTURES ON THE WEB (For designers)
Find out what plug-ins will work for you, there is a web site to advise you of your options.
VRML browser detect:
http://cic.nist.gov/vrml/vbdetect.html
STEP 2: INITIAL BROWSER PLUGIN INSTALLATION (First time viewers)
Download manually a browser plug-in such as Cortona
Cortona VRML browser plug in http://www.cortona3d.com/cortona

STEP 3: VIEWING A 3D PICTURE ON THE WEB
Open a VRML file (WRL)
e.g. there are several on the web site for this book.

IF Internet Explorer gives a cautionary notice, accept it and let ACTIVE-X do its thing.
Netscape and Internet Explorer both run Cortona well, and Cortona also works on Mozilla.
the picture may be tiny, so zoom it by clicking FIT
then PLAN to zoom back a bit
then you may then manipulate the view with STUDY!
OTHER OPTIONS: Appendix 10 covers Windows Vista, and it also offers other options for viewing vrml worlds as well as smaller objects.

Create a triangle with an apex angle of 47 degrees (2 times 23.5) representing the extremes of a light ray from winter to summer solstice. Perform a radial copy so there are about 12 of them, maybe more. And create a gnomon, in this case latitude 32 degrees. Rotate the light rays by 32 degrees similarly to match the gnomon's style angle.

Another example of a more complex kind consists of a wine glass used as a sundial. In earlier centuries, a silver or gold wine goblet was sometimes used as a sundial.

FIRST: A 2-d poly-line was created. Then if needed to get $x / y / z$ axes shown when it is selected,
 use properties to give it a 3d width. Then it is rotated or lifted up around its $X$ axis into the vertical plane so it can be transformed into a wine glass by rotating that poly-line around the Z-axis.


Rotating that poly-line is done by INSERT, 3D OBJECT, REVOLVE, and the wine glass may then be rendered. A copy of this glass is made, because it will be destroyed when we do a 3d-intersect with the solar rays in a couple of steps, just as for the horizontal dial on the previous page.

SECOND: A single solar ray is created - it is a triangle about 47 degrees at its apex, and this
 time a radial copy is performed.

The solar rays are added together with the 3d-add. Thus there are now two objects. One is the glass and its copy, the other is the solar rays. A small sphere was added at the focal point of the rays as a nodus to help with placing the rays on the wine glass, and to solve a 3d-add problem that sometimes happens (see below). A similar small sphere or nodus was also added to the wine glass rim to facilitate placing the rays on the rims consistently.

THIRD: The rays and one of the wine goblets are then used in the 3d-intersect operation, leaving just the places where the rays intersected that wine glass, which is why we made copies of the wine glass. FOURTH: The result of the intersection is moved back to the wine glass copy.


## FAIRLY ACCURATE LIGHTING IN CAD FOR HOUR LINES

TurboCAD lighting can be rather tiresome. TurboCAD lights are normally placed on the workplane, and for a directional or a spot light this is useless. And while VIEW and LIGHTS will show the light's parameters, it doesn't provide for a lighting vector setting. The secret is to use a mesh of arcs made by the arc command, turned into 3d, and saved as a group. This is the solar travel mesh on the TurboCAD page of this book's website:- "solarMesh.tcw"


This mesh depicts solar travel, and just the mesh can be copied and pasted into a CAD dial drawing, its center placed on the nodus, and then rotated for your latitude.

Alternatively, the mesh has a small dial plate and a conical gnomon whose tip is a nodus, and you may use them as a guide, or delete them and replace them with your own dial and gnomon.

For example, a dial plate was laid out, with a latitude $32.97^{\circ}$ gnomon in this case. The work-plane was set horizontal and level with the nodus (not the dial plate) before pasting the mesh model into place, and consequently before adjusting it. The solar mesh CAD model was opened, selected, and copied, and then pasted into the dial's drawing.

Alternatively you can move the mesh center to the nodus $x, y$, and $z$ coordinates. You reset any undesired coordinates manually in the inspector area. "Selector 3d properties" lets you turn on an objects coordinates in the inspector area, the coordinates may be visible there unless you do it. By now, the center of the solar mesh matches the nodus point so the solar-travel mesh is easily tilted to match the dial's latitude.

Then a light is selected, such as the directional light, and with the mouse anywhere, use a right click, select "local snap", then "nearest on facet". Move the cursor to the intersection of the hour line and the month, and wait for a mini work-plane to show that the snap is working and do a left click to anchor it. The light source is anchored on that time and date. Now do another right click, select "local snap", and select "nearest on facet". This time move the mouse to the nodus of the gnomon, wait for the mini work-plane and left click. Now the center of the light as well as the direction have been set. Don't forget to set the light color to white. You may delete the mesh also.


- In the picture to the left, the light is anchored near 9 o'clock close to the summer solstice. The light was placed on a facet of the mesh, its target on a facet on the nodus, and with the light selected, full rendering was used. This shows a spotlight and the shadow cast by the gnomon and nodus. This technique allows rapid light placement accurate for a month and hour.

The light used here was a spotlight. The light may be selected and changed to a directional light in its properties, if so desired.

## THE USE OF SPREADSHEETS ~ FORMULAE CONVERSION

Describing a formula or set of formulae (English) or formulas (American) is easier said than actually getting them into a spreadsheet for real usage. The following points all relate to a common spreadsheet, the one used for this book.

## ANGULAR MEASURE

Angular measures are commonly in degrees, 90 degrees being a right angle and there are 360 degrees in a circle. However, spreadsheets use radians, which are different to degrees. There are 2 * pi radians in a circle of 360 degrees.

Thus, every formula that uses degrees needs the measure to be converted to radians.
=RADIANS(360)
would return 6.283

Thus the trigonometric functions SIN, COS, TAN, COTAN, and so on, need the radian conversion first.
=TAN(RADIANS(45)) would return 1.0
Similarly, any function that returns an angular measure, such as ATAN, ACOS, ASIN, or ARCTAN, ARCOS, ARCSIN, also printed as TAN ${ }^{-1}, \mathrm{COS}^{-1}, \mathrm{SIN}^{-1}$, would need to be converted back to degrees, and this is done using the DEGREES function.
$=$ DEGREES(6.283) would return 359.984 or 360.0 depending on precision
Which raises another issue, that of rounding and precision. The number of significant digits on the fractional side not only determines accuracy, it also controls rounding. Thus when the above function is given one decimal position, the results is 360.0 rather than 359.9.

## ROUNDING

The number of significant digits after the decimal point is controlled by the FORMAT, CELLS, NUMBER option. Some functions do rounding of their own, and in some surprising ways.

| $=\mathrm{INT}(5.1)$ | $=\mathrm{INT}(5.9)$ | $=\mathrm{INT}(-5.1)$ | $=\mathrm{INT}-(5.9)$ | returns |
| :--- | :--- | :--- | :--- | :--- |
| 5 | 5 | -6 | -6 |  |

The INT (integer) function returns the integer part of a number on the left side of the decimal point, rounds down. And the number below 5.9 is 5 , however the number below -5.1 is -6 because this is already in the negative scale.

However, the ABS (absolute) function which removes the sign may be used if the above rounding down on the negative side is not desired.

```
=INT(ABS(5.5)) =INT(ABS(-5.5)) returns
5 5
```

If the sign must be retained, then the SIGN function can be used. Assume that cell B3 has a value of 5.5 and cell B4 has value of -5.5 then the following results can be obtained:

```
=SIGN(B3)*INT(ABS(B3)) =SIGN(B3)*INT(ABS(B3)) returns
5
-5
```


## TIME CONVERSION

Second, they show the use of methods to convert "hh.mm" for example to "hh.hh". Assume cell B3 has a value of 4.30 meaning a time but in a decimal cell, meaning to us humans, four thirty, and we wish the time in hours and decimals of hours then:

$$
=((100 * b 3-\operatorname{INT}(100 * b 3 / 100) * 100) / 60)+\mathrm{INT}(100 * b 3 / 100)
$$

returns 4.5

| Sun's apparent hour angle |  |  |  | LHA=local hour angle |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time hhmm | hh.hh | From <br> midday | LHA | Radians | sin | cos | tan | cot | PM <br> hh.hh | PM <br> hhmm |
| 4.00 | 4.00 | 8.00 | 120.0000 | 2.0944 | 0.8660 | 0.5000 | 1.7321 | 0.5774 | 8.00 | 8.00 |
| 4.15 | 4.25 | 7.75 | 116.2500 | 2.0289 | 0.8969 | 0.4423 | 2.0278 | 0.4931 | 7.75 | 7.45 |
| 4.30 | 4.50 | 7.50 | 112.5000 | 1.9635 | 0.9239 | 0.3827 | 2.4142 | 0.4142 | 7.50 | 7.30 |

The above extract from a spreadsheet shows this conversion, hh.mm is in the left most column, and hh.mm is in the second column. In this case, the hh.hh is the local hour angle:
Iha=15*hours from noon

## CONVERTING DECIMAL MINUTES TO MINUTES AND SECONDS

Always test functions that you use. For example, the integer function $=1 \mathrm{NT}$ (value) is not what you might expect. The integer of 10.1 and -10.1 is commonly held to be 10 , and -10 however the actual INT function in some spreadsheets is the integer after rounding down. Thus the integer of 10.001 may be -11 and not -10 . For example, assume cell C 7 is $\mathrm{m} . \mathrm{mm}$ or $-\mathrm{m} . \mathrm{mm}$ (such as the plus or minus EOT values in decimal):-
$=\operatorname{IF}\left(\mathrm{C} 7<0,-1^{*}\left(\operatorname{INT}(\operatorname{ABS}(\mathrm{C} 7))+(\operatorname{ABS}(\mathrm{C} 7)-\operatorname{INT}(\mathrm{ABS}(\mathrm{C} 7)))^{*} 0.6\right), \quad \operatorname{INT}(\mathrm{C} 7)+(\mathrm{C} 7-\operatorname{INT}(\mathrm{C} 7)) * 0.6\right)$

$$
\begin{array}{rrrr}
3.40 & (3.40 \text { minutes }) & \text { results in } & 3.24 \text { (3 minutes } 24 \text { seconds) } \\
-3.40 & (-3.40 \text { minutes }) & \text { results in } & -3.24 \text { (3 minutes } 24 \text { seconds) }
\end{array}
$$

This formula tests the cell's sign, uses the integer of the absolute (positive) value to get around the round down, and thus converts plus or minus minutes in decimal to minutes and seconds.

And if you wished the absolute value then you could use $=$ ABS( . . .) or $=\operatorname{SIGN(C7)})^{*}(\ldots)$, yes it is permissible to code:

$$
=A B S(I F \ldots) \quad \text { or } \quad=\operatorname{SIGN}(C 7) * A B S(I F \ldots)
$$

It is possible to end up with 6:60 meaning 7:00, that is life after computers.

## BLANKS IN FORMULAE

Some spreadsheets sometimes get upset on long formulae if blanks are used, sometimes they do not. It may say they are erroneous when all that is problematic is those blanks.

## TRIGONOMETRY AND MATH IN A SPREADSHEET

## HOUR LINE ANGLES

Horizontal or vertical dial


```
hour line angle =DEGREES(ATAN(TAN(RADIANS(15*time))*SIN(RADIANS(lat))))
```

hour line angle $H=\operatorname{atan}(\sin (l a t) * \tan (h a))$

## INTERMEDIATE VALUES

Sometimes the cell formula can get vary large and out of hand. If you make an error on entering a formula, the spreadsheet will offer a correction. Be very careful of accepting their suggestion. It is best to match the parentheses yourself. Luckily, many spreadsheets will highlight a parenthesis's matching parenthesis as you move the cursor over, I hate to say it, a parenthesis.

If a formula gets too large, consider using intermediate columns, or cells, that hold intermediate data. Intermediary values are usually kept off to one side, the right side in this case, and used as working accumulators for the final value.

For example, the latitude is copied from cell B2 into cell E3, and from then on cells look back one cell, so E4 copies from E3, E5 from E4 and so on. This makes it easier to copy a row for formula and paste them into the next line, while not having to go back and edit the formula. If all cells referred to $B 2$, then each copy and paste would change the $=B 2$ to $a=B 3$, $a=B 4$, and so on with each copy in the column.

Another technique is discussed later, using hard cell referencing, however, that makes it hard to copy and reproduce tables.

| A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- |
| 1 | LATITUDE |  | intermediate values |  |
| 2 | 74.3 |  | lat |  |
| 3 |  |  | $=\mathrm{b} 2$ | could have coded $\$$ B $\$ 2$ |
| 4 |  |  | $=3$ | which would not change when |
| 5 |  |  | $=e 4$ | copied and pasted continuously |

In the excerpt above, B2 is the "master latitude", E3 refers to it, and E4 refers to E3. When in a spreadsheet, you copy E4 to E5 then it updates the =E3 to E4, and so on as you select or even block select, copy, then past lower down in the column. If you copied cell E3 (saying =B2) to the next cell down then it would save it as =B3 which is incorrect. One can hard code cell references using the \$column\$row syntax, however the benefits may outweigh the drawbacks.

Columns can be used for intermediate values needed in complex formulae, and this is the equivalent to using accumulators in an old computer (1960's vintage) or to intermediate values in the scientific calculators.

## AN EXAMPLE OF INTERMEDIATE VALUES IN COLUMNS OFF TO THE RIGHT

DECLINATION by day and ALTITUDE by day/hour

The gnomon:chart ratio on the top right of the chart is the ratio and reciprocal of the ratio of the style length to the distance from the top of a chart of altitude curves, to the,bottom, on the lowest hour shown, typically about June 20, noon time.


Notice the use of radians instead of degrees, and the use of accumulators to the right for "td" which is an intermediate value, and how the latitude is replicated down the column on the right also, this means that the use of absolute cell references, \$column\$row is avoided. Sometimes absolute cell references are helpful, sometimes they are not. It is a question of technique.

The Julian date was hard coded because there was a problem with the DATEVALUE function in the version of spreadsheet the author used. Always double check formulae as you proceed.

NOTE: Open Office provides a compatible Excel replacement, the spreadsheet syntax is slightly different from Excel but it converts Excel spreadsheets correctly.

## SUNRISE AND SUNSET SPREADSHEET

DECLINATION by day and APPROX SUNRISE/SUNSET time STANDARD

| LATITUDE | LONG | ref:long |
| :--- | :--- | :--- |
| 51.5 |  |  |
| 0.50 |  | 0 |
| London UK |  |  |

Tables are based on the LATITUDE, LONG and time zone
reference in the boxes to the left. EOT is used and is
the 3 wave approximation. Summer time not considered.

Date Julian
1/1


| STD TIME <br> rise |  |
| :--- | ---: |
| 8.15 | 15.56 |


| Daylight <br> duration | Sunrise <br> azimuth |
| ---: | ---: |
| 7.45 | 51.0 |

First intermediate figures are EOT and declination components and the local sunrise hour angle.

| Intermediate values |  |  | LOCAL SUNRISE <br> DATA |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
| eot | td | lat | ha | hhmm |
| 3.4 | 0.00000 | 51.5 | 122.4 | 8.09 |

The next intermediate values work sunrise to completion, then sunset to completion, and each separately make the longitude then the EOT adjustment.

EOT uses formula:
-1*(9.84*SIN(RADIANS(2*(360*(julday-81)/365))) - 7.53*COS(RADIANS(360*(julday-81)/365)) $1.5 * \operatorname{SIN}(R A D I A N S(360 *(j u l d a y-81) / 365))$ )-0.3 where the Julian day in the second final data column was entered as-is, it did not use a formula.

The LOCAL hhmm of sunrise was derived using: (100*INT(localha/15)+INT(60*((localha/15INT(localha/15)))))/100 where The local hour angle of sunrise was based on: DEGREES(ACOS(TAN(RADIANS(latitude))*TAN(RADIANS(declination))))


The STD ha was equal to the LOCAL ha in the preceding intermediate value section, plus the difference of the user's longitude and the reference longitude.

Times were kept decimal until the final conversion using: INT(T8)+0.6*(T8-INT(T8)) where T8 was a decimal time.

Sunset was a similar process, sunset h.hh was found by 24 -sunrise.h.hh and this must be calculated before the EOT or longitude corrections are applied.

The EOT shown above is in mm.mm (decimal), it may be converted to mm.ss with a formula such as:-

$$
=\mathrm{IF}(\mathrm{~L} 8>0, \mathrm{INT}(\mathrm{~L} 8)+0.6 *(\mathrm{~L} 8-\mathrm{INT}(\mathrm{~L} 8)),-1 *(\mathrm{INT}(\mathrm{ABS}(\mathrm{~L} 8))+0.6 *(\mathrm{ABS}(\mathrm{~L} 8)-\mathrm{INT}(\mathrm{ABS}(\mathrm{~L} 8)))))
$$

where L8 is the time in decimal minutes (mm.mm)

## USING A SPREADSHEET GRAPHING FUNCTION TO DRAW HOUR LINE ANGLES

Using a formula or raw data from a table in the appendix, it is possible to generate a chart using the spreadsheet's charting function so that hour lines can be drawn for a dial plate.

| A | B | C | D | E | F | G | H | 1 | J |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | Enter the | latitude | 32 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |
| 4 | To have a spreadsheet generate a chart of hour line angles, eg, latitude 32 horizontal dial first get the hour line angles (from the tables or from calculation):- |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |
| 7 | pm time | 12 | 1 | 2 | 3 | 4 | 5 | 6 |  |
| 8 | angle | 0 | 8.08 | 17.01 | 27.92 | 42.55 | 63.18 | 90 |  |
| 9 |  |  |  |  |  |  |  |  |  |
| 10 | Then generate a set of differences or steps |  |  |  |  |  |  |  |  |
| 11 |  |  |  |  |  |  |  |  |  |
| 12 | difference | 0 | 8.08 | 8.93 | 10.91 | 14.63 | 20.63 | 26.82 |  |
| 13 | [formula] | $=0$ | = D 8 - C 8 | =E8-D8 | etc | etc | etc | =/8-48 |  |
| 14 |  |  |  |  |  |  |  |  |  |
| 15 | Then we need to bring the angles up to 360 degrees so we can have a pie chart of 360 degrees. To do this we add a trailing number of 360 -the sum of the steps: |  |  |  |  |  |  |  |  |
| 16 |  |  |  |  |  |  |  |  |  |
| 17 |  |  |  |  |  |  |  |  |  |
| 18 | difference | 0 | 8.08 | 8.93 | 10.91 | 14.63 | 20.63 | 26.82 | 270 |
|  |  |  |  |  |  |  |  | $\begin{aligned} & =360- \\ & \text { SUM(C18:118) } \end{aligned}$ |  |
| 20 |  |  |  |  |  |  |  |  |  |
| 21 | Select the | ells from | e left to righ | and INSER | T, CHAR | RT, PIE to | get a true | ngular cha |  |

The spreadsheet chart was captured as a JPG file, imported into a CAD program, and INSERT, MEASUREMENT, ANGULAR, options were used to measure the angles which as can be seen are well within reading tolerances.

This method has as the benefit that the aspect ratio is correct.


Another option is to convert hour line angles to $\mathrm{x}, \mathrm{y}$ pairs, then use insert chart, then $\mathrm{X}: Y$ SCATTER, then data points connected by lines. The main spreadsheet provided with this book uses that technique for horizontal and vertical dials, and the analemma depiction is another example, see chapter 25 . Such spreadsheet graphs do not maintain aspect ratios, so angles must be verified, and the h -dial and other spreadsheet graphs show one technique for doing this.

This chapter reviews numerical and other methods for effective spreadsheet usage. Spreadsheets are available on the web site named at the beginning of this book and help with most sundial tasks from the simple to the complex:- illustratingShadows.xls

## CHAPTER THIRTY ONE

## programming or use of $\sim 2 D-C A D$, and other languages

## SOME NOTES ON PROGRAMMING DELTA-CAD

Many projects in this book used Delta CAD macros whose resulting dial plates were printed, cut out, and used as templates. The reason for using Delta CAD was because it is popular, and programmable. The author uses glass, clay, and copper on his outside dials.

The author's first career before winding up as an airline pilot was in programming computers. Some work was commercial, but most work was in operating system software. The first system he programmed was an IBM 1401 in Autocoder. Then an IBM 360 in BAL (Basic Assembler Language), with a bit of PLII, and FORTRAN. All his work on the IBM 370 and later machines was in BAL and on some other machines he used C and C++. Operating systems used were BOS, MFT, and MVT on the IBM 360s, and VS1 and MVS on the IBM 370s, and GCS under VM. One pet peeve the author has is that documentation for languages is drawn up by programmers as language specifications, and the human interface is covered is somewhat academic. This tendency became worse when object oriented programming became the standard because the novice is faced with several struggles. Thus the examples in this section for the BASIC macro language for Delta CAD are intended to be "conversational" as opposed to transaction oriented. In other words less object oriented and more of a natural flow. As background, the author designed FIDO and PATCHES which were early spooling systems on IBM 360 mainframes in the 1970s, and TOTO and later SHADOW which were teleprocessing programs running under DOS and MVS on the IBM 370 and later ranges which were sold worldwide from 1972 until 1997. SHADOW in particular made over $\$ 55 \mathrm{~m}$ in sales before the author lost track and interest.

Older style conversational programs had a natural flow, assumed a sequence of events, and were less well suited to random events controlling the program flow. The next development was transaction oriented programs which used discrete pieces of code invoked when things happened. Less natural, more adaptable to random events controlling program flow, and could still easily be made to look conversational if so desired. Then came object oriented code, here "objects" were defined which had "methods" associated with them which handled "things" that affected the objects. Objects were defined by "classes" with an inheritance structure, and interrelationships between objects. Thus one simple change here would trigger many "methods" in many "objects" resulting in lots of activity. Hard to make conversational, but highly generalized and ideally suited for random events coming in (a screen input, an interrupt from an outside source), but clearly not a simple natural program flow.

## USING A CAD SYSTEM and USING CAD MACROS

A CAD system is a computer program that draws, usually better than the average human. The 2d systems such as Delta CAD are simple to use and provide professional draftings. The author prefers TurboCAD deluxe which is a full 3d modeling system used for most of the pictorials in this series of books. It also has excellent after the fact dimensioning tools. DeltaCAD leads TurboCAD in having the computer do all of the work by a simple programming system akin to BASIC. TurboCAD programming is only available in the Professional edition. Computers do what they are told and thus special techniques are needed to do simple things a human can intuitively do, such as constraining a line to the boundaries of a box. This is a summary of Delta CAD and its associated macro language with a few pointers to help write macros. Writing macros for DeltaCAD is within reach of any computer literate person. To simplify concepts, code in this chapter is extracted and reduced from the programs on the CD or website and thus may not handle some special cases. Also programmable is FreeCAD using Python, and NanoCAD using Java Script and Visual Basic Script. FreeCAD and NanoCAD are free.

This section extracts some code and explains what it is doing, and shows final resulting dial plates. There are many macros available for Delta CAD which are well worth exploring. The author has his own versions of Delta CAD macros on the web site for those who are interested, and these complement the spreadsheets which are also made available. However, these spreadsheets and macros which are fully functional are aimed at education first and foremost, and as a tool second. They are not polished works, they are not intended to produce final polished products, nor are they intended to compete with the excellent macros available.

Programs usually begin with initial setup, then they define variables to hold information being worked on, and they also ask humans what they want, and finally they produce the results.

```
''******************************************************************************
' A horizontal dial macro for Delta CAD but in conversational mode as
' opposed to the more modern object oriented mode, but with notes
' page numbers refer to Manual.pdf or Basic.pdf provided with delta cad
' *************************************************************************
Sub Main() ' main procedure is required
```



```
' Initial house keeping - clear the screen - set the drafting area unit
' ***************************************************************************
' select all objects that may exist on the screen - p223 of Manual
' then erase them all - page 189 of Manual
If (dcSelectAll) Then
    dcEraseSelObjs
End If
' set the entire future drafting area to inches or centimeters, etc
' page 43 of the Manual: 1.0 generates inches, and 2.54 is cm
dcSetDrawingScale 0.80
```

At this point, a few thoughts on hard coded numbers, specific dimensions and programming practice are relevant.

Good program practice is not to code numbers inline in a program, rather, those numbers should be in a data definition area. This makes the program easier to change, however, it also makes the program a little harder to understand.

It is similarly bad practice to define display areas, as this program does, of say 0,0 to 1,1 however, if the end result is scalable there is little harm, however good practice would still use symbolic numbers, with those defined in the data definition area.

Programming practice has been for many years to use structured coding techniques, that is IFEND IF, DO-END DO, FOR NEXT, and the like, and never to use the GOTO statements.

Another more recent architecture of programming systems has been to move towards object oriented methods, where an object not only holds the data, but also the methods for altering or reading it. And further, all objects can have inter-relationships, so if something changes that might affect one object, then that and other objects will find out and act accordingly. For example, in Windows bring up two displays of the same folder, and in one display, delete an entry. Object oriented methods are what cause the other folder display to update itself and reflect the first display's changed status.

While BASIC as supplied with Delta CAD is not fully object oriented, it does use some of its concepts, for example the structures needed to talk with humans. Next some programming structures are needed for human interaction. These are not difficult, neither are they quite as simple as elementary BASIC.

```
|************************************************************************
' A generic definition is required for a screen input area
', ****************************************************************************
Here a box on the screen for user dialog is structurally defined,
' it is only a definition of the generic area, it does not create it
', ..... Dialog aaaaa
' To create the area, there must be a Dim statement making a label
' relate to this definition
' ..... Dim bbbbb as aaaaa
' To use bbbbb there must be a ..... yyy = Dialog(xxxxx) which causes
' human interaction. So...
' create an area on the screen starting at x=20, y=20
' whose size is 200 left to right and 100 top to bottom
' whose title is "Location" - where 0,0 is top left
Begin Dialog aaaaa 20, 20, 200,100, "Location"
' the first text string starts at x=5,y=15 on the screen
' and the text string itself starts at x=60 for a height of 10
    Text 5, 15, 60,10, "Enter latitude"
', the input area starts at x=65 (further right) y=15 (same height) for a
' size of x=50, y=10
    TextBox 65, 15, 50, 10, .mylat
' the second text string starts at x=5 but now y=30, i.e. lower down
' and the text string itself starts at x=60 for a height of 10
    Text 5, 30, 60, 10, "Enter longitude"
' and the input area starts at x=65 (further right) y=30
' (same height) for a size of x=50, y=10
    TextBox 65, 30, 50, 10, .mylng
' the third text string starts at x=5 y=45
' and the text string itself starts at x=60 for a height of 10
    Text 5, 45, 60, 10, "Enter ref longitude"
' and the input area starts at x=65 y=30 for a size of x=50, y=10
    TextBox 65, 45, 50, 10, .myref
' and two buttons for what the user means, location first, button size
' next - and all such boxes must have at least one button by the way
    OKButton 65, 65, 40, 10
    CancelButton 65, 85, 40, 10
End Dialog
```

At this point, a few comments might be helpful. The Begin Dialog has nothing to do with a dialog. It is an encyclopedia definition of what you might wish to actually create.

## It is created with the Dim statement.

```
' ********************************************************************
' The generic definition must then be generated with a name
\prime *************************************************************************
'
' this defines "bbbbb" as an instance of aaaaa dialog
Dim bbbbb As aaaaa
```

And used later.....

```
| ********************************************************************
' Now define the initial general working variables
'********************************************************************
'
' define a lat and a long, and a reference longitude
Dim lat As single
Dim lng As single
Dim ref As single
```

... continued on the next page

```
continued from the last page
```

```
***********************************************************************
' Now get the lat, long, and reference longitude
' *************************************************************************
' first set the defaults - here bbbbb.mylat uses the structure
' from aaaaa
bbbbb.mylat = "32.75"
bbbbb.mylng = "108.2"
bbbbb.myref = "105.0"
```


## .........., in fact here it is being used!

```
' here the dialog is invoked and the button results returned to ccccc
' page 20 and 24 etc of Basic discusses the Dialog function
ccccc = Dialog(bbbbb)
' which causes the answer to be returned
lat = bbbbb.mylat
lng = bbbbb.mylng
ref = bbbbb.myref
- CANCEL button returns 0
' OK button returns -1
' you can determine the button with - Print ccccc, lat, lng, ref
```


## HORIZONTAL DIAL

The rest of the program is straight forward.

```
'***************************************************************************
' ok, what was returned? if "ok button" then do the program itself
' *****************************************************************************
c}\mathrm{ ccccc = -1 means the ok button was used and not the cancel button
If ccccc = -1 Then
\prime *************************************************************************
' this is the main program to draw the horizontal dial itself
'************************************************************************
' calculate hour line angles next, but first define them
Dim h, hx, hy As Single ' Delta CAD is fussy about data attributes
' the formula is... hourlineangle = atan ( sin(lat) * tan (lha) )
' almost all systems us radians
' the formula also needs adjustment for longitude displacement
' line color is 0 is black
' line type is dcsolid
' line weight is dcnormal
' set the text color, font, size, etc also
dcSetTextParms dcBLACK,"Ariel","Bold",0,8, 20,0,0 ' p321 of Manual
dccreatetext -1.25, -0.3, 0, "Hour and hour line angle H-DIAL"
dccreatetext -1.25, -0.9, 0, "Lat: "
dccreatetext -0.8, -0.9, 0, Int(lat)
dccreatetext 0.0, -0.9, 0, "Long: "
dccreatetext 0.3, -0.9, 0, Int(lng)
For hr = 6 To 18 Step 1
    ' for the hour (hr) calculate the hour line angle (h)
    h = deg(Atn(Sin(rad(lat))*Tan(rad((hr*15) +(ref-lng)))))
    ' show the time in hours
    dcSetTextParms dcBLACK,"Ariel","Bold",0,8, 21,0,0 ' p321 Manual
    dccreatetext (-1.2+((hr-6)/5)), -0.5, 0, Abs(hr)
    ' show the angle
    dcSetTextParms dcBLACK,"Ariel","Bold",0,6, 21,0,0 ' p321 Manual
    dccreatetext (-1.2+((hr-6)/5)), -0.7, 0, Int(h)
```

```
If hr < 12 Then
' morning hours ~ NOTE code for keeping lines in a boxed area
dcsetlineparms dcblue,dcsolid,dcthin ' page 228 Manual
If Abs(h) < 45 Then ' lines touch top of box
    dcSetTextParms dcBLACK,"Ariel","Bold",0,8,21,0,0 ' p321
    hx = Tan(rad((h)))
    dccreateline 0, 0, hx, 1
    dccreatetext (hx), 1.1, 0, Abs(hr) ' page 187 of Manual
Else ' lines touch side of box
            dcSetTextParms dcBLACK,"Ariel","Bold",0,8, 20,0,0 ' p321
            hy = Tan(rad((90-h)))
            dccreateline 0, 0, -1, -hy
            dccreatetext -1.1, -hy, 0, Abs(hr) ' page 187 of Manual
        End If
ElseIf hr = 12 Then
, noon hours
noon hours
... similar code which is straight forward
Else
l
    afternoon hours
... similar code except the sign of the angle goes -ve if 90
End If
```

Next h

The program concludes with drawing a couple of boxes.

```
' draw a box around everything
dccreatebox -1, 0, 1, 1 ' page 184 Manual
dccreatebox -1.2, -.2, 1.2, 1.2 ' page 184 Manual
dcviewbox -1.1, -1.1, 1.1, 1.3 ' page 225 Manual
End If
' *** this ends the entire program
' ****************************************************************************
End Sub
```

And after the end of the program, there are the DEGREES and RADIANS functions.

```
' *****************************************************************************
' Useful routines or functions - Functions must be defined at the end
' after the main program which is sub(xx) ... end sub
' ***************************************************************************
' Convert degrees to radians
Function Rad ( n As single ) As single
' page 83 basic.pdf for functions
    Rad = (n * 2 * 3.14159) / 360
End Function
' Convert radians to degrees
Function Deg ( n As single ) As single
' page 83 basic.pdf for functions
    Deg = (360 * n) / (2 * 3.14159)
End Function
```

The actual code on the web site may differ and have code to correct for certain situations, however, the objective here has been to step through most of the procedural coding. There are some coding violations, however they have been somewhat intentional in order to make the process of a complete macro, when accompanied by DeltaCAD's two books, MANUAL.PDF and BASIC.PDF, more understandable. NOTE" The Windows VISTA version of DeltaCAD has a few differences, these are covered in appendix 10.

## MERIDIAN DIAL

The horizontal dial and vertical dial methods are simple and straight forward. However, the meridian dial, uses distances along an equinox line rather than angles. And just as the horizontal and vertical dial have commonalities and differences, so do the east and west non declining meridian dials. Distance calculations use a gnomon linear height.

Things increase in complexity if calendar lines are added, and a bit more complex if those calendar lines are to terminate the hour lines.

While the horizontal and vertical dials had code to constrain the hour lines within a boundary box, this was less of a concern in the meridian dial, however, hours lines in this case would be constrained between solstice curves. The meridian dial logic allows the user to specify a gnomon linear height, and since it does, the hour lines can be far away, and exceed any boundary box. So, this meridian dial logic will not focus on constraining those hour lines and calendar lines within a box, but instead will focus on the conversion from trigonometry to sequential programming, and also show how the hour lines are constrained inside the calendar curves.

The process is straightforward.

```
' First, lets gather the x,y for the solstice lines for L.A.T. 6pm
this will be used for the solstice curves
    the distance from the equinox line to a solstice line is
    found from: tan(23.5) = dist.fr.gnomon.base / glh
    thus: dist.fr.gnomon.base = glh * tan(23.5)
    and we can use the same method for finding the x,y of
    the summer and the solstice points for 6pm L.A.T.
        * winter solstice shadow point for 6pm L.A.T.
        \ <-- the linear distance from gnomon base to solstice
            * shadow tip for 6pm L.A.T. is simply
            * gnomon base gnomon linear height * tan(23.5)
        [23.5 is solstice solar declination]
            * summer solstice shadow point for 6pm L.A.T.
winterx = - lat6distsolstc * Cos ( rad(lat)) ' minus as left of 1,1
wintery = lat6distsolstc * Sin ( rad(lat)) ' plus as above 1,1
summerx = lat6distsolstc * Cos ( rad(lat)) ' plus as right of 1,1
summery = - lat6distsolstc * Sin ( rad(lat)) ' minus as below 1,1
' the above four lines prime an x,y difference from 1,1 which is
', the top right which is the gnomon base and the top right of the
' boundary box
' the above four are differences from 1,1 so we need to remember
' that before we use them, and for fun lets put circles at those
', points which will show how the x and y differences from 1,1
' are used
winterx = 1 + winterx
wintery = 1 + wintery
summerx = 1 + summerx
summery = 1 + summery
dccreatecircle winterx, wintery, 0.01 ' page 154 Manual
dccreatecircle summerx, summery, 0.01 ' page 154 Manual
```

The logic above takes the gnomon base at $x=1, y=1$ and derives the $x, y$ coordinates for the solstice calendar points, summer and winter, and draws a couple of circles at those points, and then saves those points for the adjacent hour line. Not for the hour line location, but for (a) the end points of the hour line which will be centered on the equinox line, and (b) so a line can be drawn from 6 pm L.A.T. to the hour line, being the start of the calendar line.

That is how the calendar lines are drawn, and how the hour lines are constrained.
Rather then duplicate the code which is well commented, please refer to the web site:

## www.illustratingshadows.com

And from thence to the free DeltaCAD section which has all the code in current form.
This has not been intended to be an exhaustive treatise on Delta CAD macros, nor the BASIC language it uses, rather it is a summary of the human interface method, and the conversion from trigonometry to procedural code.

These dial plates are produced with the DeltaCAD macros available on the web site.


Hour and hour line angle VERTICAL NON DECLINER

| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 86 | -77 | -60 | -44 | -29 | -16 | -3 | 9 | 23 | 36 | 52 | 68 | 86 |

Lat: 32 Long: 108


Hour and hour line angle H-DIAL

| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 84 | -69 | -47 | -32 | -20 | -11 | -2 | 6 | 15 | 25 | 39 | 58 | 84 |

Lat: Long: 108


Note how the meridian dials clearly show the sub-style line, and how they also clearly show the style's linear height above the sub-style. A model style is especially important when working with clay dial plates, since clay shrinks approximately 10\%

The vertical dial is similar to the horizontal dial, however, the dial with calendar data, and the equatorial dial have interesting logical methods.

## CALENDAR HORIZONTAL DIAL

The logic is not that different from the horizontal dial already discussed, however the addition of calendar lines brings up some dilemmas.

Should hour lines be bounded by the solstice curves? This brings to the fore the reason that styles sometimes have a nodus, and sometimes they do not. Meridian dials do not face this dilemma, horizontal and vertical dials do.

The nodus is for calendar indications. The style is for hour line accuracy. Some horizontal dials have a tiny gnomon because of the perceived need to have the calendar lines fit on a dial plate,that is the tail wagging the dog. The two are not related.

The logic used in the calendar dial has phases. One phase draws longitude corrected hour lines, bounded by a box. And thus uses a healthy long style, the longer the better for ease of reading.

The calendar curves use local apparent time and do their own thing, independent of the hour lines. They are two independent elements, not related, and yet a lot of software goes to a great deal of trouble to accommodate a nexus that never really existed.


Hour and hour line angle H-DIAL Lat: 32 d.Long: 3

Chapter 22 discusses the logic and thus those pages are not repeated here except for some key pointers. In particular how do you limit the calendar line size compared to the rest of the dial plate? Several ideas are...

- Draw each calendar line segment and whenever an "x" or "y" ordinate exceeds the boundaries, then derive a new pair that starts at a boundary.
- The method used in the program provided with this book is not to draw any line segment if any of the $x$ or $y$ coordinates of a line exceeds any boundary
- Use limiting hours

And so on.

EQUATORIAL DIAL
On the face of it, what could be difficult programming a dial whose hour lines are a fixed 15 degrees apart. Well, first, those line have to be drawn and limited to a circle or a box. That immediately employs special logic depending if the lines drawn from the boundary center, located at 0,0 hit the top, left, right, or bottom side. Then comes the circles for the calendar data, followed by the sunrise and sunset line, and like the meridian dials, the gnomon length is critical. And finally, the rotation of the south facing plate is the opposite of the north facing plate, and similarly the numbers go clockwise for one, counter clockwise for the other.


To the right is an azimuth dial longitude corrected. This program or macro can use the spline (curved line) or straight line method of showing the hour point curve by calendar date. This allows an albeit stylistic DXF file to be exported and subsequently imported into engraving software such as Dr Engrave, as supplied with the Roland engraving printers.

To the left is the result of the equatorial dial macro provided on the CD as well as available on the website. This is a plate facing north in the northern hemisphere as can be seen by the sunrise/set line being above the nodus, whereas for the other half of the dial it would be below, the hours reversed, and the longitudinal correction rotation would also be different.



To the right is a shepherd's dial which does not use the spline function, rather it calculates the sun's declination every two days and draws the curves of over 180 small two day line segments.

To the left is a polar dial also longitude corrected. This macro used a series of lines rather than a spline function.


The sample DeltaCAD programs or macros, available on the CD accompanying this book, as well as on the web site, also provide for animation. For example some dial macros allow

- the hour lines and calendar lines to be animated as the latitude changes, or
- the gnomon shadow is animated through the day for a given declination.

DeltaCAD was not designed for animation, but the results of these macros available on the CD and web site can be highly educational. So, how can one make animation work in DeltaCAD?

One way would probably be to use drawing layers. One layer might hold the hour lines, and another layer hold the shadow of the gnomon. The gnomon shadow holding layer could be deleted and re-generated with the next shadow. However that is easier to do by hand on the work surface than by programming since the layer must be empty before deletion if done programmatically.

Alternatively, one layer per shadow could be used, with all but the desired active layers turned off. However, that would be expensive in terms of resources.

Another method would be to redraw the shadow lines with a null color thus deleting a line's pixels, all work would then be done in one shadow layer separate from the hour line layer, but there is no null color available.

Another approach would be to redraw the shadow lines in white, but that would still obliterate the underlying lines, unless a separate layer were used.

Or, the shadow lines could be unique, and then all objects that held that uniqueness could be deleted, leaving only the permanent lines. And so on...

Regardless of the method used, it appears that actual drawing and displaying are asynchronous with the dc-commands or functions. Hence lines still seem to be clobbered, sometimes the drawing stops until the end if the computer is busy. And that can happen if any programs are open even if they are not doing anything. So, "yer pays yer money and takes yer choice".

The simple fact is that animation can be an enormous benefit in understanding how a dial changes with latitude or by time of day.

As a reminder, these are conversational programs and do not use object oriented features, they are as simple as they can be.

In keeping with this book, these DeltaCAD macros or programs are for their educational benefit more than for being polished stellar examples of a finished product. However the dial plates they produced were actually used by placing the paper dial plates on clay, and thus were fashioned the clay dials seen in this book. If you use any software by anyone at all for actual dial plate design, always double check the resulting dial plate, angular, and linear measurements for appropriateness.

NOTE: The command STOP causes a BASIC SCRIPT ERROR, use EXIT FUNCTION or some equivalent instead.

NOTE: DeltaCAD for Windows Vista formalized some syntax that was looser in prior versions. For example function calls require parameters not required in prior versions. Also, sloppy coding of DeltaCAD features, such as DCCREATETEXT no longer works, rigid parameter coding is required. These changes are simple and enforce better programming methods.

PRINTING TECHNIQUES ~ ~ ~ FOR DIAL PLATES LARGER THAN PRINT PAPER SIZE
Once you have a DeltaCAD macro executed and a dial plate depicted, printing is easy.


The above process can be used to print dial plates using DeltaCAD for dials larger than the print paper size. CAUTION: Some screen capture programs do not accurately preserve aspect ratios, so be careful if you use a screen captured layout and paste it into a word processor.

Vertical declining dials facing almost east or west often need a method of printing just a part of a dial depiction. Once you have the dial plate drawn, as shown to the right, use DeltaCAD VIEW (top row, not third row of tabs in DeltaCAD) and then zoom to get the area you desire. If you use screen capture programs, be aware that some capture programs may distort DeltaCAD angles (aspect ratios). The angles in DeltaCAD are correct as tabulated.

Because of hour line bunching, use the following print technique:

VIEW (third row, not top row) then...
VIEW OBJECT IN RECTANGLE and draw its rectangle
 FILE then SET PRINT REGION and in that box...
in PRINTAREA do SET TO CURRENT WINDOW -you may enter print scale numbers also FILE then PRINT PREVIEW and then you may print


The above process can be used to print dial plates using DeltaCAD for dial plates smaller than the displayed size. CAUTION: Some screen capture programs do not preserve aspect ratios.

## NanoCAD, FreeCAD, and Powerdraw:

NanoCAD is free but licensed, and programmed using Java Script and Visual Basic Script. FreeCAD is an open source CAD program available at no cost, programmed using Python. Powerdraw is free and uses a Pascal variant. The book Programming Shadows discusses programming in these systems. Available macros may include horizontal, vertical, vertical declining, polar, meridian, shepherd, winged azimuth, and Cappucin dials, calendar/declination curves, and an almanac. These macros are on the website, and on the CD.

## SOME NOTES ON PROGRAMMING IN BASIC [In this case JustBASIC]

Available at www.justbasic.com is a free BASIC true IDE (integrated development environment). A more extensive version called Liberty Basic is available for a small fee. This system is used in some of the programs on this book's web site as well as on the CD provided with these books.

This BASIC system is different from DeltaCAD, but very close. What follows is a simple program to display data for a simple horizontal dial.

```
' ============================================================================
' A simple horizontal dial with longitude correction
' by definition it can be a vertical dial for co-latitude
```

' Ask for longitude and latitude and the longitude for the legal time zone
print "---------------------H-DIAL------------------------"
print "If run as an .EXE file, please read [please Read JB.txt]"
print " sws-bas-h-dial.bas -or-- sws-bas-h-dial.tkn"
print " Last revision: Jan 27, 20070932 mst"
print $\qquad$
print
input "Enter design latitude: [eg 32.75] "; zla
input "Enter design longitude: [eg 108.2] "; zln
input "Enter legal meridian longitude: [eg 105] "; zle
' if null input then setup defaults
if zla $=0$ then zla=32.75
if $z \ln =0$ then $z \ln =108.2$
if zle $=0$ then zle=105
' Calculate the correction due to longitude, and display the correction
cor = zln-zle $\quad$ ' This is the correction in degrees
print "Lat: ", zla
print "long: ", zln, "Ref: ", zle
print "corr: ", 4*cor, "minutes" ' This is the correction in minutes
print "----------------------------------------------------------
' Loop the range of hours and show the hour and its angle

```
print ""
print "If the sign for the earlier hours differ from the later hours"
print " then it means the hour line went past 90, adjust accordingly."
print ""
print "Hr: ", "no corr: ", "long corr: "
for n=05 to 19 step 1 ' Start with am on left, to pm on right
    zhr = n - ( 4*cor/60) ) 'This is the correction in hours
    ' One could use IF, ELSEIF, etc however JustBASIC does not support ElseIF
        if n< 12 then
            if n=11 then
                                    print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr)), "+ve is to the left"
            else
                        print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr))
            end if
end if
```

```
    if n=12 then
    print "- - - -"
    if zln >= zle then
        print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr)), "to the left of sub style"
    else
        print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr)), "to the right of sub style"
    end if
    print "- - - -"
    end if
    if n> 12 then
    if n=13 then
        print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr)), "-ve is to the right"
    else
        print n, shrink(hdial(zla,zln,12-n)), shrink(hdial(zla,zln,12-zhr))
    end if
    end if
    ' repeat the loop
next n
print "Hr: ", "no corr: ", "long corr: "
print ""
print "If the sign for the later hours differ from the earlier hours"
print " then it means the hour line went past 90, adjust accordingly."
[dial]
print "-
    "------------------------------------------------------
input "end - hit ENTER to terminate"; zzz
end
' define a function for returning n hour angle
'H = atan ( sin(lat) * tan (lha))
function hdial(lat,long,hr)
    hdial = atn((sin(lat*2*3.1416/360) * tan(hr*15*2*3.1416/360)))*360/(2*3.1416)
end function
' Shrink to 2 significant digits
function shrink(a)
    shrink = int(a*100)/100
end function
' *** END ***
```


## SOME NOTES ON PROGRAMMING IN "C" (CPP or C++)

There is a free C or C++ compiler and IDE available with which this program has been tested.

- http://www.windows8downloads.com/win8-dev-c--wdoxnrth/
- http://sourceforge.net/projects/orwelldevcpp/?source=dlp

The release works well on Windows 8:
Dev-Cpp 5.5.3 TDM-GCC x64 4.7.1 Setup.exe

A program for horizontal dials is shown below. This is a conversational program and does not use graphics nor a GUI (graphic user interface). This does not use object oriented features, it is as simple as it can be. Object oriented code is available on the website.

```
#include <stdio.h>
#include <math.h>
// http://www.bloodshed.net/devcpp.html about the C products
// http://www.bloodshed.net/dev/devcpp.html download options
#define degtorad ((2 * 3.1416) / 360)
#define radtodeg (360 / (2 * 3.1416))
int main () // main procedure
    int i , ii ; // working integers
    float j; // working float
    float lat; // latitude and
    float sinlat; // and sin of latitude
    float hlat ; // hour line angle
    float lng; // longitude
    float ref; // reference meridian
    float corh; // correction for longitude
    float hla; // hour line angle
    printf ("*------------- Horizontal Dial ---------------\\n");
    printf ("www.illustratingshadows.com cpp-hdial.cpp\n");
    printf ("*----------- March 7, 2007 0838------------\n");
    printf ("Enter latitude
    scanf ("%f", &lat) ;
    printf ("Enter longitude ");
    scanf ("%f", &lng);
    printf ("Enter legal meridian ");
    scanf ("%f", &ref);
    printf ("Design latitude: %7.2f \n", lat);
    printf ("Design longitude: %7.2f \n", lng);
    printf ("Reference meridian: %7.2f \n", ref);
    corh = (4*(lng-ref)) / 60 ;
    printf ("Correction to shadow %7.2f hours \n", corh);
    printf ("Correction to shadow %7.2f minutes \n", corh*60);
    printf ("\n");
    printf ("Corrected hour line angles \n" ) ;
    printf ("morning hours first\n");
```

```
    for (i = - 6; i <7 ; i++ )
    { if (i==0) {printf ("\nnoon\n");};
    sinlat = sin( degtorad*lat ) ;
    // get the hour angle of the sun
    ii = (-1)*i ;
    j = 15 * (ii+corh) ;
    /* get the resulting hour line angle ~ atan(sin(lat)*tan(hr*15) */
    hlat = sinlat * tan(degtorad*j) ;
    /* get the hour line angle back to degrees */
    hla = radtodeg*( atan(hlat) ) ;
    printf ("Hour: %3i hour angle: %6.2f Angle: %6.2f \n", i, j, hla );
    }
    printf ( "afternoon hours last\n" );
    printf ( " \n");
    printf ( "*** END *** (enter any letter and return to quit)\n") ;
    scanf ("%f", &ref); // a pause using any old variable
    return 0;
}
```

"C" was the original language in the "C" series of languages. It was structured, in that the IF THEN program flow was strongly supported, and the GOTO was diminished.

As programming moved from sequential code towards event driven code, the concept of "objects" as opposed to simple variables or structures of variables came into play. And further, the concept came into existence of building code into the object to handle standard work that these objects might need done for them. These code segments, directly tied to the objects, were known as methods. This concept was used for many years in the IBM mainframe world where interrupt driven code was not new. However, as the UNIX world moved into more mainstream activity, the UNIX community considered these to be new concepts.

Be that as it may, the development of objects and their methods was formalized as a concept in the development of $\mathrm{C}++$, a language derivative of " C ". The object and its methods were abstracted further into classes.

A class was a definition of an object, or rather what it would be like. Thus a class was more of a dictionary, and the objects themselves were constructed from that dictionary. This was seen in the DeltaCAD definition of a human interaction window, and its subsequent generation or construction, and its subsequent use.

However, in a true object oriented system, the methods became inter-related. That interaction allowed simple events to cause the updating of many objects. One example is the Windows display of a folder of files, and how that display is updated automatically when in another window a file is added or dropped from that folder.

## SOME NOTES ON PROGRAMMING IN FORTRAN

A free FORTRAN compiler/ linker is available at::
And notes about it are available at:
And the compiler download link is at:
click on the INSTALLER link
http://gcc.gnu.org/fortran/ http://gcc.gnu.org/wiki/GFortran http://gcc.gnu.org/wiki/GFortranBinaries

The compiler is about 14 mb , and is a compiler and a linker, based on command line options.
A good language manual is available at:
http://h21007.www2.hp.com/dspp/files/unprotected/Fortran/docs/lrm/dflrm.htm
Place the source in "C:\Documents and settingsluser name" otherwise the RUN, COMMAND will not be able to access the directory, unless placed in the drive's root folder, such as C:IZFTN

In windows XP do RUN, COMMAND, and do a compile first, the following .BAT file is a poor man's IDE:-

```
edit program.txt
gfortran -c -x f95 program.txt
pause
gfortran -o executable program.o
program
```

edit fort. $1 \quad[$ look at the output ]

The program output for a certain dial was:-

The program output that was tested was in a file called "fort.1" and is shown below:-

```
Latitude: 32.75
Longitude: 108.20
Reference: 105.00
hour.corr: 0.21
hour hr.ang line.angle
morning hours
    -6 -93.20 84.10
    -5 -78.20 -68.88
    -4 -63.20 -46.96
    -3 -48.20 -31.18
    -2 -33.20 -19.49
    -1 -18.20 -10.09
noon (-3.20 -1.73
noon
    1 11.80 6.45
        2 26.80 15.28
        3 41.80 25.81
        4 56.80 39.58
        5 71.80 58.71
        6 86.80 84.10
afternoon hours
www.illustratingshadows.com
```



## SOME NOTES ON PROGRAMMING IN PASCAL

There are three free Pascal compilers available with which the Illustrating Shadows programs have been tested and work. Both of these systems have been used with the programs on this web site as well as on the CD provided with these books.
www.bloodshed.net/devpascal.html
an 8 mb downloadable fully integrated development environment (IDE) whose windows are more compatible with Widows
www.freepascal.org/download.var
a 28 mb downloadable fully integrated development environment (IDE) whose windows are more compatible with DOS. This has some useful documentation files usable in both versions of the Pascal IDE.
rtl.pdf Run time library Has the functions and the required "uses" for them ref.pdf Reference Has the language structures such as IF... and so on

Some trigonometric functions need the MATH library, some use the default or SYSTEM library. This is why the program says "uses math;", however the default "SYSTEM" library has no such "uses" statement since it is implied.

An excellent online tutorial is found at http://www.taoyue.com/tutorials/pascal/contents.html
This has better examples of some of the useful things not well explained in RTL.PDF or REF.PDF

A GUI Pascal system with programmable forms is Open Source and called Lazarus. The "alldials" program as well as the IBM 1401 and 360 simulators were written using Lazarus.

The vertical dial which is longitude corrected and supports wall declination, somewhat stripped down, is shown below. There are some complex IF statements whose syntax is critical.

This is a conversational program and does not use graphics nor a GUI (graphic user interface). This does not use object oriented features, it is as simple as it can be.

```
program VerticalDial;
{ Educational purposes only }
uses math;
var lng : single ;
    ref : single ;
    corh: single ;
    lat : double ;
    dsw : single ; dec : single ;
    zhr : single ;
    sd : single ;
    sh : single ;
    hla : float ; { hour line angle itself }
var xxx : string ;
var i : integer ;
    ii : integer ;
    j : float ;
```

```
begin
    writeln ('*------------ Vertical (dec) Dial ---------------*');
    writeln ('www.illustratingshadows.com v-dec-dial-pgm.pas');
    writeln ('*------------ March 6, 2007 0903 --------------*')
    writeln ('Enter latitude ');
    readln (lat);
    writeln ('Enter longitude ');
    readln (lng);
    writeln ('Enter legal meridian');
    readln (ref);
    writeln ('Enter declination (+ve S..W, -ve S..E)');
    readln (dsw);
    writeln ('Design latitude: ', lat:6:2);
    writeln ('Design longitude: ', lng:6:2);
    writeln ('Reference meridian: ', ref:6:2);
    writeln ('Correction to shadow ', 4*(lng-ref):6:2, ' minutes.');
    corh := (4*(lng-ref)) / 60 ;
    writeln ('Correction to shadow ', corh:6:1, ' hours.');
    writeln (' ');
    { a complex IF statement }
    { display the style distance SD and style height SH }
    if dsw = 0 then
        begin
        sd := 0 ;
        sh := 0 ;
        writeln ('non declining dial') ;
        end
    else
        begin
        sd := radtodeg(arctan(sin(degtorad(dsw))/tan(degtorad(lat)))) ;
        sh := radtodeg(arcsin(cos(degtorad(lat))*cos(degtorad(dsw)))) ;
        writeln ('this is a vertical declining dial') ;
        writeln ('SD [style distance from vertical]: , sd:5:2);
        writeln ('SH [style height fr vertical dialplate]: ' sh:5:2);
        end ;
    { end of the IF statement }
    lat := 90 - lat ; { make co-latitude }
    writeln ('Latitude set to co-latitude as vertical dial: ',lat:6:2);
    writeln ( 'Corrected hour line angles follow' ) ;
    writeln ( 'morning hours first' );
    for i := - 6 to +6 do
    begin
        if i=O then writeln ('noon');
        if i=1 then writeln ('noon');
        { get the hour angle of the sun }
        ii := - 1*i ;
        j := 15 * (ii+corh) ;
        { Vertical decliner dial function }
        { complex IF }
        dec := dsw * -1 ;
        zhr := i - corh ;
        if zhr = 0 then
            begin
            hla := 0 ;
            end
        else
            begin
            hla := radtodeg(arctan((cos(degtorad(90-lat)) /
                        (sin(degtorad(dec))*sin(degtorad(90-
                    lat))+(\operatorname{cos}(\operatorname{degtorad}(\operatorname{dec}))/tan(degtorad(15*(12-zhr)))))))) ;
            end ;
        { end of complex IF }
        writeln ('Hour: ', i, ' hour angle: ' , j:6:2 , ' Angle: ', hla:6:2 );
    end ;
    writeln ( 'afternoon hours last' );
    writeln (' ');
    writeln ( '*** END ***') ;
    readln (xxx);
end.
```


## LAZARUS ~ OPEN SOURCE VARIANT OF DELPHI (A PASCAL IDE)

Lazarus is a Pascal IDE (Integrated Development Environment), open source, running on multiple operating systems. Applications can be based on user defined forms.

Lazarus site:
wiki link:
download link:
use the Win32 file:
and NOT:
www.lazarus.freepascal.org/
wiki.lazarus.freepascal.org/
select: Lazarus Windows 32 bits
lazarus-0.9.26-fpc-2.2.2-win32.exe
lazarus-qt-0.9.26-fpc-2.2.2-win32.exe
as you will get very frustrated trying to locate: qtcore4.dll

The download is one single file, and it installs and runs first time. Lazarus executable programs can be very large as they have debugging information stored in them. For Windows Vista win64 systems the only compression available is to use Winzip or a similar program. For win32 versions which run on Windows Vista win64 as well as on 32 bit systems, STRIP and UPX together will reduce an executable's size from around 12 mb to under 1mb. STRIP and UPX will not compress a win64 executable. Sometimes Lazarus waits after RUN on Vista (32 bit version), click RUN, RESET DEBUGGER and it will work the next time.

Lazarus can be compared to DELPHI, which is 332 mb , and the pre-reqs another 234 mb . Delphi can be found at:http://www.turboexplorer.com/delphi

Lazarus is an Open Source system, based on PASCAL, and is somewhat compatible with Delphi. One of the shortcomings of modern object oriented languages is their type conversion issues. The graphical program for sundials highlights this problem, namely converting from floating point (graphical coordinate calculations) to integer (graphical package needs) requires an intermediate string conversion! Harking back to IBM's PL/I language, some lessons that the new language developers could learn emerge. First: PL/I had as one of its values the concept that if a programmer could write something that made sense to him or her, then the PL/I compiler should also be able to make sense of it also. All these newer languages or language adaptations are very weak on real world needs of commercial programmers, and seem to be more suited to those who delight in getting around complexities of a language. LAZARUS is no exception, and the documentation is designed for those who already know the system. Second: PL/I had the ability for almost any data type to be converted implicitly either at compile or at execution time, so that a programmer could take in a string of characters that contained numbers and implicitly convert them to an integer, or floating point number, and vice versa.

Illustrating Shadows has several programs written using Lazarus. The IBM 1401 and IBM System 360 simulators, language assemblers, and debugging aids were written using Lazarus. An "ALLDIALS.EXE" program was also written using Lazarus. The CD with the Illustrating Shadows books has detailed notes on programming with Lazarus, and the general principles are similar to Visual Basic. Lazarus code is more portable than the Visual Microsoft products. ALLDIALS is a good example of graphics programming in Lazarus.


## SHORT REVIEW ON PROGRAMMING Excel SPREADSHEETS

IMPORTANT NOTE: Open Office is a free Microsoft Office equivalent. In the case of Excel, the syntax for formulae and the conditional formatting are not the same.

Angular measure uses radians, there are 2 * pi radians in a circle of 360 degrees. The trigonometric functions SIN, COS, TAN, COTAN, and so on, need the radian conversion first. Any function that returns an angular measure, such as ATAN, ACOS, ASIN, also printed as TAN ${ }^{-1}$, $\mathrm{COS}^{-1}, \mathrm{SIN}^{-1}$, would need to be converted back to degrees using the DEGREES function. The number of significant digits after the decimal point is controlled by the FORMAT, CELLS, NUMBER option. Some functions do rounding of their own. The INT (integer) function returns the integer part of a number on the left side of the decimal point, rounds down. And the number below 5.9 is 5 , however the number below -5.1 is -6 because this is already in the negative scale. However, the ABS (absolute) function which removes the sign may be used if the above rounding down on the negative side is not desired. If the sign must be retained, then the SIGN function can be used. Sometimes the cell formula can get vary large and out of hand. If you make an error on entering a formula, the spreadsheet will offer a correction. Be very careful of accepting their suggestion. It is best to match the parentheses yourself. If a formula gets too large, consider using intermediate columns, or cells, that hold intermediate data. Intermediary values are usually kept off to one side, the right side in this case, and used as working accumulators for the final value. One can hard code cell references using the \$column\$row syntax. And finally, when you choose a formula, from whatever source, always test it at extreme as well as in between values. For example, there are two versions of the formulae for azimuth in publication, only one works for all hours. Bellow a simple spreadsheet is depicted with comments.


In the above, the top left cell is B8, the top right cell is B12. Cells B11~D11, B12~F12, and B13~B14 are shown in italics purely for reference.

## SOME MORE NOTES ON DRAWING AND BUILDING WITH TURBOCAD

TurboCAD as such does not have a readily available programming system, its strength lies in its 3d abilities. These notes review key concepts that trip up new users. First, a review of 3d.

With TurboCAD, a key concept is the work plane. Things happen on THE work plane. For example, if you wished to place an object on the slope of the wedge, you could use the original work plane, create the object, tilt it, move it higher above the original plane, and hope. Or you could change the work plane to match the facet, i.e. the slope, and then use the new work plane.


The WORKSPACE, WORK PLANE, BY FACET was used with the mouse going close to the slope, and upon clicking the facet, the work plane and the grid lines changed. The new object, a cylinder, was built and it aligned perfectly with that slope.


Another confusion point is moving an object, and which coordinates are used. The cylinder object was selected and moved along the X axis, the default is that the entire system's coordinates are used. Thus the cylinder displaces along the entire system's $X$ axis and not on the object's $X$ axis. The shift was achieved with the POS-X field on the bottom of the TurboCAD screen while the object itself was selected.


Another confusion point is rotating an object. Rotating around the X axis is done by grabbing the Y axis handle and rotating. The Y -axis uses the Z , and rotating around the $Z$ uses the $X$ axis handle.

Lighting in 3d is another problem area. With nowhere to "hang" a light, it is very hard to provide good lighting. Key points are that a light when added is not white, it is grey. Second, a light can be anchored to an object such as a solar mesh as depicted to the right, with LOCAL SNAP and ANCHOR ON FACET, and the light's focus can also be anchored somewhere. The result is calendar and
 time accurate lighting. Use:- solarMesh.tcw shown to the right.

Several chapters in this book cover much more extensively the advanced 3d techniques such as 3d subtract, intersection, and 3d add. Of course, 3d add should not be confused with a similar beneficial concept of the group, and both should be thoroughly understood.

TurboCAD 2d drafting is simplicity itself. The system does not have a programmable interface such as DeltaCAD. However its final drawings are very well formed. And 2d pictorials can be converted to solid 3d objects and rotated in 3d. The only area of confusion lies in specifying a base for an angular measure and the direction that angle will take. This applies when drawing lines at an angle from a base line, which itself may be at any angle. Measuring angles is simplicity itself with DIMENSION, ANGULAR MEASURE, and then enter ALT-a and then place the mouse at the intersection of the two lines, and then select each of those two lines, and the angle is displayed.

Drawing (as opposed to measuring) a line at an angle to another requires that the angle base as well as direction be established. OPTIONS and then ANGLE opens the control panel for setting the system's defaults.

Assuming a base line from which others will be measured is at say $15^{\circ}$ then the system is given that as a base, and in this example a direction of clockwise.

From then on, lines can be drawn from that angled base line.

Drawing lines clockwise is now simple, and done by completing the line while observing the displayed angle. To draw angles counter clockwise, the OPTIONS, ANGLE panel is used and the counter clockwise box is checked.



A line then drawn at a steeper angle from the original line will show as an angle starting from 0 . This simple method allows any base line to be used, and then any lines to drawn offset by a simple angle, without the need for subtracting the desired angle from 360. This simple process saves much frustration. Alternatively lines can be rotated, which has the benefit that an angle can then be typed in.

Another problem area is selecting and resizing any object. The aspect ratio must be set and double checked by a right click on the selected object. Otherwise hours of frustration may ensue.

The manual provided with TurboCAD is extremely thorough, however the above insights may make it a lot easier to use the system. Sadly, that same book has little on their SDK (programming system), see the next two pages.

## SOME NOTES ON PROGRAMMING TURBOCAD USING THE SDK (VBS)

TurboCAD, like DeltaCAD, offers a programming methodology. However, rather than use one third party system as does DeltaCAD, TurboCAD elected to use the set of languages available from Microsoft. Further, the interface is only available with the Professional version, not with the more affordable Deluxe installation. Thus, DeltaCAD is likely to remain the CAD system of choice for the average diallist, and has probably passed critical mass for that reason.

The TurboCAD VBS (Visual Basic Script) macros are to be stored in:c: 1 Program Files

IMSIITCWP11 or later release, such as TCWP12 SDKISamplesIVBSIWshScriptPack (Wsh means: GUI Windows Script Host )

In current versions of TurboCAD Professional (version 11 and later), folders are found for:-
C\#, CPP, DELPHI, VB NET, VBASIC, VBS, vc, and finally vcnet
Several problems come to light at this juncture.

1. Microsoft languages come and go, and the languages themselves are not stable as they are enhanced, and some features are dropped.
2. Most have either an involved build processes or collections of libraries that must go with the program, VBS (visual basic script) is luckily an exception.
3. While TurboCAD has an excellent book for the hands on CAD user, the two or three pages referring to their SDK are in no way enlightening.
4. The computer readable documentation is of use to someone already knowing the system. There is no obvious cross reference between simple CAD functions and their calls, functions, or methods. Thus finding how to draw a line is not intuitively obvious.
5. Luckily there are some third party web sites that are a great help.

While VBS is the slowest interface, it does not require a host of installation libraries, nor a complex build. Instead, a simple text editor (WordPad for example) can build the program and save it in the appropriate folder, and in TurboCAD Professional, clicking the MACRO tool allows the user to then click on FILE FOLDERS, and from thence the VBS program (script) may be run.

These scripts are executed by loading TurboCAD Professional, with a NEW drawing area, and clicking on the macro tool (if visible), or if not visible, by clicking on VIEW, MacroRecorderPalette. Then locate the file folder, and if needed, set the file type to VBS (so you can see the scripts or macros).

which brings up a sub panel, click on files

and then

click on the file you wish to run. NOTE: the drawing area must exist, so do a NEW when bringing up TurboCAD, otherwise the following Windows Script Host message appears.


Useful references sources are:-
http://msdn2.microsoft.com/en-us/library/ms950396.aspx
http://www.tyharness.co.uk/tcvbacrashcourse/tcvbacrashcourse.htm
http://www.tyharness.co.uk/cad.htm
tcsdk.chm in the Docs folder of the TurboCAD SDK, is not without merit. Cryptic at times but it can provide clues. And the SEARCH function, once a function or method has been identified, is very helpful.

## VBS IN TURBOCAD - KEY POINTS TO KNOW

The implementation is object oriented. Thus a graphics area is acquired, and things added to that area, or, object within that area. And objects have methods (eg: drawing a line) and properties (eg: methods to alter the appearance of a line)

```
InputBox get a line of input (forms are more involved)
AddLineRectangle adds a rectangle
AddCircleCenterAndPoint adds a circle
AddLineSingle adds a line
AddText adds text into the drawing
```

graphic elements built, such as a line, have methods (properties) that can be invoked Properties("Penstyle").Value $=$ "DASHED"
Properties("Pencolor").Value $=\operatorname{RGB}(0,255,0)$

Other functions exist including trigonometric functions, however ASN (arcsin) has to be programmed by the programmer, as does the "to" and "from" degrees/radians. This is a common deficit in many BASIC based systems.

The CD with this book, and this book's web site have functioning dial plate TurboCAD scripts.

## TURBOCAD PRO AND THE "PARAMETRIC PART SCRIPTING"

TurboCAD has several programmable options. One uses VBS (Visual Basic Script). The current versions of TurboCAD (not 15.1 nor earlier), supports Parametric Script entities. In essence, a script is written that inserts an entity into a drawing, and from then on, that entity may be selected and its parameters may be modified to generate a whole new layout.

The Parametric Script language is interesting in that the "Output" statement is executed last, and other statements wherever they may be, are executed first. Case is critical, thus "output" is not executable, "Output" is the correct spelling. Trigonometric functions use degrees, however there is no INT nor ABS provided.

Some documents are important to have. As happens with IMSI, documentation is not made obvious. Searches of the internet are essential for clues to a new feature, and the parametric scripting feature is no exception.

Essential notes on parametric script building:
This explains how the editor marks syntax errors:-
http://www3.turbocadcommunity.com/tiki-index.php?page=PPM+Scripting
http://www3.turbocadcommunity.com/tiki-index.php?page=PPM+Scripting+Reference
Useful but incomplete notes on parametric scripts:-
http://downloads.imsidesign.com/HelpFiles/Scripting\ Parametric\ Parts\ Feb\ 29,\ 2008.pdf
It is only through the above two documents that scripts can easily be written. The Parametric Script Editor is invoked by "VIEW" and then "Parametric Script Editor Palette", and the script can be easily typed, or loaded from a file in any folder.

The Editor uses color coding:

| black | variables |
| :--- | :--- |
| brown | text |
| blue | reserved words |
| magenta | is an error |
| The status line of the palette indicates the error reason for the error. |  |
| green | comments |


"Output(plate);" has "plate" in magenta and thus says "plate" is wrong. Plate has a syntax error in that two ")" are missing.


Once corrected, then "Output(plate);" will show in normal text.


A fabulous feature of these script created objects is that once a parametric part has been built on the drawing area, it can be selected (as with any entity) and then VIEW, SELECTION INFO will enable you to see and change those parameters (set with the "Parameter" statement).


The parameters were changed from lat 52, long 108.2 and 105 to a new set of $32.75,101.3$ and 105. And the entire drawing redrew itself.

## Typical output from a script, and accuracy

The TurboCAD Parametric Script included here generated the following dial layout. The hour line angles were measured with INSERT, DIMENSION, ANGULAR, then right click and select ANGLE NODE and then follow the prompts.


h -dial and calendar using gnomon linear height
Lat: 32.8 d.Long: 03.2
$\begin{array}{lllllllllllll}6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18\end{array}$ $\begin{array}{lllllllllllll}84.1 & -68.9 & -47.0 & -31.2 & -19.5 & -10.1 & -1.7 & 06.4 & 15.3 & 25.8 & 39.6 & 58.7 & 84.1\end{array}$

Hours below horizontal use the 90 reference line below horizontal.

The angles are very close to the same dial plate's hour line angles produced by another CAD system, DeltaCAD.

## The Parametric Script (program)

```
// HORIZONTAL DIAL ~ This seems to test correctly, but use as
// a basis for your own code. Not guaranteed.
// ~ TCAD-hDial-3-adv.ppm [advanced h-dial]
// www.illustratingshadows.com Simon Wheaton-Smith Dec 10, 2008
//
//
// You enter lat, lng, and ref in the three parameter statements below
//
lat = Parameter("Latitude", 32.75, ANGULAR, Interval(20,80));
lng = Parameter("Longitude", 108.2, ANGULAR, Interval(0,360));
ref = Parameter("LegalStd", 105.0, ANGULAR, Interval(0,360));
//
// you change the values of latitude, longitude, and reference
// longitude above
// -----------------------------------------------------------------
// state the vertical and horizontal limit size
xyl = 10;
// and the multiplier for the outer border box
scl = 0.2;
scz = 1+scl;
// derive the hours of correction for hour lines
dif = 4*(lng - ref)/60;
// define the hours
y06 = atan(sin(lat)*tan(15*(6+dif)));
ha07 = atan(sin(lat)*tan(15*(5+dif)));
    x07 = IF( (ha07)>=45, -xyl, -xyl*tan(ha07) );
    y07 = IF( (ha07)>=45, xyl*tan(90-ha07), xyl );
ha08 = atan(sin(lat)*tan(15*(4+dif)));
    x08 = IF( (ha08)>=45, -xyl
    y08 = IF( (ha08)>=45, xyl*tan(90-ha08),
ha09 = atan(sin(lat)*tan(15*(3+dif)));
    x09 = IF( (ha09)>=45, -xyl, -xyl*tan(ha09) );
    y09 = IF( (ha09)>=45, xyl*tan(90-ha09), xyl );
x10 = atan(sin(lat)*tan(15* ( 2+dif)));
x11 = atan(sin(lat)*tan(15* ( 1+dif)));
x12 = atan(sin(lat)*tan(15* ( 0+dif)));
x13 = atan(sin(lat)*tan(15* (-1+dif)));
x14 = atan(sin(lat)*tan(15* (-2+dif)));
ha15 = - atan(sin(lat)* tan(15*(-3+dif))) ;
    x15 = IF( (ha15)>=45, xyl, xyl*tan(ha15) );
    y15 = IF( (ha15)>=45, xyl*tan(90-ha15), xyl );
ha16 = -atan(sin(lat)*tan(15*(-4+dif))) ;
    x16 = IF( (ha16)>=45, xyl,
    y16 = IF( (ha16)>=45, xyl*tan(90-ha16), xyl );
ha17 = -atan(sin(lat)*tan(15*(-5+dif))) ;
    x17 = IF( (ha17)>=45, xyl,
                                    xyl*tan(ha17) );
    y17 = IF( (ha17)>=45, xyl*tan(90-ha17), xyl );
y18 = atan(sin(lat)*tan(15* (-6+dif)));
// display the dial plate
plate=Polyline(
        Point(0,0) , Point(-xyl, xyl*tan(90-y06)), // 0600
        Point(0,0), Point( x07, y07) , // 0700
        Point(0,0) , Point( x08, y08) , // 0800
        Point(0,0) , Point( x09, y09) , // 0900
        Point(0,0) , Point(-xyl*tan(x10),xyl) , // 1000
        Point(0,0) , Point(-xyl*tan(x11),xyl) , // 1100
        Point(0,0) , Point(-xyl*tan(x12),xyl) , // noon
        Point(0,0), Point(-xyl*tan(x13),xyl) , // 1300
        Point(0,0) , Point(-xyl*tan(x14),xyl) , // 1400
        Point(0,0) , Point( x15, y15) , // 1500
```

```
    Point(0,0) , Point( x16, y16) , // 1600
    Point(0,0) , Point( x17, y17) , // 1700
    Point(0,0), Point(xyl, -xyl*tan(90-y18)), // 1800
// now the inner boundary box
    Point(0,0)
    Point(-xyl,0)
    Point(-xyl,xyl) ,
    Point(xyl,xyl) ,
    Point(xyl,0)
// back at 0,0
    Point(0,0)
    );
//
box=Polyline(
// now the outer boundary box
    Point( 0, -scl*xyl) ,
    Point(-scz*xyl,-scl*xyl) ,
    Point(-scz*xyl, scz*xyl) ,
    Point(scz*xyl, scz*xyl) ,
    Point(scz*xyl, -scl*xyl) ,
    Point( 0, -scl*xyl) );
//Output (plate,box);
Output (plate);
Output (box);
//
```

There are several problems with the above script, one is that it does not handle shifting from a fixed $x$ or $y$-value with a generated $x$ or $y$-value, except for 0700-0800 and 1500-1700 hours.

Since few diallists use TurboCAD, and fewer still use the TurboCAD Professional version, the effort to go to those extremes was not made.

The final display of the above program is shown to the right.

NOTE: If you save a TCW file with an entity created with these scripts, and later open it, you can modify the parametric parms for the object and it will redisplay itself This has the potential for being a significant benefit beyond other CAD systems.


## Older methods ~ DL, Nomograms, Slide rules, and Dialing scales

## The long forgotten "DL" or difference in longitude of the vertical decliner

In some earlier books the term DL was derived for the vertical decliner. Such books were written when calculators weren't around but slide rules and nomograms were. It made sense to display a formula in logarithm form. Logarithms, or logs, are a means whereby multiplication is done by addition, and division by subtraction. The horizontal dial formula for an hour line's angle (hla):-

| hla | $=\operatorname{atan}(\sin (\mathrm{lat}) * \tan ($ hour angle) ) | whi |
| :---: | :---: | :---: |
| tan(hla) | $=\quad \sin (\mathrm{lat}) * \tan ($ hour angle) | becomes when using logs... |
| log tan(hla) | $=\log \sin ($ lat $)+l o g \tan ($ hour angle) |  |

Logs make nomograms easy to develop for sundials, and help people who have no calculators use simple addition or subtraction in place of multiplication and division. Tables are used to get the log of a number, and its reverse the anti-log as well. The vertical decliner can be created using a formula based on a surrogate equatorial dial, the most common formula in use is:-

$$
\text { hla }=\operatorname{atan}(\cos (l a t) \quad /(\cos (d e c) * \cot (h a)+\sin (d e c) * \sin (l a t)) \quad)
$$

a formula producing the same results but based on a surrogate horizontal dial's hour lines is:-

$$
\text { hla }=\operatorname{atan}(\sin (\text { hdialsHla }) / \sin (90-\text { hdialsHla }- \text { dec }) * \tan (\text { lat }) \quad)
$$

or

$$
\text { hla }=\operatorname{atan}\left[\frac{\sin (\operatorname{atan}(\sin (\text { lat }) * \tan (\text { lha }))}{\tan (\text { lat }) * \sin (90-\operatorname{dec}-\operatorname{atan}(\sin (\text { lat }) * \tan (\text { lha })))}\right]
$$

But another alternative exists. A horizontal dial with a latitude equal to the vertical decliner dial's SH can be slid over the vertical decliner's dial plate such that 12 noon local apparent time on the surrogate horizontal dial plate matches the SD on the real vertical decliner. This process is discussed in chapter 16 (vertical decliner), chapter 23 (calendar or declination curves), and chapter 25 (the analemma). The hour lines on the real vertical decliner, and the hour lines on the pseudo horizontal surrogate dial will not normally match. However they can be made to match by adjusting the hour lines on the surrogate dial plate to match those on the vertical decliner, this is performed by using a longitude adjustment. That adjustment has little to do with the real vertical decliner's design longitude. It is a value that will simulate a longitude enabling the surrogate dial's hour lines to match the real dial. The surrogate dial's latitude is of course SH (style height). This magical longitude difference is called "DL" or difference in longitude, and is derived from a simple formula from the vertical decliner (see derivation in chapter 16). NOTE: SD is an hour line angle, and DL is a surrogate equatorial dial's hour angle that produces that DL .

$$
\text { DL }=\operatorname{Atan}(\operatorname{Tan}(\operatorname{dec}) / \operatorname{Sin}(\text { lat }) \quad)+\text { vDecLongitude }- \text { legalMeridianLongitude }
$$

Use this DL as the surrogate dial's longitude and reduce its magnitude by 15 degrees if it was larger than 15, until the result is 15 degrees or less. And each reduction of 15 degrees will require the hour line labels to be adjusted by one hour, since one hour is represented by 15 degrees. Then use a legal meridian of 0 in the horizontal dial plate process. Since the hours on a horizontal dial rotate in the opposite direction to a vertical dial (chapter 13), the sense of the angles must be reversed. The resulting hours will then produce hour lines that match the real vertical decliner. The nomogram for a vertical decliner uses this method since the "DL" formula has two terms compared to many more for the other formulae. "DL" is an almost forgotten value, and yet it still has a use and should be remembered. NOTE: a vertical dial of latitude " $90-\mathrm{SH}$ " can be used in place of a horizontal dial, when the sense of the hours would not need reversing.

## The nomogram and the sun dial (sundial nomograms are in the appendices)

The nomogram, popularized in 1880 by Philbert d'Ocagne, has many forms. The simplest is two variables represented by vertical lines left and right, with the solution represented by a vertical line mid way between the two (method 1a below). There are a number of web sites that discuss nomogram theory and design. The JAVA alternative Python has software for drafting nomograms. Illustrating Shadows provides DeltaCAD and Lazarus/Pascal nomogram aids for dialing needs.

Nomograms are graphical solutions to mathematical problems. The problem inputs and the solution are represented by values placed along lines or curves. The derivation of the shapes and the marking of values on the lines are based on rules of triangles. For the purposes of sundials and other simple problems, certain nomogram shapes are best for certain kinds of formulae. For the purposes of explanation, "L" means the left line, "R" the rightmost line, and these are both inputs, while " $C$ " represents the central line which provides the solution.

$C=L+R$
can multiply and divide with logs

$C=L / R$
logs not needed

$C=L \times R$ logs not needed

$1 / C=1 / L+1 / R$ logs not needed

The method 1 nomograms above do not show multiplication. Since multiplication can be done by adding the "log" or logarithm of the numbers, method 1 can perform multiplication if log values are used. Similarly division can be done by subtracting the log of the numbers so again method 1 can be used. Method 2 does not use logs since division is implied, and method 3 implies multiplication, so again, logs are not used. The Illustrating Shadows DeltaCAD macro "nomogram.bas" uses natural or Naperian logs based on 2.71828 since the base 10 log function did not perform as the documentation suggested, the Lazarus/Pascal version uses base 10.

Method 1a: A horizontal dial, two inputs (latitude and time), one solution (hour line angle): the nomogram's general formula: $C=L+R$
$\begin{array}{ll}\text { hour line angle } & =\text { atan }(\sin (l a t) * \tan (\text { hour angle })) \\ \log \tan (\text { hla }) & =\log \sin \text { lat }+\log \tan \text { hour angle }\end{array}$

In the example to the right, the horizontal dial nomogram was derived manually.

The left scale is the logarithm of the sine of the latitude, the right is the logarithm of the tangent of the hour from noon (times 15 of course), with the center vertical line being the logarithm of the tangent of the hour line angle on the dial plate, compressed by a factor of two. The axes are labeled with latitude, hour, and hour line angle (functional scales) and not their logarithmic equivalents (natural scales). The end result is a nomogram whereby a line can be drawn from the latitude (left) to the hour from noon (right) and the dial plate's hour line angle read in the middle.

A sample line for latitude 32 and 0900 or 1500 is shown, and the hour angle in the middle is seen to be close to 27.92 degrees which it should be.


This is not longitude corrected, you make the longitude correction before using the tool by converting the longitude difference to an hour angle adjustment. The Hour Line Angle and the Hours From Noon scales get smaller from the bottom as you move up, but then get bigger again. This is because initially the TAN is slowly increasing while the log decreases more rapidly, however later, the TAN increases far more than the log decreases. The first step in building nomogram is to look at the extremes of the three variables because they must be placed on the nomogram's vertical lines. Their magnitudes should be reasonable and should be selected so that extremes are omitted if the nomogram scales would be impractical.

| "L" line |  | "R" line |  | "C" line |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| LATITUDE | log sin(lat) | HOURS FF | log $\tan (\mathrm{hr})$ | HOUR LINE ANGLE | log(tan) of resulting hour line angle |
| 0 | \#NUM! | 0.00 | \#NUM! | 0 | \#NUM! |
| 1 | -1.76 | 0.25 | -1.18 | 1 | -1.76 |
| 2 | -1.46 | 0.50 | -0.88 | 5 | -1.06 |
| 5 | -1.06 | 0.50 | -0.88 | 10 | -0.75 |
| 10 | -0.76 | 0.75 | -0.70 | 15 | -0.57 |
| 15 | -0.59 | 1.00 | -0.57 | 20 | -0.44 |
| 20 | -0.47 | 1.25 | -0.47 | 25 | -0.33 |
| 25 | -0.37 | 1.50 | -0.38 | 30 | -0.24 |
| 30 | -0.30 | 1.75 | -0.31 | 35 | -0.15 |
| 35 | -0.24 | 2.00 | -0.24 | 40 | -0.08 |
| 40 | -0.19 | 2.50 | -0.12 | 45 | 0.00 |
| 45 | -0.15 | 3.00 | 0.00 | 50 | 0.08 |
| 50 | -0.12 | 3.50 | 0.12 | 55 | 0.15 |
| 55 | -0.09 | 4.00 | 0.24 | 60 | 0.24 |
| 60 | -0.06 | 5.00 | 0.57 | 65 | 0.33 |
| 65 | -0.04 | 6.00 | 16.21 | 70 | 0.44 |
| 70 | -0.03 |  |  | 75 | 0.57 |
| 75 | -0.02 | NOTE: $\log$ | (0) is something you | 80 | 0.75 |
| 80 | -0.01 | neve | get | 85 | 1.06 |
| 85 | 0.00 | 倍 | versus | 90 | 16.21 |
| 90 | 0.00 | NOTE: a lo | g(1) results in 0 |  |  |

$\begin{array}{lll}\text { "L" left line: } & \text { INPUT 1: The practical latitude range is: } & 0.00 \text { to }-1.76 \\ \text { " } R \text { " right line: } & \text { INPUT 2: The practical hour range from noon is: } & -1.18 \text { to } 0.57 \\ \text { " } \mathrm{C} \text { " center line: } & R E S U L T: \text { The practical dial plate's hour line angle range is } & -1.76 \text { to } 1.06\end{array}$
NOTE: The "C" scale is actually compressed in half, see NOTE below.
NOTE: $\log$ TAN(hour line angle) is used, see the formula development below.
hla $=\operatorname{atan}(\sin$ (lat) * $\tan$ (hour angle) ) thus
$\tan (\mathrm{hla}) \quad=\sin (l a t) * \tan ($ hour angle $)$ thus
$\log \tan (\mathrm{hla}) \quad=\log \sin (\mathrm{lat})+\log \tan ($ hour angle)
The next step is to build a set of three parallel lines, equidistant for simplicity, with equally spaced number scales (natural scales) that cover the ranges of the inputs and the output. Thus a nomogram with three lines ranging from -1.76 to 1.06 would be appropriate. All three have 0 in the same place. BUT NOTE: The center line, when equidistant between the other two lines, has its scale compressed in half, i.e. the center solution scale can accommodate numbers twice as large as can the other scales. This is because the center scale is the sum of the two outer scales.

NOTE: The natural scale is linear, and we use the log of our value on that line, make a marker for that logarithm, but label it with the original number (input) or final number (result), i.e. 30 for latitude 30, being the functional scale.

Some key points must be reemphasized at this time.

First, for these type1 and type 1 a nomograms, they all have a raw data baseline of 0 starting at the same horizontal place on all three lines. This is shown to the right.

Second, the center scale is compressed in half when equidistant between the left and right lines.

Third, when looking at the horizontal, vertical, or any other nomogram, why does "0" not seem to be in the same place on all the lines? The answer was given on the preceding page, however it will be reiterated. The label on a point on a line may not be the data point itself. For example, assume the input is for "latitude".

Latitude 0 becomes sin(lat) which is also 0 , but the logarithm of 0 is unusable.

So a latitude is selected of say latitude 1. The $\sin (1)$ becomes 0.017452 . And the $\log$ of that is usable. In the case of DeltaCAD and the macros using natural logs, the log base 2.71828, and the key point is to use logs, not what their base is. And the log is what is used along the vertical scales, because it is the logs we want to add, and we want to add the logs because that is how we multiply.

Even so, we do not print the log value as that is useless, what we print is the original latitude, or whatever.

So the nomogram appears as if the zero values are not in alignment, when in reality they are.

This is shown to the right.
starts at 4 only to demonstrate that its 0 is still with the other scale's 0 .


The above linear scales are the "natural scales", they share the common " 0 ". These scales are seldom displayed.
The below non linear scales are the "functional scales" which do not share "0" except by formula coincidence. Functional scales are actually displayed and used.

HORIZONTAL DIAL www.illustratingshadows.com


What if a nomogram looks too unusable. The next step is to change the line separation, and the scale of one of the lines. In other words instead of equidistant lines with the left and right lines having the same scale, and the center line having half that scale, they can be changed:-


The " $x$ " values are positions along the $X$ scale, for example:-

| $x L$ | $=$ | 0 |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $x C$ | $=$ | so | $a=x C-x L$ | $=0.8$ |
| $x R$ | $=$ | so | $b=x R-x C$ | -1.0 |

and the " $m \mathrm{R}$ " is the scale factor (modulus) of the right line. And with the "a" and "b" separations and the modulus of the scale, "mR", we can derive a scale factor (modulus) for the other two lines. For example, assume the right scale factor is:-

```
mR = 0.5
```

then we derive the modulus (scale factor) for the left and center lines:-

| mL | $=$ | mR | $*$ | $\mathrm{a} / \mathrm{b}$ |
| :--- | :--- | :--- | :--- | :--- |
| mC | $=$ | mL | $*$ | mR |

The above logic allows the scales and line separation to be manipulated to achieve a more usable nomogram. Below are two nomograms for the same problem, the sole difference is line spacing (left to center to right), and a change in one line's scale (modulus). The one thing that cannot be changed is the fact that all scales have a real 0 in common even if not obvious, it is the compression and separation that are manipulated.


Be creative when designing a nomogram with a complex formula. Consider the vertical decliner, one way would be to use one of the vertical decliner formulae:-

$$
\text { hour line angle }=\quad \operatorname{atan}(\cos (l a t) /(\cos (\operatorname{dec}) * \cot (h a)+\sin (d e c) * \sin (l a t))
$$

However a nomogram to deduce the vertical decliner's hour line angles would be a very busy nomogram because of the many terms, in essence several nomograms would be combined. Interestingly, the vertical decliner dial hour lines can be derived another way. One such way is to take the SH (style height), and use that as the latitude of a surrogate horizontal dial. If that is done, then how do the surrogate horizontal dial hour lines become adjusted so they are correct?

The horizontal dial's hour lines are adjusted by an assumed longitude of "DL". This technique is discussed earlier in this chapter and proved in chapter 16. To summarize the analemma notes of chapter 25, "a surrogate h-dial is provided with a latitude of SH and a longitude of DL from the vertical decliner derived figures, with a legal reference meridian of 0 , to be aligned along the vertical decliner's SD. Longitudes greater than 15 are reduced by 15 until the result is 15 or less, while adjusting the hour line labels by 1 for each 15 degrees."


Since a nomogram for a vertical decliner dial's hour line angles would be somewhat involved due to the many terms in the formula, and since "DL" only has two terms, this method can easily be employed.

The nomogram to the left derives this "DL", and the vertical decliner dial plate so derived, is duplicated below from chapter 25 which discusses the concept in more detail, as does the beginning of this chapter.


Method 2: Consider using a different nomogram method for the same formula: $\quad \mathbf{C}=\mathbf{L} / \mathbf{R}$ Sometimes, not much can be done to better align a nomogram that uses the three parallel lines, this is because of the common " 0 " reference line, especially when logarithms are involved.

To the right is a normal three line nomogram for a meridian or a polar dial.

For any hour from local noon (polar) or local 6 am or 6 pm (meridian) it gives the distance from the sub style to the hour line, this is a simple tan of the hour (times 15).

However, in the center line we get for a given declination, the distance along an hour line for that declination point. This assumes a style linear height of 1.

This is fully usable, but it takes up paper space and has longer solution lines that are desirable.

So, while the three vertical lines work for the formula:-

$$
C=L+R
$$

and if logs are used, it also works for multiplication and division

$$
C=L \times R \quad \text { and } \quad C=L / R
$$


and since the polar and meridian dials have a formula using division, rather than have the nomogram handle division by subtracting logarithms, why not use a nomogram designed just for division. And the $C=L / R$ nomogram, commonly called the $N$ or $Z$ nomogram does just that.


For equations like $C=L / R$ the $L$ and $R$ scales are linear, although information portrayed may appear non linear as when trigonometric or other functions are used. The C scale is not linear. The C scale is derived as follows:-
pq [along the diagonal] $=\mathrm{pr} I((\mathrm{~m} 2 / \mathrm{m} 1)+\mathrm{v}) \quad$ where " v " is the value of the data point

NOTE: the m 2 and m 1 are the scale multipliers, not the range of numbers, nor the tan or cos of a value. So for dist=tan(dec)/cos(hr) with dec=0 to 24 , $\mathrm{hr}=0$ to 6 , neither the 24 nor 6 are used, nor the $\tan (24)$ nor $\cos (6$ hours), but rather the scale multipliers if any. In the DeltaCAD macro the left scale was multiplied by 4 , the right scale multiplied by 2 , so $\mathrm{m} 2 / \mathrm{m} 1$ would be $2 / 4$ in this case.

NOTE: the left side scales have 0 at their vertex, and the diagonal connects the 0 of both vertical scales. The 0 is for the value used in the formula. So if $R$ were say $\cos (v a l u e)$, then that 0 would be cos(value) (i.e. represent 90 degrees) and not "value".


Another point of interest. In using the three parallel vertical line nomogram, type 1, the declination began at 1 . This was because the formula for the meridian or polar declination point on an hour line is:-
calendar point on an hour line is $=\quad \operatorname{sh} * \tan (\mathrm{dec}) / \cos ($ Iha $)$
and because this has division, the three parallel line nomogram uses logs. And log for a declination 0 is unusable, so the declination had to start at 1.

With the $N$ or $Z$ nomogram, logs are not used, so the declination can start at zero.

Method 3: Consider using a different nomogram method for the same formula: $\quad \mathbf{C}=\mathrm{L} \times \mathrm{R}$ A circular nomogram handles multiplication without logs, so using an h -dial as an example.

$$
\tan (\mathrm{hla})=\sin (\mathrm{lat}) * \tan (\mathrm{ha}) \quad \text { this is the standard h-dial formula }
$$

from $x$ L the $x$ value for the top HOUR ANGLE (HA is 0 to 90 , and 1 hour is 15 degrees) circle is:-

$$
x=S /(1+\tan * * 2(h a)) \quad \text { see: THE NOMOGRAM by Allcock and Jones }
$$

from $x L$ the $x$ value for the lower LATITUDE circle is:-

$$
x=\mathrm{S} /(1+\sin * * 2(\text { lat })) \quad \text { see: } \text { THE NOMOGRAM by Allcock and Jones }
$$

from $x L$ the $x$ value for the horizontal HOUR LINE ANGLE line is:-

$$
x=S /(1+\tan (\text { hla })) \quad \text { note: not } \tan ^{2}
$$

Developing a usable $x$ and $y$ for the circle: Having an "x" value along the horizontal line is nice, but how do we get a " $y$ " value where a vertical line extended at " $x$ " meets the circle's perimeter.
by definition, the scale " S " is the semi-circle's diameter so the radius is $\mathrm{S} / 2$, thus:-
$\operatorname{big} X=S / 2-x \quad$ where $\operatorname{big} X$ is the $X$ from the semicircle center and where $x$ is the value derived above
$x \mathrm{~L}$ is the starting reference point for " $x$ " for all " $x$ " variables


from the semi circle's center we have an x value of "bigX", and we have the radius which is "S/2" thus by Pythagoras Theorem

$$
\begin{aligned}
& (S / 2) \text { * }(S / 2)=\operatorname{big} X * \operatorname{big} X+y \text { * } y \quad \text { now re-arrange the terms } \\
& y^{*} y=(S / 2) *(S / 2)-\operatorname{big} X * \operatorname{big} X \quad \text { but } \operatorname{big} X=S / 2-x \text {, so } \\
& y=\operatorname{sqrt}(S * S / 4-\operatorname{big} X * \operatorname{big} X) \text { then } \\
& y=\operatorname{sqrt}\left(S * S / 4-(S / 2-x)^{\star}(S / 2-x)\right) \quad \text { then multiply parentheses } \\
& y=\operatorname{sqrt}\left(S^{*} S / 4-\left(S^{*} S / 4-S^{*} x / 2-x^{*} S / 2+x^{*} x\right)\right. \text { ) } \\
& y=\operatorname{sqrt}\left(S^{*} S / 4-\left(S^{*} S / 4-2^{*} S^{*} x / 2+x^{*} x\right)\right. \text { ) } \\
& y=\operatorname{sqrt}\left(S^{*} S / 4-\left(S * S / 4-S^{*} x+x^{*} x\right)\right. \text { ) } \\
& \left.y=\operatorname{sqrt}\left(\quad S^{*} x \quad-x^{*} x\right)\right) \\
& y=\operatorname{sqrt}\left(S^{*} x-x^{*} x\right) \quad \text { final answer for " } y \text { " }
\end{aligned}
$$

DeltaCAD and Lazarus have an h-dial using this method. Yes, nomograms still have life in them. Supplemental Shadows has additional information on nomograms.

## The slide rule and the sun dial

The sad reality is that the slide rule has become a way of the past. I still have two modern rules from my college days, my two wood and ivory ones having long since sprouted wings. Little is to be gained by pages of slide rule usage, however, the design of a horizontal dial with a slide rule may be of interest.

Assume a latitude of 32.75 degrees (lat), and the sun's local hour angle of 15 degrees (lha). The hour line angle (hla) formula is:-

$$
\text { hla }=\operatorname{atan}(\sin (l a t) * \tan (l h a))
$$

On most slide rules the lower fixed part has the "X" scale first, then the "SIN" next, and the "TAN" below that.

To the right, above 32.75 is found the SIN value of 0.54 , and this number has to be remembered.

Then the cursor is moved to above the tangent of the local hour angle for say 11 am or 1 pm , i.e. 15 degrees in this case, the right side of the lower picture.

And the 10 (representing 1) is placed above the $\tan (15)$. see right side below.


Then go to the remembered sin of the latitude of 32.75 , namely 0.54 , upper left below, and look down to the "TAN" scale which is by definition now acting as ATAN, showing 8.25 degrees.


And a spreadsheet confirms the result with 8.2478 degrees to the slide rule's 8.25 .

The process is repeated with other hours and with longitude correction.

| latitude ${ }^{\text {radians }}$ |  | $0.5410 \sin$ (lat) |  |
| :---: | :---: | :---: | :---: |
| 32.75 | 0.5716 |  |  |
| 15 | 0.2618 | 0.2679 | tan(lha) |
| atan(sin(lat) | n(lha) | 0.1440 |  |
| degrees is |  | 8.2478 |  |

## The Dialing Scales and the sun dial

latitude scale

|  |  |  |  |  | 1 | - |  |  |  |  |  | I ${ }^{\text {l }}$ | 1 | $\square 1$ |  |  |  |  | Midudeton | ales - w | www | ding | had | vs.com |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | latitud | de sca | ale | 10 |  |  |  |  |  |  |  |  | 40 |  | 50 | , |  | \$\% |  |  |  |  |  |  |
|  | time sc | scale |  | E |  |  |  | E |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | E |
|  | , |  |  |  | \% |  |  |  |  |  |  |  |  |  |  |  |  |  |  | I |  |  |  |  |
| $\square$ |  | $1]$ | 11 | T | 'T] | 11 | 1 | 1 | 11 |  |  |  |  |  | 1 |  |  |  | 11 | 7 | T\| |  |  | 尚 |

hour scale
Frank Cousins popularized the scales made by E. C. Middleton which were based on Samuel Foster's work in 1638, made practical by George Serle in 1657. These scales may be used to design vertical, horizontal, polar, meridian, and declining dials.

## Horizontal dial design

A dial plate is drawn with the $6 \mathrm{am} / \mathrm{pm}$ and noon line, establishing a dial center. The latitude scale is placed with 0 degrees latitude at dial center, and the new dial's latitude marked along the 6 am line, latitude 40 in this case, and copied an equal distance on the other half of the dial, 6 pm .


The hour scale is placed with the 6 am point on the 6 am line where the latitude was marked, and the noon point on the noon line, or the 6 pm point may be placed on the 6 pm line with the noon point on the noon line. Hour lines are drawn from dial center to the hour point on the hour scale.

## Vertical dial design

A vertical dial is drawn using the co-latitude. The hour line angles of a horizontal dial match the angles for a vertical dial at the co-latitude, and vice versa. Thus, proceed as above, however in place of latitude, use 90 -latitude. Of course, the co-latitude is the angle between the style and the sub-style.

## Polar and Meridian (true east or true west) dial design

If the latitude scales were unfolded to be two sides of a triangle, the time scale being the third, then the hour marks would each be 15 degrees from where the two latitude scales meet. In this case a simple protractor may be easier!


## Vertical decliner design

A vertical decliner has the gnomon rotated by an amount called the style-distance based on the latitude and wall declination. Assume a latitude 30 dial with 10 degrees east wall declination.

Style Distance:
Style Height: style and sub style angle is:

$$
\begin{array}{ll}
\mathrm{SD}=\operatorname{atan}(\sin (\mathrm{dec}) / \tan (\mathrm{lat}) \quad) & -16.7 \\
\mathrm{SH}=\operatorname{asin}\left(\cos (\mathrm{lat})^{*} \cos (\mathrm{dec})\right) & +58.5
\end{array}
$$

Having calculated the style height (SH) and style distance (SD), draw a dial plate for the vertical decliner as shown below, the dashed line being angled from the vertical noon line by SD. At 90 degrees to that, draw another line as shown. [See Tables A5.1, A5.2 and formulae A8.22, A8.23.]


Using the two dashed lines, pretend they are noon and the 6 am and 6 pm lines (which they are not). But in so pretending, use the dialing scales as for the vertical dial design, except use SH instead of the co-latitude ( SH is the colatitude when the wall declination is zero). The dialing scales create construction points.

Then use the hours scale and draw the two edges as shown below. Note that the latitude used is the Style Height (SH). SH is the colatitude if the wall declination was zero. Note where the real noon or vertical line intercepts the hours scale, and note the time difference from noon. These pictorials are not drawn to scale. Below, the real noon is at $1: 15$ from the "lets


Again, assuming that the 1:15 time on the clock scale represents the real noon, then 2:15 on the clock scale will represent $1 \mathrm{pm}, 3: 15$ on the clock scale represents 2 pm , and so on, at least on that side of the scale.


However, on the other side of noon, backing off one hour places 11 am (real) at 12:15 on the dialing scale. So 10:00 am would be at the 11:15 on the dialing scale, 9 am real would be at 10:15 on the dialing scale, and so on. This shows the symmetry of a declining dial around the extended substyle.


## CHAPTER THIRTY THREE

## EXTRA THINGS

## SHADOW DEFINITIONS

The shadow cast by a gnomon to a dial plate may be sharp or ill defined.



Full shadow

If the ratio of the gnomon diameter (gd) to the distance of the gnomon from the dial plate (pd) is less than 0.01 then the shadow will be almost unusable, and if the ratio is better than 0.04 then the shadow will be well defined. A plate as opposed to a rod has the benefit of increasing the size of the full shadow which in turn avoids two penumbras approaching and thus making shadow lines less fuzzy.

## ERRORS IN SUN DIALING

Just as shadow fuzziness can generate errors, so can other factors. The spreadsheet integer (INT) function rounds down, thus -5.7 becomes -6 whereas 5.7 becomes 5 . A dial plate not level introduces errors that may be quite substantial, and North south alignment errors, especially on a vertical dial can be significant. Altitude dials whose gnomon length is used (Shepherd but not Capuchin) may introduce errors. The number of significant digits of a result can be significant, 1 digit versus 2 digits can have a significant effect on a result. Any formula that is an approximation or series (EOT, Declination), and data derived from them (azimuth, altitude) can produce differing results, aggravated by leap year and other approximations. Hence why different authors have different tabular data. This book uses two differing algorithms and thus may seem self inconsistent!

## SOME FIGURES OF THE UNIVERSE AND SOLAR SYSTEM

The Earth is tilted from its orbit by 23.5 degrees, and wobbles around with a cycle of 25,800 years, which as a result cycles through a different constellation about every 2150 years.

The moon and the sun both have a diameter of about 0.5 degrees from the Earth.
The moon's orbit around the Earth is actually offset by about 5 degrees from the ecliptic which is the plane of the Earth's orbit around the sun and is discussed more thoroughly in chapter 27. The Earth's polar axis is offset by about 23.5 degrees from its orbit around the sun.

## IMPORTANT DATES FOR THE DIALIST, and thus a few rules of thumb:-

February 11th
March 21
April 15
May 13th and 14th
June 15
June 21
July 25th and 26th
September 1
September 23
early November
December 21
December 25

EOT max: sun is slow and the EOT is +14 minutes 12 seconds vernal equinox
EOT = 0
an EOT peak: sun is fast, so the EOT is -3 minutes 39 seconds EOT = 0
(some use June 14)
summer solstice
an EOT peak: sun is slow with an EOT of +6 minutes 30 seconds. EOT = 0
autumnal equinox (some use September 21 by convention)
EOT max: sun is fast and the EOT is then -16 minutes 22 seconds
winter solstice (some use December 22)
EOT = 0
(some use December 24)

The use of the 21st for the solstices and equinoxes is often used as an approximation, and easy to remember. However the actual date may vary based on the year, leap year, and so on. In the year 1246 the EOT smaller deviations were both 4 minutes 58 seconds, and the maxima were both 15 minutes 39 seconds. The February minimum is shrinking about 12 seconds a century, the May maximum is shrinking by about 9 seconds a century, the July minimum is growing about 13 seconds a century, and the November maximum is growing by about 5 seconds a century.

## LIMITING, UNUSABLE AND USABLE HOURS ON A DIAL PLATE

There are limiting or unusable hours, so when designing a sundial there is no point in marking up lines that can't be used. If the dial is large then time and materials are wasted as is real estate on the dial plate because the shadow of the style or nodus will never touch those unusable hours or calendar information. For all dials, it is better to eliminate unusable space and increase the size of the remaining hour lines and calendar information, thereby increasing their accuracy.

Displayable hours depend on latitude and dial type. For example, on the equator every day is an equinox thus the most a dial can display is 6 am to 6 pm local apparent time (L.A.T.). At the poles, night lasts months, day last months, or somewhere in between. Thus a horizontal dial can display 24 hours. Software with shadow simulators can help visualization of usable shadow, one such example is SHADOWS, a software program sometimes used by the author but not affiliated with this book. Alternatively, chapter 30 has a CAD solar-travel mesh that can help visualize hours of use. In summary form, with times being local apparent time, the rules of thumb are:-

| Armillary |
| :--- |
| Equatorial |
| Horizontal |
| Ceiling |


| can display from sunrise to sunset, however, an armillary |
| :--- |
| dial plate can interfere with itself near the equinox, and |
| equatorial dials may indicate nothing at the equinox |

Vertical facing the equator the pole

Vertical decliner | no early or late hours, no summer hours at low latitudes |
| :--- |
| 6 am to 6 pm |
| but to 6 am |
| but after the latest sunset nor before the earliest sunrise |
| 12 hours maximum |
| not hours above the horizontal line for the nodus |
| roughly one hour shift for each 15 degrees of wall declination |
| requires deep thought |

## DISORIENTED DIALS ~ relocating azimuth and altitude dials

What is a dial plate? A dial plate is nothing more than a display or a presentation device for shadows. For altitude, azimuth, or hour angle dials two points usually define a useful shadow, a dial center and a nodus. For azimuth dials the shadow points to an hour point. For altitude dials the nodus shadow depicts a time. For hour angle dials, the angle of that shadow on the dial plate is how time is portrayed.

Consider an azimuth dial such as the winged azimuth dial. At a typical latitude, such as latitude 32, the dial plate to the right shows azimuths that diverge rapidly around solar noon in the summer but tend to be more linear towards the winter.

At the equator the azimuths show a reversal at the equinoxes because in June the sun is north of the equator, whereas in
 December it is south, see left picture. The equator has two winters and two summers each year. The two solid lines with arrows show 1 pm for the June and December solstices,
 their angle is equal but in opposite hemispheres, and similarly 9 am is shown with dashed lines. Remember, the hour curves are azimuths, and not hour lines, they show angles.

The equator's azimuth chart matches a polar dial at the equator because azimuth is an angle, and not a shadow tip. The azimuth curves are deceiving, those calendar circles are purely an artificial display aid.


If, while retaining the true north south alignment, the latitude 0 dial was moved to latitude $32^{\circ}$ and tilted by $32^{\circ}$ towards the south, could it possibly still show the correct times throughout the season? Could the reversing equator azimuth dial possibly work?

Both pictures to the side have the equator's azimuth dial tilted by $32^{\circ}$ so it emulates being on the equator, and a normal $32^{\circ}$ latitude azimuth and a $32^{\circ}$ horizontal dial.

The left shows almost 4 pm at the summer solstice, all three dials concur. To the right is about 3:40 pm on the winter solstice, again all three dials concur. And what about the north pole you might ask? Please see the next page.


At the north pole, the azimuth chart becomes an equatorial dial, a set of 15 degree arcs. The azimuth and hour angle all clearly relate because when indicated on a dial plate, the dial plate is merely a display screen to hold an image of the sun on a gnomon, nodus, or style.

Clearly the pole's azimuth dial, being an equatorial dial, works at all latitudes!

So an azimuth dial can be tilted to work at a new latitude, however that tilt must be in the north south direction.

This is why the azimuth can be used for a declining dial. A declining dial simulates a dial at another latitude, and that latitude is the style height or SH .


Think a little further out, about 93 million miles, for that is where the sun is in relation to Earth. And put a dial somewhere on planet Earth,

It shows a time.
Remove planet Earth, and the dial still works as it has no knowledge of latitude or longitude. What it does have is a set of angles defining its orientation to the sun.

Take this a stage further, latitude does not really mean latitude, but rather an angle, or rather a co-angle, from a rod in space that happens to be parallel to the Earth's polar axis.

Azimuth dials can be ported from one latitude to another, the requirement is that they be tilted, and that the tilt be towards or from the north south line.

There is some discussion that this is not true, so what cannot be tilted and why?
The Italian and Babylonian hour lines, or any line depicting the sunrise or sun set, cannot be corrected by tilting. The reason is that in their case, latitude does not just mean an orientation in space but it also means its position with respect to the horizon and location from whence the sun will rise or set. Sunrise and sunset related lines are fiercely latitude dependant because they depend on the spherical geometry, or horizon circle, to determine whether the sun is or is not visible.

In conclusion, an azimuth dial can be latitude corrected provided the tilt to correct it is towards the north or south.

If an hour angle dial and an azimuth dial can be tilted for latitude correction, what about an altitude dial?

For any given day at the pole, the sun's altitude does not vary by more than half a degree.


What would happen if such a dial were moved south from the north pole to another latitude. One extreme would be to the equator where it would be on its side. You would set the gnomon to the month as usual, and rotate the cylinder until the shadow paralleled the rotation axis of this horizontal dial, as usual. And the nodus shadow would always be on that month's part of the curve. Regardless of the time, on any given day of the year on the equator the sun has a fixed altitude compared to a north south line. The sun is moving in a cone around that horizontal north south line.

S-DIAL Lat: 0
Let us take another extreme, an altitude dial for the equator and move it somewhere else.

To the right is an altitude chart for the equator, it shows some interesting things. In March and September the sun's altitude goes to 90 degrees, they have two summers there. And in December and June the sun is at its lowest, they have two winters there.

This is because the sun moves from -23.5 to the +23.5 declination.

If this dial were moved to another latitude, such as latitude 32, how could it possibly work. The answer is that it must be titled, and like the azimuth and hour angle dial it must be tilted towards the true south or true north. And that north/south alignment now removes the key benefit of an altitude dial, namely, no
 compass needed. Failure to do that alignment will cause the dial to be in error, and hence the belief that altitude dials cannot be latitude corrected by tilting.

So at latitude 32 this dial would tilt to the south by 32 degrees. And it must is turned around its own rotational axis, and while being so turned it retains a fixed tilt to the south of 32 degrees.

This dial still works! For the summer at latitude 32, the sun will be orbiting in a cone whose center is this dial and the shadows will only appear on its upper side. For the winter the reverse is true. Just as for an equatorial dial.

And taking another extreme, that same dial at the north pole would be horizontal. And yet it would still work! It works because the horizontal shepherds dial when rotated on its now horizontal axis will be using the north pole's azimuth of the sun to generate an equivalent equatorial altitude. You might think that this fails for the winter when there is no sun, but no, the Earth itself is blocking the sun, but those altitudes still exist were the Earth to vanish, it is the Earth's curvature causing that sun blocking.

Do not confuse azimuth and altitude with sunrise and sunset and the Italian Babylonian times which are not thusly portable. In summary, all this is why a declining dial's calendar lines can use another latitude's altitude and azimuth.

## THE APPENDICES

The full appendices are located in the appendix supplement to: Illustrating Time's Shadow available on: http://www.illustratingshadows.com however critical selections of key information follow

| Appendix 1 | Geometry and trigonometry <br> Trigonometric functions and geometrical rules and circular measures <br> Tables of trig values - sin, cos, and tan |
| :--- | :--- |
| Appendix 2 | Tables independent of latitude or longitude <br> Julian day <br> Jeclination of the sun by the day |
|  | The equation of time, generic and astronomical, and century comparisons <br> Sun's apparent hour angle, and standard time to hour angle |
|  | Longitude to time |
| Aids to olocating north, compass correction and astro compass, noon transit time |  |
| Latitude, Longitude, Magnetic declination information |  |

TRIGONOMETRIC FUNCTIONS

| degrees | radians | sin | cos | $\tan$ | cotan | degrees | radians | sin | cos | tan | cotan |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.017 | 0.0175 | 0.9998 | 0.0175 | 57.2900 | 46 | 0.803 | 0.7193 | 0.6947 | 1.0355 | 0.9657 |
| 2 | 0.035 | 0.0349 | 0.9994 | 0.0349 | 28.6363 | 47 | 0.820 | 0.7314 | 0.6820 | 1.0724 | 0.9325 |
| 3 | 0.052 | 0.0523 | 0.9986 | 0.0524 | 19.0811 | 48 | 0.838 | 0.7431 | 0.6691 | 1.1106 | 0.9004 |
| 4 | 0.070 | 0.0698 | 0.9976 | 0.0699 | 14.3007 | 49 | 0.855 | 0.7547 | 0.6561 | 1.1504 | 0.8693 |
| 5 | 0.087 | 0.0872 | 0.9962 | 0.0875 | 11.4301 | 50 | 0.873 | 0.7660 | 0.6428 | 1.1918 | 0.8391 |
| 6 | 0.105 | 0.1045 | 0.9945 | 0.1051 | 9.5144 | 51 | 0.890 | 0.7771 | 0.6293 | 1.2349 | 0.8098 |
| 7 | 0.122 | 0.1219 | 0.9925 | 0.1228 | 8.1443 | 52 | 0.908 | 0.7880 | 0.6157 | 1.2799 | 0.7813 |
| 8 | 0.140 | 0.1392 | 0.9903 | 0.1405 | 7.1154 | 53 | 0.925 | 0.7986 | 0.6018 | 1.3270 | 0.7536 |
| 9 | 0.157 | 0.1564 | 0.9877 | 0.1584 | 6.3138 | 54 | 0.942 | 0.8090 | 0.5878 | 1.3764 | 0.7265 |
| 10 | 0.175 | 0.1736 | 0.9848 | 0.1763 | 5.6713 | 55 | 0.960 | 0.8192 | 0.5736 | 1.4281 | 0.7002 |
| 11 | 0.192 | 0.1908 | 0.9816 | 0.1944 | 5.1446 | 56 | 0.977 | 0.8290 | 0.5592 | 1.4826 | 0.6745 |
| 12 | 0.209 | 0.2079 | 0.9781 | 0.2126 | 4.7046 | 57 | 0.995 | 0.8387 | 0.5446 | 1.5399 | 0.6494 |
| 13 | 0.227 | 0.2250 | 0.9744 | 0.2309 | 4.3315 | 58 | 1.012 | 0.8480 | 0.5299 | 1.6003 | 0.6249 |
| 14 | 0.244 | 0.2419 | 0.9703 | 0.2493 | 4.0108 | 59 | 1.030 | 0.8572 | 0.5150 | 1.6643 | 0.6009 |
| 15 | 0.262 | 0.2588 | 0.9659 | 0.2679 | 3.7321 | 60 | 1.047 | 0.8660 | 0.5000 | 1.7321 | 0.5774 |
| 16 | 0.279 | 0.2756 | 0.9613 | 0.2867 | 3.4874 | 61 | 1.065 | 0.8746 | 0.4848 | 1.8040 | 0.5543 |
| 17 | 0.297 | 0.2924 | 0.9563 | 0.3057 | 3.2709 | 62 | 1.082 | 0.8829 | 0.4695 | 1.8807 | 0.5317 |
| 18 | 0.314 | 0.3090 | 0.9511 | 0.3249 | 3.0777 | 63 | 1.100 | 0.8910 | 0.4540 | 1.9626 | 0.5095 |
| 19 | 0.332 | 0.3256 | 0.9455 | 0.3443 | 2.9042 | 64 | 1.117 | 0.8988 | 0.4384 | 2.0503 | 0.4877 |
| 20 | 0.349 | 0.3420 | 0.9397 | 0.3640 | 2.7475 | 65 | 1.134 | 0.9063 | 0.4226 | 2.1445 | 0.4663 |
| 21 | 0.367 | 0.3584 | 0.9336 | 0.3839 | 2.6051 | 66 | 1.152 | 0.9135 | 0.4067 | 2.2460 | 0.4452 |
| 22 | 0.384 | 0.3746 | 0.9272 | 0.4040 | 2.4751 | 67 | 1.169 | 0.9205 | 0.3907 | 2.3559 | 0.4245 |
| 23 | 0.401 | 0.3907 | 0.9205 | 0.4245 | 2.3559 | 68 | 1.187 | 0.9272 | 0.3746 | 2.4751 | 0.4040 |
| 24 | 0.419 | 0.4067 | 0.9135 | 0.4452 | 2.2460 | 69 | 1.204 | 0.9336 | 0.3584 | 2.6051 | 0.3839 |
| 25 | 0.436 | 0.4226 | 0.9063 | 0.4663 | 2.1445 | 70 | 1.222 | 0.9397 | 0.3420 | 2.7475 | 0.3640 |
| 26 | 0.454 | 0.4384 | 0.8988 | 0.4877 | 2.0503 | 71 | 1.239 | 0.9455 | 0.3256 | 2.9042 | 0.3443 |
| 27 | 0.471 | 0.4540 | 0.8910 | 0.5095 | 1.9626 | 72 | 1.257 | 0.9511 | 0.3090 | 3.0777 | 0.3249 |
| 28 | 0.489 | 0.4695 | 0.8829 | 0.5317 | 1.8807 | 73 | 1.274 | 0.9563 | 0.2924 | 3.2709 | 0.3057 |
| 29 | 0.506 | 0.4848 | 0.8746 | 0.5543 | 1.8040 | 74 | 1.292 | 0.9613 | 0.2756 | 3.4874 | 0.2867 |
| 30 | 0.524 | 0.5000 | 0.8660 | 0.5774 | 1.7321 | 75 | 1.309 | 0.9659 | 0.2588 | 3.7321 | 0.2679 |
| 31 | 0.541 | 0.5150 | 0.8572 | 0.6009 | 1.6643 | 76 | 1.326 | 0.9703 | 0.2419 | 4.0108 | 0.2493 |
| 32 | 0.559 | 0.5299 | 0.8480 | 0.6249 | 1.6003 | 77 | 1.344 | 0.9744 | 0.2250 | 4.3315 | 0.2309 |
| 33 | 0.576 | 0.5446 | 0.8387 | 0.6494 | 1.5399 | 78 | 1.361 | 0.9781 | 0.2079 | 4.7046 | 0.2126 |
| 34 | 0.593 | 0.5592 | 0.8290 | 0.6745 | 1.4826 | 79 | 1.379 | 0.9816 | 0.1908 | 5.1446 | 0.1944 |
| 35 | 0.611 | 0.5736 | 0.8192 | 0.7002 | 1.4281 | 80 | 1.396 | 0.9848 | 0.1736 | 5.6713 | 0.1763 |
| 36 | 0.628 | 0.5878 | 0.8090 | 0.7265 | 1.3764 | 81 | 1.414 | 0.9877 | 0.1564 | 6.3138 | 0.1584 |
| 37 | 0.646 | 0.6018 | 0.7986 | 0.7536 | 1.3270 | 82 | 1.431 | 0.9903 | 0.1392 | 7.1154 | 0.1405 |
| 38 | 0.663 | 0.6157 | 0.7880 | 0.7813 | 1.2799 | 83 | 1.449 | 0.9925 | 0.1219 | 8.1443 | 0.1228 |
| 39 | 0.681 | 0.6293 | 0.7771 | 0.8098 | 1.2349 | 84 | 1.466 | 0.9945 | 0.1045 | 9.5144 | 0.1051 |
| 40 | 0.698 | 0.6428 | 0.7660 | 0.8391 | 1.1918 | 85 | 1.484 | 0.9962 | 0.0872 | 11.4301 | 0.0875 |
| 41 | 0.716 | 0.6561 | 0.7547 | 0.8693 | 1.1504 | 86 | 1.501 | 0.9976 | 0.0698 | 14.301 | 0.0699 |
| 42 | 0.733 | 0.6691 | 0.7431 | 0.9004 | 1.1106 | 87 | 1.518 | 0.9986 | 0.0523 | 19.081 | 0.0524 |
| 43 | 0.750 | 0.6820 | 0.7314 | 0.9325 | 1.0724 | 88 | 1.536 | 0.9994 | 0.0349 | 28.636 | 0.0349 |
| 44 | 0.768 | 0.6947 | 0.7193 | 0.9657 | 1.0355 | 89 | 1.553 | 0.9998 | 0.0175 | 57.290 | 0.0175 |
| 45 | 0.785 | 0.7071 | 0.7071 | 1.0000 | 1.0000 | 90 | 1.571 | 1.0000 | 0.0000 | inf | 0.0000 |

## SUN'S DECLINATION

from appendix 2

|  | Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -23.1 | -17.3 | -7.9 | 4.2 | 14.8 | 21.9 | 23.2 | 18.2 | 8.6 | -2.9 | -14.2 | -21.7 |
| 2 | -23.0 | -17.1 | -7.5 | 4.6 | 15.1 | 22.1 | 23.1 | 18.0 | 8.2 | -3.3 | -14.5 | -21.8 |
| 3 | -22.9 | -16.8 | -7.1 | 5.0 | 15.4 | 22.2 | 23.0 | 17.7 | 7.8 | -3.6 | -14.8 | -22.0 |
| 4 | -22.8 | -16.5 | -6.7 | 5.4 | 15.7 | 22.3 | 23.0 | 17.5 | 7.5 | -4.0 | -15.1 | -22.1 |
| 5 | -22.7 | -16.2 | -6.3 | 5.8 | 16.0 | 22.5 | 22.9 | 17.2 | 7.1 | -4.4 | -15.5 | -22.3 |
| 6 | -22.6 | -15.9 | -6.0 | 6.2 | 16.3 | 22.6 | 22.8 | 16.9 | 6.7 | -4.8 | -15.8 | -22.4 |
| 7 | -22.5 | -15.6 | -5.6 | 6.5 | 16.6 | 22.7 | 22.7 | 16.6 | 6.4 | -5.2 | -16.1 | -22.5 |
| 8 | -22.3 | -15.3 | -5.2 | 6.9 | 16.9 | 22.8 | 22.6 | 16.4 | 6.0 | -5.6 | -16.4 | -22.6 |
| 9 | -22.2 | -14.9 | -4.8 | 7.3 | 17.1 | 22.9 | 22.5 | 16.1 | 5.6 | -6.0 | -16.7 | -22.7 |
| 10 | -22.1 | -14.6 | -4.4 | 7.7 | 17.4 | 23.0 | 22.4 | 15.8 | 5.2 | -6.3 | -16.9 | -22.8 |
| 11 | -21.9 | -14.3 | -4.0 | 8.0 | 17.7 | 23.0 | 22.2 | 15.5 | 4.9 | -6.7 | -17.2 | -22.9 |
| 12 | -21.8 | -14.0 | -3.6 | 8.4 | 17.9 | 23.1 | 22.1 | 15.2 | 4.5 | -7.1 | -17.5 | -23.0 |
| 13 | -21.6 | -13.6 | -3.2 | 8.8 | 18.2 | 23.2 | 22.0 | 14.9 | 4.1 | -7.5 | -17.8 | -23.1 |
| 14 | -21.4 | -13.3 | -2.8 | 9.1 | 18.4 | 23.2 | 21.8 | 14.6 | 3.7 | -7.8 | -18.0 | -23.2 |
| 15 | -21.3 | -13.0 | -2.4 | 9.5 | 18.7 | 23.3 | 21.7 | 14.3 | 3.3 | -8.2 | -18.3 | -23.2 |
| 16 | -21.1 | -12.6 | -2.0 | 9.8 | 18.9 | 23.3 | 21.5 | 14.0 | 3.0 | -8.6 | -18.6 | -23.3 |
| 17 | -20.9 | -12.3 | -1.6 | 10.2 | 19.1 | 23.4 | 21.3 | 13.7 | 2.6 | -9.0 | -18.8 | -23.3 |
| 18 | -20.7 | -11.9 | -1.3 | 10.5 | 19.4 | 23.4 | 21.2 | 13.4 | 2.2 | -9.3 | -19.1 | -23.4 |
| 19 | -20.5 | -11.6 | -0.9 | 10.9 | 19.6 | 23.4 | 21.0 | 13.0 | 1.8 | -9.7 | -19.3 | -23.4 |
| 20 | -20.3 | -11.2 | -0.5 | 11.2 | 19.8 | 23.4 | 20.8 | 12.7 | 1.4 | -10.1 | -19.5 | -23.4 |
| 21 | -20.1 | -10.8 | -0.1 | 11.6 | 20.0 | 23.5 | 20.6 | 12.4 | 1.0 | -10.4 | -19.8 | -23.4 |
| 22 | -19.9 | -10.5 | 0.3 | 11.9 | 20.2 | 23.5 | 20.4 | 12.0 | 0.6 | -10.8 | -20.0 | -23.4 |
| 23 | -19.6 | -10.1 | 0.7 | 12.3 | 20.4 | 23.5 | 20.2 | 11.7 | 0.2 | -11.1 | -20.2 | -23.4 |
| 24 | -19.4 | -9.7 | 1.1 | 12.6 | 20.6 | 23.4 | 20.0 | 11.4 | -0.1 | -11.5 | -20.4 | -23.4 |
| 25 | -19.2 | -9.4 | 1.5 | 12.9 | 20.8 | 23.4 | 19.8 | 11.0 | -0.5 | -11.8 | -20.6 | -23.4 |
| 26 | -18.9 | -9.0 | 1.9 | 13.3 | 21.0 | 23.4 | 19.6 | 10.7 | -0.9 | -12.2 | -20.8 | -23.4 |
| 27 | -18.7 | -8.6 | 2.3 | 13.6 | 21.2 | 23.4 | 19.4 | 10.3 | -1.3 | -12.5 | -21.0 | -23.3 |
| 28 | -18.4 | -8.3 | 2.7 | 13.9 | 21.3 | 23.3 | 19.2 | 10.0 | -1.7 | -12.9 | -21.2 | -23.3 |
| 29 | -18.2 |  | 3.1 | 14.2 | 21.5 | 23.3 | 18.9 | 9.6 | -2.1 | -13.2 | -21.4 | -23.3 |
| 30 | -17.9 |  | 3.5 | 14.5 | 21.7 | 23.2 | 18.7 | 9.3 | -2.5 | -13.5 | -21.5 | -23.2 |
| 31 | -17.6 |  | 3.9 |  | 21.8 |  | 18.5 | 8.9 |  | -13.9 |  | -23.1 |

Different declination charts may disagree, factors affecting them would be leap year approximations, and the formula employed. Many formulae are approximations.

## EQUATION OF TIME

from appendix 2

| EQUATION OF TIME TABLE EO |  |  |  |  |  |  |  | minutes:seconds |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| 1 | 3:09 | 13:34 | 12:17 | 3:49 | -2:54 | -2:09 | 3:51 | 6:17 | 0:02 | -10:21 | -16:21 | -10:52 |
| 2 | 4:07 | 13:42 | 12:05 | 3:32 | -3:01 | -1:59 | 4:03 | 6:13 | -0:22 | -10:40 | -16:22 | -10:29 |
| 3 | 4:34 | 13:48 | 11:52 | 3:14 | -3:07 | -1:50 | 4:14 | 6:08 | -0:41 | -10:59 | -16:22 | -10:06 |
| 4 | 5:02 | 13:54 | 11:39 | 2:57 | -3:13 | -1:39 | 4:25 | 6:03 | -1:01 | -11:17 | -16:22 | -9:42 |
| 5 | 5:28 | 13:59 | 11:26 | 2:39 | -3:18 | -1:29 | 4:35 | 5:57 | -1:21 | -11:35 | -16:20 | -9:17 |
| 6 | 5:55 | 14:03 | 11:12 | 2:22 | -3:23 | -1:18 | 4:45 | 5:51 | -1:41 | -11:53 | -16:18 | -8:52 |
| 7 | 6:21 | 14:06 | 10:58 | 2:06 | -3:27 | -1:07 | 4:55 | 5:43 | -2:01 | -12:10 | -16:15 | -8:27 |
| 8 | 6:46 | 14:09 | 10:43 | 1:49 | -3:30 | -0:56 | 5:04 | 5:36 | -2:22 | -12:27 | -16:11 | -8:00 |
| 9 | 7:11 | 14:10 | 10:28 | 1:33 | -3:33 | -0:44 | 5:13 | 5:27 | -2:43 | -12:43 | -16:06 | -7:34 |
| 10 | 7:35 | 14:11 | 10:13 | 1:16 | -3:35 | -0:32 | 5:22 | 5:19 | -3:03 | -12:59 | -16:00 | -7:07 |
| 11 | 7:59 | 14:12 | 9:57 | 1:01 | -3:37 | -0:20 | 5:30 | 5:09 | -3:24 | -13:15 | -15:54 | -6:39 |
| 12 | 8:22 | 14:11 | 9:42 | 0:45 | -3:38 | -0:08 | 5:37 | 4:59 | -3:46 | -13:30 | -15:46 | -6:11 |
| 13 | 8:44 | 14:10 | 9:25 | 0:30 | -3:39 | 0:04 | 5:44 | 4:48 | -4:07 | -13:44 | -15:38 | -5:43 |
| 14 | 9:06 | 14:08 | 9:09 | 0:15 | -3:39 | 0:16 | 5:51 | 4:37 | -4:28 | -13:58 | -15:29 | -5:15 |
| 15 | 9:27 | 14:05 | 8:52 | 0:00 | -3:38 | 0:29 | 5:57 | 4:26 | -4:49 | -14:12 | -15:19 | -4:46 |
| 16 | 9:48 | 14:01 | 8:35 | -0:13 | -3:37 | 0:42 | 6:03 | 4:13 | -5:11 | -14:25 | -15:09 | -4:17 |
| 17 | 10:08 | 13:57 | 8:18 | -0:27 | -3:36 | 0:55 | 6:08 | 4:01 | -5:32 | -14:37 | -14:57 | -3:48 |
| 18 | 10:27 | 13:52 | 8:01 | -0:40 | -3:34 | 1:08 | 6:13 | 3:48 | -5:53 | -14:49 | -14:45 | -3:19 |
| 19 | 10:45 | 13:47 | 7:43 | -0:53 | -3:31 | 1:21 | 6:17 | 3:34 | -6:15 | -15:00 | -14:31 | -2:49 |
| 20 | 11:03 | 13:40 | 7:25 | -1:06 | -3:27 | 1:34 | 6:20 | 3:20 | -6:36 | -15:10 | -14:17 | -2:19 |
| 21 | 11:20 | 13:33 | 7:08 | -1:18 | -3:24 | 1:47 | 6:23 | 3:05 | -6:57 | -15:20 | -14:02 | -1:50 |
| 22 | 11:36 | 13:26 | 6:50 | -1:30 | -3:19 | 2:00 | 6:26 | 2:50 | -7:18 | -15:29 | -13:47 | -1:20 |
| 23 | 11:52 | 13:18 | 6:32 | -1:41 | -3:14 | 2:13 | 6:28 | 2:34 | -7:39 | -15:38 | -13:30 | -0:50 |
| 24 | 12:06 | 13:09 | 6:14 | -1:52 | -3:09 | 2:26 | 6:29 | 2:18 | -8:00 | -15:46 | -13:13 | -0:20 |
| 25 | 12:20 | 13:00 | 5:56 | -2:03 | -3:03 | 2:39 | 6:30 | 2:02 | -8:21 | -15:53 | -12:55 | 0:08 |
| 26 | 12:33 | 12:50 | 5:37 | -2:12 | -2:57 | 2:51 | 6:30 | 1:45 | -8:41 | -15:59 | -12:36 | 0:38 |
| 27 | 12:45 | 12:39 | 5:19 | -2:22 | -2:50 | 3:04 | 6:29 | 1:28 | -9:02 | -16:05 | -12:17 | 1:07 |
| 28 | 12:57 | 12:28 | 5:01 | -2:31 | -2:43 | 3:16 | 6:28 | 1:10 | -9:22 | -16:09 | -11:57 | 1:37 |
| 29 | 13:07 |  | 4:43 | -2:39 | -2:35 | 3:28 | 6:26 | 0:52 | -9:42 | -16:14 | -11:36 | 2:06 |
| 30 | 13:17 |  | 4:25 | -2:47 | -2:27 | 3:40 | 6:24 | 0:34 | -10:01 | -16:17 | -11:14 | 2:35 |
| 31 | 12:26 |  | 4:07 |  | -2:18 |  | 6:21 | 0:16 |  | -16:19 |  | 3:03 |

## ITALIAN AND BABYLONIAN HOUR LINES from appendix 6

Babylonian and Italian values by latitude. Times are hh.mm Local Apparent Time Solstice Sunrise and Sunset (equinox is $6 \mathrm{am} / \mathrm{pm}$ ). No longitude correction. No EOT correction.

## Winter solstice:

Declination:
$-23.5$

| latitude | Rise | Set | Day length hrs |
| :---: | :---: | :---: | :---: |
| 30 | 6.58 | 17.02 | 10.04 |
| 31 | 7.00 | 17.00 | 10.00 |
| 32 | 7.03 | 16.57 | 9.54 |
| 33 | 7.05 | 16.55 | 9.50 |
| 34 | 7.08 | 16.52 | 9.44 |
| 35 | 7.10 | 16.50 | 9.40 |
| 36 | 7.13 | 16.47 | 9.34 |
| 37 | 7.16 | 16.44 | 9.28 |
| 38 | 7.19 | 16.41 | 9.22 |
| 39 | 7.22 | 16.38 | 9.16 |
| 40 | 7.25 | 16.35 | 9.10 |
| 41 | 7.28 | 16.32 | 9.04 |
| 42 | 7.32 | 16.28 | 8.56 |
| 43 | 7.35 | 16.25 | 8.50 |
| 44 | 7.39 | 16.21 | 8.42 |
| 45 | 7.43 | 16.17 | 8.34 |
| 46 | 7.47 | 16.13 | 8.26 |
| 47 | 7.51 | 16.09 | 8.18 |
| 48 | 7.55 | 16.05 | 8.10 |
| 49 | 8.00 | 16.00 | 8.00 |
| 50 | 8.04 | 15.56 | 7.52 |
| 51 | 8.09 | 15.51 | 7.42 |
| 52 | 8.15 | 15.45 | 7.30 |
| 53 | 8.20 | 15.40 | 7.20 |
| 54 | 8.27 | 15.33 | 7.06 |
| 55 | 8.33 | 15.27 | 6.54 |
| 56 | 8.40 | 15.20 | 6.40 |
| 57 | 8.48 | 15.12 | 6.24 |
| 58 | 8.56 | 15.04 | 6.08 |
| 59 | 9.05 | 14.55 | 5.50 |
| 60 | 9.15 | 14.45 | 5.30 |

Summer solstice:
Declination:
$+23.5$

| latitude | Rise | Set | Day length hrs |
| :---: | :---: | :---: | :---: |
| 30 | 5.01 | 18.59 | 13.58 |
| 31 | 4.59 | 19.01 | 14.02 |
| 32 | 4.56 | 19.04 | 14.08 |
| 33 | 4.54 | 19.06 | 14.12 |
| 34 | 4.51 | 19.09 | 14.18 |
| 35 | 4.49 | 19.11 | 14.22 |
| 36 | 4.46 | 19.14 | 14.28 |
| 37 | 4.43 | 19.17 | 14.34 |
| 38 | 4.40 | 19.20 | 14.40 |
| 39 | 4.37 | 19.23 | 14.46 |
| 40 | 4.34 | 19.26 | 14.52 |
| 41 | 4.31 | 19.29 | 14.58 |
| 42 | 4.27 | 19.33 | 15.06 |
| 43 | 4.24 | 19.36 | 15.12 |
| 44 | 4.20 | 19.40 | 15.20 |
| 45 | 4.16 | 19.44 | 15.28 |
| 46 | 4.12 | 19.48 | 15.36 |
| 47 | 4.08 | 19.52 | 15.44 |
| 48 | 4.04 | 19.56 | 15.52 |
| 49 | 3.59 | 20.01 | 16.02 |
| 50 | 3.55 | 20.05 | 16.10 |
| 51 | 3.50 | 20.10 | 16.20 |
| 52 | 3.44 | 20.16 | 16.32 |
| 53 | 3.39 | 20.21 | 16.42 |
| 54 | 3.32 | 20.28 | 16.56 |
| 55 | 3.26 | 20.34 | 17.08 |
| 56 | 3.19 | 20.41 | 17.22 |
| 57 | 3.11 | 20.49 | 17.38 |
| 58 | 3.03 | 20.57 | 17.54 |
| 59 | 2.54 | 21.06 | 18.12 |
| 60 | 2.44 | 21.16 | 18.32 |

Sunset (true time or local apparent time) occurs the same number of hours after noon that sunrise happens before. An 0605 sunrise is 5 hours 55 minutes before noon, thus sunset is 5 hours 55 minutes after it, or 1755 . Except for on the hour or half hour, the minutes do not match. For standard time, then the time is shifted by the longitude correction and then by the equation of time, thus the March and September equinoxes do not match due to differing EOT values.

Italian hour lines are commonly used to indicate the number of hours until sunset. For Italian hour lines, exclude the EOT. And exclude the longitude correction unless the dial already has considered longitude in its design. The spreadsheets on the web site allow you to do this, and of course, the above table provides all the data needed for Italian and Babylonian lines. The equinox is not shown since true sunrise and sunset occurs at 6am and 6pm L.A.T..

## COLLECTION OF FORMULAE

COMMON ABBREVIATIONS ARE:- The sun's declination is "dec", the latitude is "lat", the sun's local hour angle is "Iha" being 15 times the hour from noon (or 6 am or 6 pm for meridian dials), and "hla" is the hour line angle on a dial, "sh" is style height usually angular, "sd" is style distance.

## SUNS DECLINATION FOR ANY GIVEN DAY OF THE YEAR

Source: http://eande.lbl.gov/Task21/C2/algo1/1-11.html
Day number, J J=1 on January 1, 365 on 31 December, February has 28 days, "pi" is 3.14159

| Jan | Feb | Mar | Apr | May | Jun |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 31 | 59 | 90 | 120 | 151 |
| Jly | Aug | Sep | Oct | Nov | Dec |
| 181 | 212 | 243 | 273 | 304 | 334 |

Day angle: $\quad \mathrm{da}=2$ * $\mathrm{pi}^{*}(\mathrm{j}-1) / 365 \quad$ (is an intermediate figure in radians)
A8.1

Sun Declination: $\quad d e c=$ degrees $\left(0.006918-0.399912^{*} \cos (d a)+0.070257 * \sin (\mathrm{da})\right.$ $-0.006758^{*} \cos (2 * d a)+0.000907^{*} \sin (2 * d a)$ $-0.002697 * \cos \left(3^{*} d a\right)+0.001480 * \sin (3 * d a)$
alternative formula: $\quad \operatorname{dec}=\left(23.45^{*} \sin ((0.9678(j-80)))\right)$ source: Claude Hartman

A8.2a

A8.2b

## SUNS ALTITUDE AND AZIMUTH ON ANY GIVEN HOUR GIVEN THE SUNS DECLINATION

The sun's declination is "dec", and the latitude is "lat", and the sun's local hour angle is "lha".
ALTITUDE: The sun's altitude is its angle when looked at face on

$$
\text { alt }=\operatorname{ASIN}(\operatorname{SIN}(\operatorname{dec}) * \operatorname{SIN}(\text { lat })+
$$

$\cos ($ dec $) * \cos ($ lat $) * \operatorname{COS}($ lha $))$
AZIMUTH:

$$
\begin{aligned}
\text { azi }= & \operatorname{ATAN}(\operatorname{SIN}(\text { lha }) /((\operatorname{SIN}(\text { lat }) * \operatorname{COS}(\text { lha })) \\
& -(\operatorname{COS}(\text { lat }) * \operatorname{TAN}(\operatorname{dec})))
\end{aligned}
$$

A8.3

A8.4
note: Some authors present two different formulae. They agree in all aspects except for 6am and thus also 6pm, when using the author's spreadsheet.

## SUNRISE AND SUNSET TIME FORMULA

The sun's declination is "dec", and latitude is "lat". The sun's local hour angle is "lha". For the setting or rising sun, the azimuth and the sun's hour angle from noon are:-

$$
\begin{aligned}
& \text { aziRiseSet }=180-\arccos (\sin (\mathrm{dec}) / \cos (\text { lat })) \\
& \text { IhaRiseSet }=\arccos (\tan (\text { lat }) * \tan (\mathrm{dec}))
\end{aligned}
$$

## A8.6

## EQUATORIAL DIAL CALENDAR AND SUNRISE/SUNSET LINE DATA

Calendar lines are arcs whose radius = gnomon linear height / tan( dec )
A8.7
Horizontal sunrise/set line from gnomon distance = gnomon linear height * tan( lat )

HORIZONTAL DIAL HOUR LINE ANGLE
A8.8
hla $=\operatorname{atan}(\sin ($ lat $) * \tan ($ lha $))$ also the reverse: $\quad$ Iha $=\operatorname{atan}(\tan ($ hla $) / \sin (l a t))$

POLAR DIAL Times are from noon. "sh" = style linear height.
from style to hour line $\quad=$ sh * $\tan$ ( Iha ) distance up an hour line to calendar line $=s h * \tan (\mathrm{dec}) / \cos$ (Iha)

## A8.9

A8.10

MERIDIAN DIAL ~ EAST/WEST VERTICAL NON DECLINING DIAL
"Iha" time here is from 6am or 6pm, not from noon. "sh" = style linear height.
from style to hour line $\quad=$ sh * tan( Iha from 6pm or 6am ) distance up an hour line to calendar line $=s h * \tan (\mathrm{dec}) / \cos$ (Iha)

## SOUTH VERTICAL NON DECLINER HOUR LINE ANGLE

hla $=\operatorname{atan}(\tan (\mathrm{lha}) * \sin (90-$ lat $) \quad$ angle hour line makes with 12 o'clock line

A8.13
hla $=\operatorname{atan}(\tan ($ lha $) * \cos ($ lat $) \quad$ ) since $\cos 0=\sin 90$ etc

ANALEMMATIC DIAL
"lat" is latitude, "Iha" is the sun's local hour angle from noon (15 times the hour from local noon)
$m=M$ * $\sin ($ lat $) \quad m=$ semi minor axis, you select the " $M$ " value
A8.14
an hour point's horizontal distance:-

$$
C^{\prime}=M \text { * } \sin (\text { Iha })
$$

to get an hour point's vertical distance:-


A8.15

A8.16

$$
h^{\prime}=M * \sin (\text { lat }) * \cos (\text { lha })
$$

or to get an hour point by angle from C assuming an ellipse has been drawn

$$
x=\arctan (\tan (\operatorname{lna}) / \sin (\text { lat }))
$$

to get the analemmatic points for the gnomon up or down the " $m$ " scale

$$
z=M * \tan (d e c) * \cos (\text { lat })
$$

legal time = L.A.T. + EOT.corr + west.long.corr + 1 if summer or standard - east.long.corr

A8.19 time

## STANDARD TIME TO MARK AN HOUR LINE (as in calibrating hour lines using empirical dialing)

| dial hour point | $=$ | clock time for that hour point | + EOT i.e. |
| :--- | :--- | :--- | :--- |
| legal time to | $=$ | desired L.A.T. | + EOT | mark

This is not inconsistent. If the EOT were -10 minutes, then when the dial reads 1400 , the legal time would be 1350 . So, at 1350, with an EOT of -10 , the sun's shadow will indicate the 1400 hour point. This may appear inconsistent with the rules of algebra, however it is correct. Because of this apparent inconsistency, the dialist is advised to draft a table of times and the hour point they would thus indicate before marking a dial empirically.

## SOLAR TIME FROM STANDARD TIME (as in finding true north)



Solar noon indicates true north because the sun is at its highest point. Thus the shadow produced at solar noon will point to true north. Solar noon happens at the standard time adjusted as follows:-
legal time $\quad=12: 00: 00+$ EOT +1 (if summer) + longitude correction

This is not inconsistent. It may appear that signs should be reversed, however we are in fact achieving the correct arithmetic rules.


## VERTICAL DECLINER ~ HOUR LINE ANGLES \& GNOMON ANGLES

The hour line angles are: $\quad$ hla $=\operatorname{atan}(\cos (l a t) /(\cos (\mathrm{dec}) * \cot (\operatorname{lha})+\sin (\mathrm{dec}) * \sin (\mathrm{lat})))$

A8.21

A8.21a

Gnomon rotation simplifies calendar and analemma drafting, and uses the following formula:-
Gnomon offset from vertical is: sd=atan( $\sin (\mathrm{dec}) / \tan (\mathrm{lat})$ ) $\quad$ Style Distance
A8.22

Style and sub style angle is: $\quad \mathbf{s h}=\operatorname{asin}(\cos (l a t) * \cos (d e c)$ ) Style Height
A8.23


A design for South $x x$ degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. If longitude correction is applied, care must be applied as hour lines shift. The North facing decliner gnomons are inverted, and the vertical is midnight.

DL, "Difference in Longitude" derived for vertical decliners, is a "longitude" for a surrogate horizontal dial of latitude SH , which then enables this surrogate dial's analemma and other features to be placed on the original vertical decliner.

$$
\mathrm{DL} \quad=-(\operatorname{Atan}(\operatorname{Tan}(\operatorname{dec}) / \operatorname{Sin}(\text { lat })))-(\operatorname{lng}-\text { ref })
$$

A8.24

## VERTICAL EAST OR WEST DIAL THAT SLOPES OR INCLINES/RECLINES

The latitude of the "design dial" is:- $\quad=90-$ the latitude of the actual dial and
The declination of the "design dial" is:- $=90-$ the reclination of the actual dial or $\quad=$ the inclination of the actual dial


## BIFILAR SUNDIAL

There are variations on this dial design that make it universal. However, the following two formula are for the bifilar dial designed for a specific latitude.

The north south wire can be any height, the east west wire height is equal to:-

$$
\text { east west wire height }=\text { height of the north south wire * } \sin (\text { lat })
$$

A8.27

While the north south wire is placed over the noon line, the east west wire is placed at a distance from the dial center that is equal to:-
dist from dial center = height of the north south wire * cos( lat )
A8. 28
reference: http://www.de-zonnewijzerkring.nl/eng/index-bif-zonw.htm
Beware that the shadow of the cross-hair can be off the dial plate for early hours or winter hours.

## EQUATION OF TIME

A formula derived from Frans Maes from data by Savoie producing the EOT in minutes and using two sine waves is used for some spreadsheets, e.g. A2.1b, A2.1c. The values in the sin(...) function result in radians, so the formula is spreadsheet ready as-is. Value $d=1$ to 365

$$
E=7.36 * \operatorname{Sin}(2 * 3.1416 *(d-4.21) / 365)+9.92 * \operatorname{Sin}(4 * 3.1416 *(d+9.9) / 365)
$$

Another formula using the sum of three sine waves is used for some spreadsheets, e.g. A2.1d, A2.1e. The $\sin (\ldots)$ values result in degrees hence the required indicated radian conversion.

$$
\begin{aligned}
& \mathrm{E}=-1 \star\left(9.84^{*} \operatorname{SIN}\left(\operatorname{RADIANS}\left(2 *\left(360^{*}(\mathrm{~mm} 1+\mathrm{dd}-81) / 365\right)\right)\right)-\right. \\
& 7.53^{*} \operatorname{COS}\left(\operatorname{RADIANS}\left(360^{*}(\mathrm{~mm} 1+\mathrm{dd}-81) / 365\right)\right)- \\
&\left.1.5^{*} \operatorname{SIN}\left(\operatorname{RADIANS}\left(360^{*}(\mathrm{~mm} 1+\mathrm{dd}-81) / 365\right)\right)\right)-0.3
\end{aligned}
$$

where: $\quad \mathrm{mm} 1 \quad$ is the number of days prior to this month's day 1 , So Jan is 0 , Feb is 31 , Mar is 59 , April is 90, etc, assuming a non leap year. For leap years add 1 for March to December.

| Jan | Feb | Mar | Apr | May | Jun | Jly | Aug | Sep | Oct | Nov | Dec |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | 31 | 59 | 90 | 120 | 151 | 181 | 212 | 243 | 273 | 304 | 334 |

dd is the day of the month, being 1 to 31
i.e. $\quad \mathrm{mm} 1+\mathrm{dd}$ is the Julian day of the year

Another three wave formula is:

$$
\begin{array}{r}
\mathrm{E}=7.5^{*} \operatorname{SIN}(\operatorname{RADIANS}(\mathrm{~d}-5))-10.2^{*} \operatorname{SIN}\left(\operatorname{RADIANS}\left(1.93^{*}(\mathrm{~d}-80)\right)\right)+ \\
0.5^{\star} \operatorname{SIN}\left(\operatorname{RADIANS}\left(1.5^{*}(\mathrm{~d}-62)\right)\right)
\end{array}
$$

Another formula derived from the work of Frank Cousins uses the sum of seven sine waves, produces the EOT in seconds, however this book does not use it in any programs:-

$$
\begin{aligned}
& \mathrm{E}=-\left(-97.8^{\star} \mathrm{SIN}(\mathrm{SL})-431.3^{*} \operatorname{COS}(\mathrm{SL})+596.6^{*} \operatorname{SIN}\left(2^{*} \mathrm{SL}\right)-1.9^{*} \operatorname{COS}\left(2^{\star} \mathrm{SL}\right)+4^{*} \operatorname{SIN}\left(3^{*} \mathrm{SL}\right)+19.3^{*} \operatorname{COS}\left(3^{\star} \mathrm{SL}\right)-\right. \\
& \left.12.7^{*} \operatorname{SIN}(4 * \mathrm{SL})\right)
\end{aligned}
$$

A8.29d
where "SL" is the solar longitude, being $\operatorname{SL=}=\left(-1^{*}((356 / 365.2422) * 360-270)\right)+$ julian day of year the values in $\sin (\ldots)$ result in degrees, so the RADIANS( ) function (not shown) is required for a spreadsheet.

Every approximation is just that, and this book uses several methods for the EOT to demonstrate the real world of approximations, with their benefits as well as drawbacks. Even established published tables vary by almost a minute. Part of this is explained by the year within a leap year cycle, part by the decade the table was printed, and so on. The most accurate formulae use the astronomical Julian day. This is somewhat involved and discussed in more detail in the earlier part of this book. Please refer to appendix 10 which has some other sources of formulae.

## NOMOGRAM STYLES AND ASSOCIATED FORMULAE


$C=L+R$
$C=L+R$
can multiply and divide with logs

$C=L / R$
logs not needed

$C=L$ *
logs not needed
method 4

$1 / C=1 / L+1 / R$ logs not needed

DIALING SCALES (Foster, Serle, Middleton)


For a unit length on the hours scale of 10, the distances from 9 am or 3 pm (1500) are:-

| Time h.hh |  | Dist | Time h.hh |  | st |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6.00 | 12.00 | 5.00 | 9.00 | 15.00 | 0.00 |
| 6.25 | 12.25 | 4.38 | 9.25 | 15.25 | -0.33 |
| 6.50 | 12.50 | 3.84 | 9.50 | 15.50 | -0.66 |
| 6.75 | 12.75 | 3.34 | 9.75 | 15.75 | -0.99 |
| 7.00 | 13.00 | 2.89 | 10.00 | 16.00 | -1.34 |
| 7.25 | 13.25 | 2.47 | 10.25 | 16.25 | -1.70 |
| 7.50 | 13.50 | 2.07 | 10.50 | 16.50 | -2.07 |
| 7.75 | 13.75 | 1.70 | 10.75 | 16.75 | -2.47 |
| 8.00 | 14.00 | 1.34 | 11.00 | 17.00 | -2.89 |
| 8.25 | 14.25 | 0.99 | 11.25 | 17.25 | -3.34 |
| 8.50 | 14.50 | 0.66 | 11.50 | 17.50 | -3.84 |
| 8.75 | 14.75 | 0.33 | 11.75 | 17.75 | -4.38 |
| 9.00 | 15.00 | 0.00 | 12.00 | 18.00 | -5.00 |
| dist $=$ scale[i.e. 10] * tan ( 15 * hours) |  |  |  |  |  |

And the distances for latitude are:

| Latitude | Distance | Latitude | Distance | Latitude | Distance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00 | 28 | 4.24 | 57.50 | 6.38 |
| 2 | 0.35 | 30 | 4.46 | 60.00 | 6.48 |
| 4 | 0.70 | 32 | 4.66 | 65.00 | 6.65 |
| 6 | 1.04 | 34 | 4.86 | 70.00 | 6.80 |
| 8 | 1.38 | 36 | 5.04 | 75.00 | 6.91 |
| 10 | 1.71 | 38 | 5.21 | 80.00 | 7.00 |
| 12 | 2.04 | 40 | 5.37 | 85.00 | 7.05 |
| 14 | 2.35 | 42 | 5.52 | 90.00 | 7.07 |
| 16 | 2.66 | 44 | 5.66 |  |  |
| 18 | 2.95 | 46 | 5.79 |  |  |
| 20 | 3.23 | 48 | 5.91 |  |  |
| 22 | 3.50 | 50 | 6.02 |  |  |
| 24 | 3.76 | 52.50 | 6.15 |  |  |
| 26 | 4.01 | 55.00 | 6.27 |  |  |

dist $=$ scale[i.e. 10] * $\sin (l a t) / \operatorname{sqrt}\left(1+\sin ^{2}(\right.$ lat $\left.)\right)$
A8. 31

A spreadsheet to calculate the scales is available in:-
illustratingShadows.xls
Note: $\sin ^{2}{ }^{2}$ (lat) means:- $\sin (l a t) ~ * ~ \sin (l a t)$
Note: Dialing scales can be used not only for horizontal, vertical, and vertical decliner, but also for polar and meridian dials.

Style Distance from noon

$$
\left.\begin{array}{cc}
\text { SDn } \left.=\quad \operatorname{atan}\left(\begin{array}{c}
\left(\sin (\mathrm{dec})^{*} \sin (\mathrm{inc}) *\right. \\
((\sin (\mathrm{inc}) * \cos (\mathrm{dec})-\tan (\mathrm{lat}) * \cos (\mathrm{inc}))) \\
(\cos (\mathrm{inc})+\tan (\mathrm{lat}) * \cos (\mathrm{dec}) * \sin (\mathrm{inc})))
\end{array}\right)\right)
\end{array}\right)
$$

## A8.32

A8.33

The SD and hour line angles are from the noon line. They are adjusted with the angle from the horizontal to noon distance, and the resulting number subtracted from 90 degrees provides the same data with the vertical as a reference.

Horizontal to noon $=\operatorname{atan}(\tan (\operatorname{dec}) * \cos ($ inc $) \quad)$
A8.34
The hour line angles from noon are:-

$$
\begin{aligned}
&=\quad \operatorname{atan}\left(\quad \left(\left(\left(\begin{array}{l}
(\cos (\text { lat }) * \sin (\mathrm{inc}))- \\
\\
\sin (\text { lat }) * \cos (\mathrm{inc}) * \cos (\mathrm{dec})) * \tan (\text { lha }))+ \\
\cos (\mathrm{inc}) * \sin (\mathrm{dec})) / /
\end{array}\right.\right.\right.\right. \\
&(\cos (\mathrm{dec})+\sin (\mathrm{dec}) * \sin (\text { lat }) * \tan (\text { lha })))
\end{aligned}
$$


hour lines from noon line SH SDn
horizontal to noon
for longitude corrected dials, angles are still from local apparent noon

The formulae used in the spreadsheet are:-
Style Distance from noon
SDn = DEGREES(ATAN((sin(dec)*sin(inc) * ((sin(inc)* $\cos (\mathrm{dec})$
$\left.\left.\left.\left.\left.-\tan (\text { lat })^{\star} \cos (\mathrm{inc})\right) /\left(\cos (\mathrm{inc})+\tan (\mathrm{lat})^{\star} \cos (\mathrm{dec})^{\star} \sin (\mathrm{inc})\right)\right)\right)\right)\right)$
Style Height SH

$$
\mathrm{SH}=\mathrm{DEGREES}\left(\operatorname{ASIN}\left(\left(\cos (l a t) * \sin (\mathrm{inc})^{*} \cos (\mathrm{dec})\right)-\left(\sin (l a t){ }^{\star} \cos (\mathrm{inc})\right)\right)\right)
$$

A8.37

Vertical to noon = DEGREES(ATAN(tan(dec) * cos(inc)))

## A8.38

The hour line angles from the vertical are:-

$$
\begin{aligned}
& \left.=\text { DEGREES(ATAN( ((( (cos(lat)*} \sin ^{\star}(\mathrm{inc})\right)- \\
& \left.\left.\left.\quad \sin (\text { lat })^{\star} \cos (\text { inc })^{\star} \cos (\mathrm{dec})\right)^{\star} \tan (\mathrm{ha})\right)+\cos (\mathrm{inc})^{\star} \sin (\mathrm{dec})\right) \\
& \\
& \left.\left(\cos (\mathrm{dec})+\sin (\mathrm{dec})^{\star} \sin (l a t)^{\star} \tan (\mathrm{lha})\right)\right)
\end{aligned}
$$

## A8.36

## CALENDAR LINE/CURVE FORMULAE (Declination curves, Date curves)

see the page preceding table A4.4 these pages have the descriptive text see the page preceding table A4.5 and pictorials

## CEILING DIALS but also usable for large horizontal dials (even with inaccessible dial center)

Assume that: $\quad \mathrm{vd}=$ vertical distance from the mirror to the ceiling
or
vd = gnomon linear height, from the nodus to the dial plate immediately below


Then from a point on the ceiling immediately above the mirror, or a point on the dial plate immediately below the nodus of a horizontal dial to a point on the north south meridian line would be:-
ceiling above mirror to winter solstice or
below nodus to winter solstice $=v d * \tan (l a t+23.44)$
A8.41a
ceiling above mirror to equinox or
below nodus to equinox $=v d$ * $\tan (l a t)$
A8.41b
ceiling above mirror to summer solstice or
below nodus to summer solstice $\quad=v d * \tan (l a t-23.44)$
A8.41c
ceiling above mirror to dial center o
below nodus to dial center $\quad=\mathrm{vd}$ * ( tan(lat) $+1 / \tan$ (lat) )

And the distance from the north south meridian line, along the equinox line, for an hour line, given an hour line angle is:-

$$
\text { hr.dist }=\tan (\text { hour line angle) } * v d *(\tan (\text { lat })+1 /(\tan (\text { lat }))
$$

A8.41d
NOTE: distance "vd" is depicted at distance "mb" in the appendix of proofs and derivations.

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Simon Wheaton-Smith
ISBN 978-0-9960026-0-8
Library of Congress Control Number: 2014904839



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