Sept 10, 2006

CASE STUDY POLAR DIAL WITH ITALIAN HOUR LINES AND CALENDAR CURVES

This dial will be for Silver City, NM, whose coordinates are:

location lat:	32.75° N	
location long:	108.2° W	
magnetic declination:	10.6° E	

The dial plate from 7 am to 5 pm will be a maximum of 16 inches east to west and 8 inches top to bottom (approximately)

Hour lines will be shown and so also will there be Italian hour lines. Italian hour lines indicate the time since the last sunset, however common practice is to use them to show the time until the upcoming sunset. Italian hour lines, and their cousin the Babylonian hour lines showing time since sunrise, are highly latitude specific. In other words the trick of tilting a dial plate will correct the time of day hours for a latitude shift but such is not the case for Italian and Babylonian hour lines.

A spreadsheet was constructed as below, using the trigonometric formulae, however this data is also available for a gnomon relative linear height of 1 in the appendices.

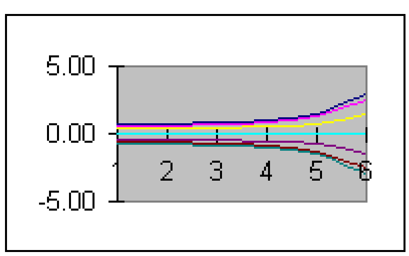
POLAR DIAL - hour line distance as well as calendar points on the hour lines

				GNOMON HT ~ HOUR LINE DISTANCE				
hour	degrees	radians	tan	1	1.5	1.75	2	2.25
1	15	0.2618	0.2679	0.27	0.40	0.47	0.54	0.60
2	30	0.5236	0.5774	0.58	0.87	1.01	1.15	1.30
3	45	0.7854	1.0000	1.00	1.50	1.75	2.00	2.25
4	60	1.0472	1.7321	1.73	2.60	3.03	3.46	3.90
5	75	1.3090	3.7321	3.73	5.60	6.53	7.46	8.40

hour line distance = style linear height * tan(15*time)

	Gnomon height	1.75	Hours from noon					
calendar data:	decl	radians	0	1	2	3	4	5
June solstice	23.50	0.4102	0.76	0.79	0.88	1.08	1.52	2.94
May and July	20.00	0.3491	0.64	0.66	0.74	0.90	1.27	2.46
August and April	12.00	0.2094	0.37	0.39	0.43	0.53	0.74	1.44
September and March equinox	0.00	0.0000	0.00	0.00	0.00	0.00	0.00	0.00
October and February	- 12.00	-0.2094	- 0.37	- 0.39	- 0.43	-0.53	-0.74	- 1.44
November and January	- 20.00	-0.3491	- 0.64	- 0.66	- 0.74	-0.90	-1.27	- 2.46
December solstice	- 23.50	-0.4102	- 0.76	- 0.79	- 0.88	-1.08	-1.52	- 2.94

calendar height on hour line = style linear height * tan (declination) / cos (15*time)



The spreadsheet shows that a 1.75 inch gnomon linear height would generate a dial plate of 6.53 length on either side of noon, or 13.1 inches which fits easily in the 16 inch limitation. Additionally, the largest calendar line spread would be twice 2.94 inches or 5.8 inches which also fits within the 8 inch limitation.

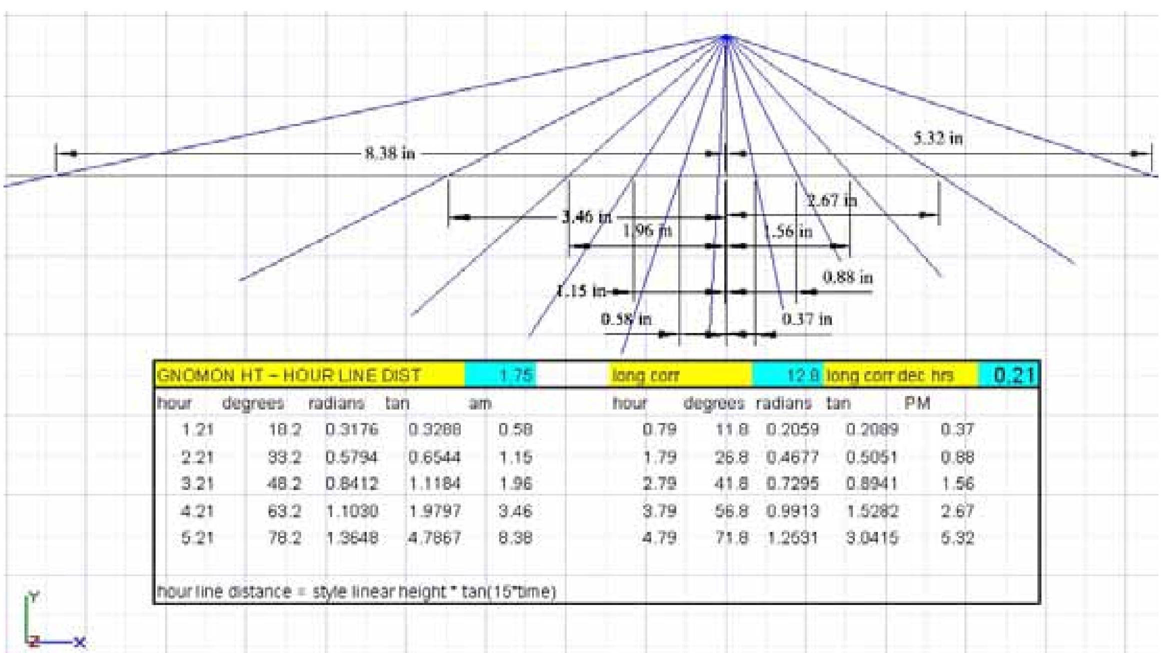
However, the above does not consider longitude corrections, which is about 13 minutes. Those 13 minutes came from 4 minutes per degree of longitudinal distance from the dial (longitude 108.2 west) and the legal time meridian's longitude of 105. The distance is thus 3.2 degrees, placing the actual correction at 12.8 minutes or 12 minutes 48 seconds.

The spreadsheet below shows the new hour distances.

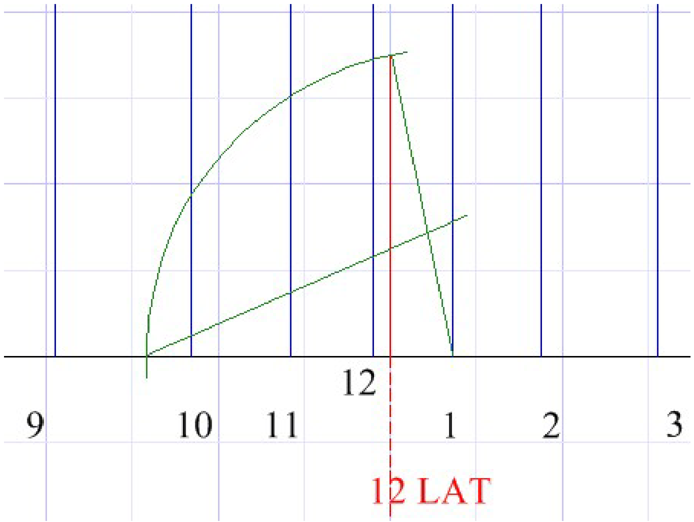
GNOMON HT ~ HOUR LINE DIST				1.75		long corr		12.8	long corr dec hrs	0.21
hour	degrees	radians	tan	am		hour	degrees	radians	tan	PM
1.21	18.2	0.3176	0.3288	0.58		0.79	11.8	0.2059	0.2089	0.37
2.21	33.2	0.5794	0.6544	1.15		1.79	26.8	0.4677	0.5051	0.88
3.21	48.2	0.8412	1.1184	1.96		2.79	41.8	0.7295	0.8941	1.56
4.21	63.2	1.1030	1.9797	3.46		3.79	56.8	0.9913	1.5282	2.67
5.21	78.2	1.3648	4.7867	8.38		4.79	71.8	1.2531	3.0415	5.32
hour line distance = style linear height * tan(15*time)										

Similarly the calendar line distances on each hour line would be different, and the spreadsheet could accommodate that also.

However, the geometric method is worth addressing and will be used in this case study, with the spreadsheet as a cross check. The geometric method will be implemented using CAD as a drafting tool.



The geometric methods which used CAD as a drafting tool, agreed with the spreadsheet on hour distances. A benefit of CAD is that the final drawing may be printed and used as a template for the actual dial.



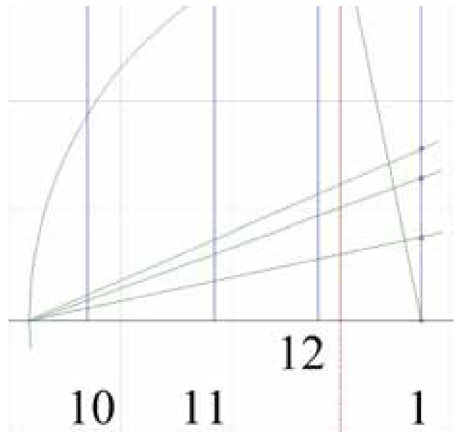
An hour line is selected, in this case 1 pm legal time.

A line is drawn from the base to the top of the style.

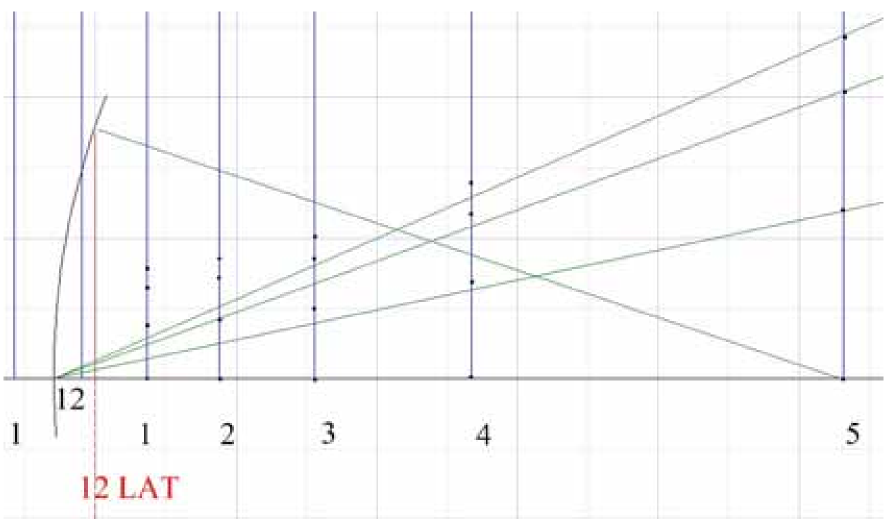
That line is rotated, an arc was used, down to the dial plate.

From where that arc intercepted the dial plate, a line was drawn at the declination, 23.5 in this case, and where that line intersected the original hour line, there is the point for that calendar line for that declination.

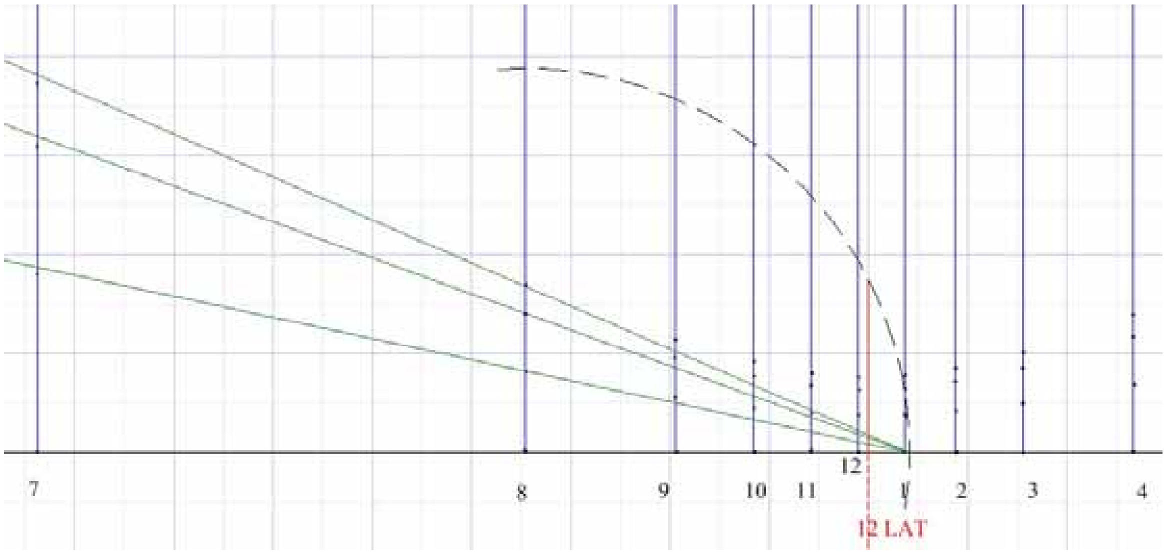
To the right is a close-p of the above, except that lines have been drawn for 12 and 20 degrees, and for completeness the 0 degree equinox has a circle on it, albeit somewhat superfluous.



The CAD system used was TurboCAD whose deluxe version is available for around $100, and also provides 3d modeling. This system allows lines to be grouped, and this facilitates the rest of the calendar lines because once those declination lines have been drawn, their base can be moved and calendar points drawn quickly, it saves re measuring those calendar based declination angles.

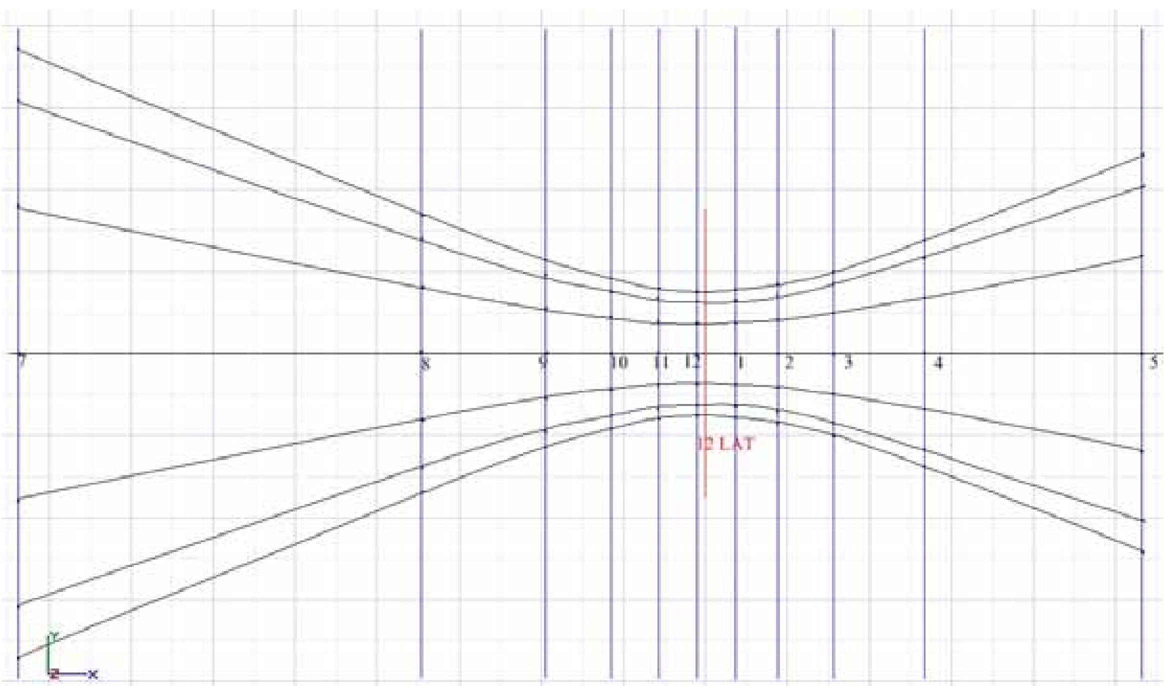


The results of shifting the group of declination angles is shown to the left.



The morning hour calendar lines have been added.

An arc is shown for 0800 standard legal time, together with the 0800 three calendar declination angle lines.



The calendar dots are drawn with a curve drawing function, in this case the Bezier function. And the entire upper half was selected and mirror copied around the equinox line.

Italian lines for the hours until sunset need to be added. Italian lines are actually hours since the last sunset, however in the real world they are used to show hours until the next sunset. This is latitude 32.75, and either the formulae may be used or the tables in the appendix.

Hour angle of rising/setting sun: hsr = arccos(tan(lat) * tan(decl)) from noon

where

Sun Declination: decl = degrees (0.006918 – 0.399912*cos(da) + 0.070257*sin(da)
– 0.006758*cos(2*da) + 0.000907*sin(2*da)
– 0.002697*cos(3*da) + 0.001480*sin(3*da)

where da = Day angle:

da = 2 * pi * (j-1) / 365 (in radians, is an intermediate figure)

where J is the day of the year

J=1 on 1 January, J=365 on 31 December. February being 28 days.

Jan	*Feb*	*Mar*	*Apr*	*May*	*Jun*
0	*31*	*59*	*90*	*120*	*151*
Jly	*Aug*	*Sep*	*Oct*	*Nov*	*Dec*
181	*212*	*243*	*273*	*304*	*334*

alternative formula:

decl = DEGREES = (23.45*sin(radians(0.9678(jd-80)))) source: Claude Hartman

Or the spreadsheet may be used.

Babylonian and Italian values by latitude. Times are hh.mm Local Apparent Time
Solstice Sunrise and Sunset (equinox is 6am/pm). No longitude correction. No EOT correction.

Winter solstice: Declination:			-23.5	Summer solstice: Declination:			+23.5
Lat	Rise	Set	Day length hrs	Lat	Rise	Set	Day length hrs
30	6.58	17.02	10.04	30	5.01	18.59	13.58
31	7.00	17.00	10.00	31	4.59	19.01	14.02
32	7.03	16.57	9.54	32	4.56	19.04	14.08
33	7.05	16.55	9.50	33	4.54	19.06	14.12

Either way, sunset is local apparent time of:-

1655 winter solstice standard time
1906 summer solstice time

Sunset times are local apparent time. With a dial that is not longitude corrected, i.e. showing LAT, then there is no problem. However if the hour lines are longitude corrected then those hour lines cannot be used as the foal points for the Italian lines unless the sunset times are also longitude corrected. Since the hour lines were longitude corrected, the sunset times for the dial's location will be later as the dial is west of the legal time meridian.

The correction is 12.8, almost 13 minutes. So the legal time for sunset will need adjusting:-

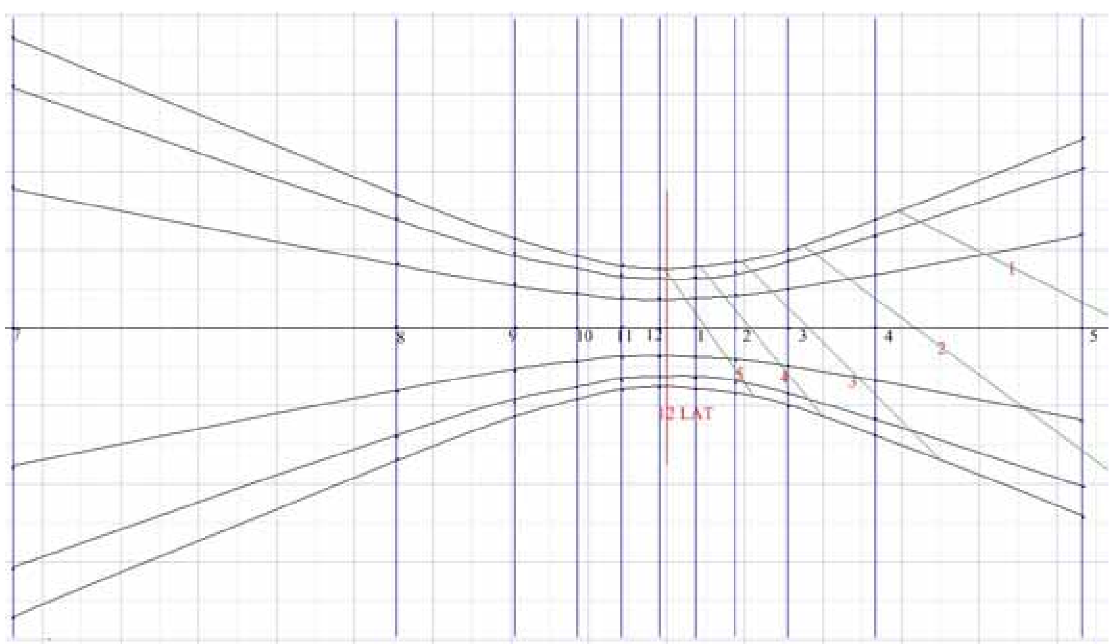
1708	winter solstice standard time
1919	summer solstice time

The sunset time at the equinox is 1800 (6 pm) local time, but that also needs adjusting, so it is now

1813	March or September equinox

A line is drawn from 1708 winter, through 1813 on the equinox to 1919 on the summer solstice line, this is the sunset time. That sunset line will not fit on this dial plate. So now it is backed off one hour to:-

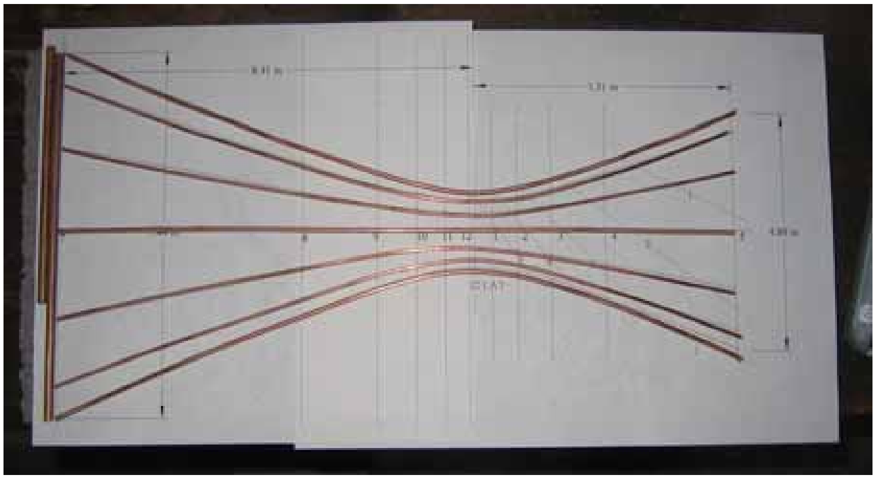
1608	1713	1819	for the 1 hour to sunset Italian line
1508	1613	1719	for the 2 hour to sunset Italian line
1408	1513	1619	for the 3 hour to sunset Italian line
1308	1413	1519	for the 4 hour to sunset Italian line
1208	1313	1419	for the 5 hour to sunset Italian line



The CAD version is then printed out and the dial plate and gnomon constructed.

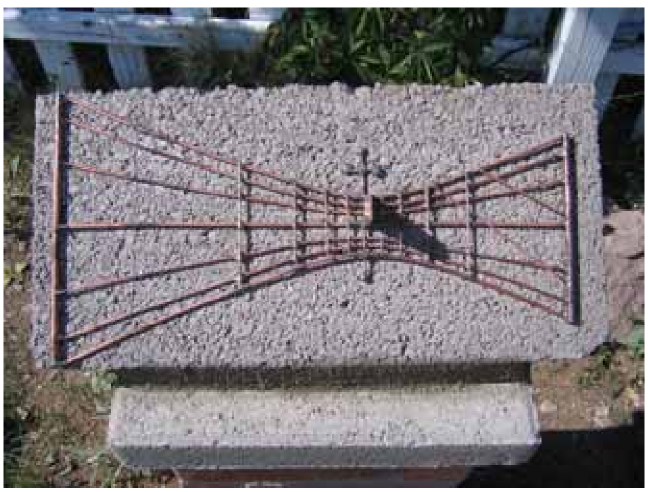
A key point about Italian lines that can never be repeated too many times is that they are latitude dependent. While a polar dial, or any hour angle based dial, can be tilted to correct for a dial's actual latitude compared to its design latitude, this is not true for Italian or Babylonian hour lines. They work for one latitude only.

These Italian lines were adjusted to accommodate the longitude. They result in exactly the same placement were no longitude correction applied to both them and the dial plate hour lines. This is because they are indicating the time right now, here, until the next sunset



The dial plate was constructed. This dial plate was to be copper wire attached to a paver. The ends were of small copper pipe, each of those two end pipes had a 10 gauge copper wire attached to it, and these were set in holes drilled in the concrete paver.

However, before that the hyperbolic calendar lines and the straight equinox line were cut from cupper wire and shaped. Similarly with the hour lines, and the Italian lines.



If the calendar lines are closest to the concrete paver then the hour lines would rest above them, and to keep proportions, the gnomon linear height would be from the top of the hour line wires. The shadow of the gnomon and the hour lines would be read on the concrete paver. Alternatively, the hour lines could be closest to the concrete paver with the calendar lines above them, in which case the gnomon linear height would be from the concrete paver.

The dial was secured by four copper rods to holes in the concrete paver.

The final dial rested on the 12 o'clock pillar of a large garden analemmatic dial.

Foot note: because the tangent of 45 is 1, and because the 3 pm or 9 am line is 45 degrees, for non longitude corrected dials one of the rules of thumb is that the gnomon linear height is the same as the sub-style to 3 pm or 9 am distance. If the dial is longitude corrected, as this one is, then the 3 pm and 9 am rule of thumb does not apply.