

## **ILLUSTRATING TIME'S SHADOW**

### **The Appendices**

**by Simon Wheaton-Smith**



a pine woodcut print

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Simon Wheaton-Smith  
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# THE ILLUSTRATING SHADOWS COLLECTION

Illustrating Shadows provides several books or booklets:-

Simple Shadows	Build a horizontal dial for your location. Appropriate theory.
Cubic Shadows	Introducing a cube dial for your location. Appropriate theory.
Cutting Shadows	Paper cutouts for you to make sundials with.

Illustrating Times Shadow

the big book	Illustrating Times Shadow ~ Some 400 pages covering almost every aspect of dialing. Includes a short appendix.
Appendices	Illustrating Times Shadow ~ The Appendices ~ Some 180 pages of optional detailed appendix material.
Supplement	Supplemental Shadows ~ Material in the form of a series of articles, covers more on the kinds of time, declination confusion, other proofs for the vertical decliner, Saxon, scratch, and mass dials, Islamic prayer times (asr), dial furniture, and so on!

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CAD	DeltaCAD ~ macros for almost all dialing needs in BASIC NanoCAD ~ dial macros written in VBS and Java Script FreeCAD ~ dial macros written in Python Powerdraw ~ dial macros in a Pascal subset ProgeCAD ~ dial macros written in LISP TurboCAD ~ dial macros written in VBS, and parametric part scripts also Blender ~ dial macros written in Python
Languages	Programs in the languages are discussed in Programming Shadows
Spreadsheets	illustratingShadows.xls simpleShadows.xls cubicShadows.xls



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# THE APPENDICES

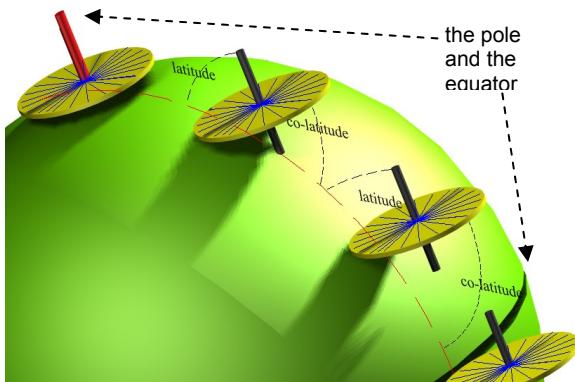
Appendix 1	Geometry and trigonometry Trigonometric functions and geometrical rules and circular measures Tables of trig values – sin, cos, and tan
Appendix 2	Tables independent of latitude or longitude Julian day Declination of the sun by the day The equation of time, generic and astronomical, and century comparisons Sun's apparent hour angle, and standard time to hour angle Longitude to time Aids to locating north, compass correction and astro compass, noon transit time Latitude, Longitude, Magnetic declination information
Appendix 3	Location dependant ~ miscellaneous dial hour line data Hour line angles for horizontal and vertical dials latitude 30 to 60 Reflecting or ceiling dial information. Polar and Meridian dial tables because they are not latitude dependant Equatorial calendar line radii and sunset line distance data
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Appendix 8	Formulae Constants and variables Spreadsheet formulae issues Actual and apparent inconsistencies in the three "classic books" on sundials
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Appendix 10	Bibliography ~ and cross references ~ and various guides Books, software, other references, sundial parts, sundial numerals often used Miscellaneous dates, Limiting hours by dial type, Trivia, Safety tips Testing a sundial at a location other than the design location. Summer time depiction

This document is part of the [www.illustratingshadows.com](http://www.illustratingshadows.com) series of books, booklets, and software. These appendices mostly match those earlier books, however some sections have been re-organized, moved around, renumbered, and formulae standardized. Formulae involving dates use approximations thus the tables herein may disagree with other tables and sources.

Please check [www.illustratingshadows.com](http://www.illustratingshadows.com) for updates to this document. This document may be copied or given to others provided credit is retained to [www.illustratingshadows.com](http://www.illustratingshadows.com) and Simon Wheaton-Smith, and no charge made.

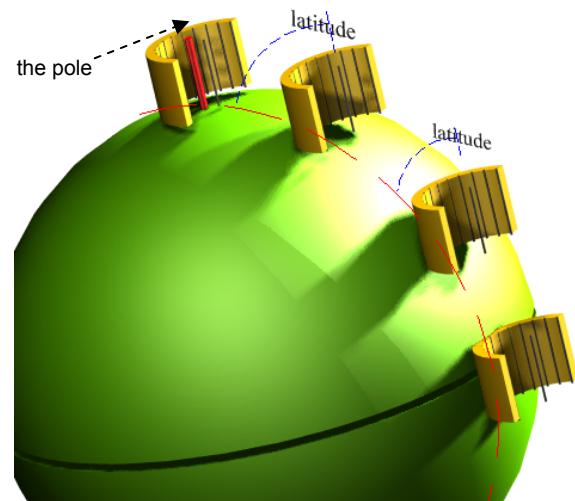
## APPENDIX 1 – TRIGONOMETRY AND GEOMETRY

### THE EQUATORIAL DIAL



The equatorial dial plate, as far as hours go, is independent of latitude. Latitude comes into play only for the sunset line, and setting the gnomon to latitude, with the dial plate holding the hour lines, being set to co-latitude, or  $90^\circ$ –latitude. Longitude and EOT must be applied. The dial plate parallels the equator, which is why this is called an equatorial dial. Some people call the armillary dial an equatorial dial. The time cannot be read as the year approaches an equinox. The hours are marked by  $15^\circ$  radials.

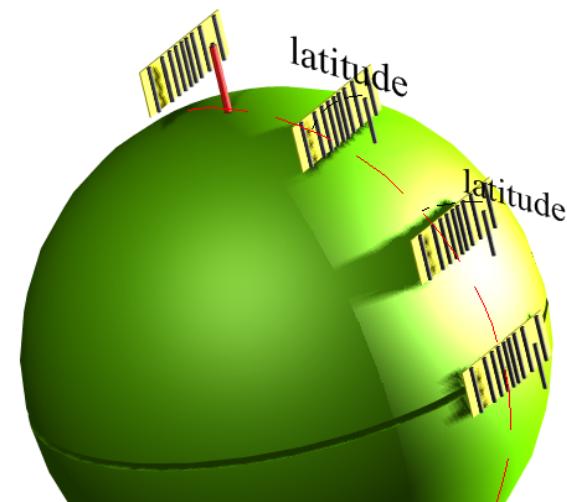
### THE ARMILLARY DIAL



The armillary dial is often called, incorrectly, an equatorial dial. The word armillary relates to an “arm” bracelet surrounding the polar axis. Some equatorial dials do include an armillary at their edge, this allows the time to be read at the equinoxes. Longitude and EOT must be applied. The gnomon and the dial plate are set to latitude. The hours are marked by  $15^\circ$  radials.

Some armillary dials have a number of circles added, they show the solstices, and a depiction of the ecliptic, which is the plane that the earth floats on as it orbits the sun.

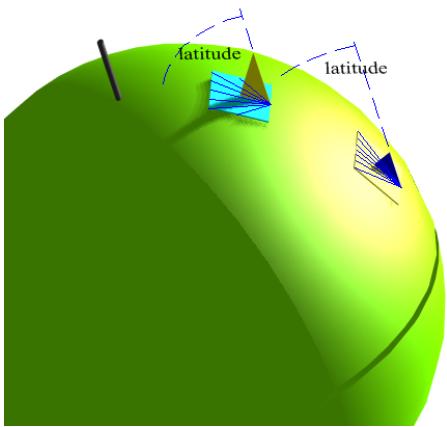
### THE POLAR DIAL



The polar dial plate parallels the polar axis, it is an armillary dial that has been flattened out. It faces the equator. Longitude and EOT must be applied. The gnomon and the dial plate are set to latitude. The hours are marked by the trigonometric “tangent” of the hour time  $15^\circ$ .

A polar dial can be rotated  $90^\circ$  90 degrees around the gnomon, then the dial plate, while still paralleling the polar axis, also parallels the meridian. It is then called a meridian dial.

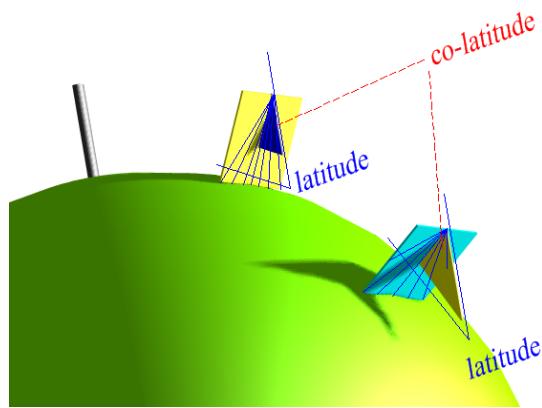
## THE HORIZONTAL DIAL



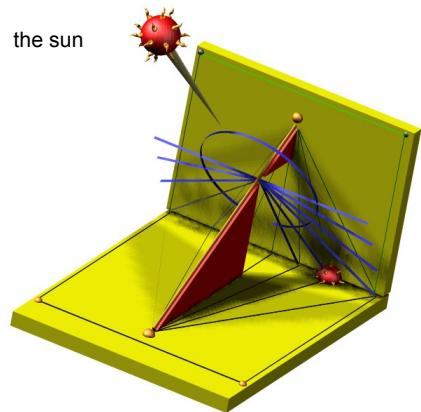
The horizontal dial has a dial plate that parallels the earth's surface, hence it is horizontal. See the picture to the left. Longitude and EOT must be applied. The gnomon is set to latitude. The hour lines are marked by the trigonometric "tangent" of the hour time  $15^\circ$  multiplied by the trigonometric "sine" of the latitude.

## THE VERTICAL DIAL

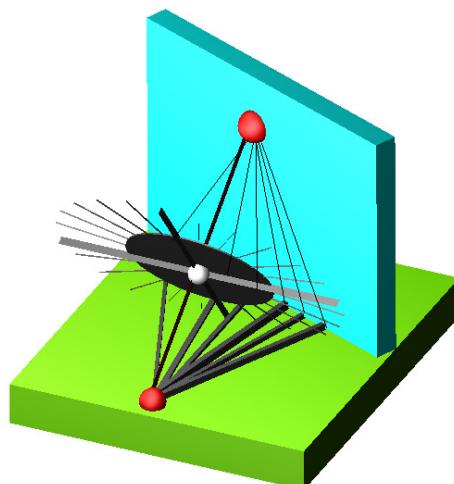
A vertical dial facing the equator is the same design as a horizontal dial for the co-latitude, or,  $90^\circ$ -latitude. See the picture to the right. But if a longitude correction is built in, then it is in the opposite direction because the shadow moves in the opposite direction compared to a horizontal dial. It all works out well.



Vertical dials may sometimes not face true south, they are called vertical decliners since the wall on which the dial is mounted declines. In the extreme, they parallel the meridian, when they become meridian dials.



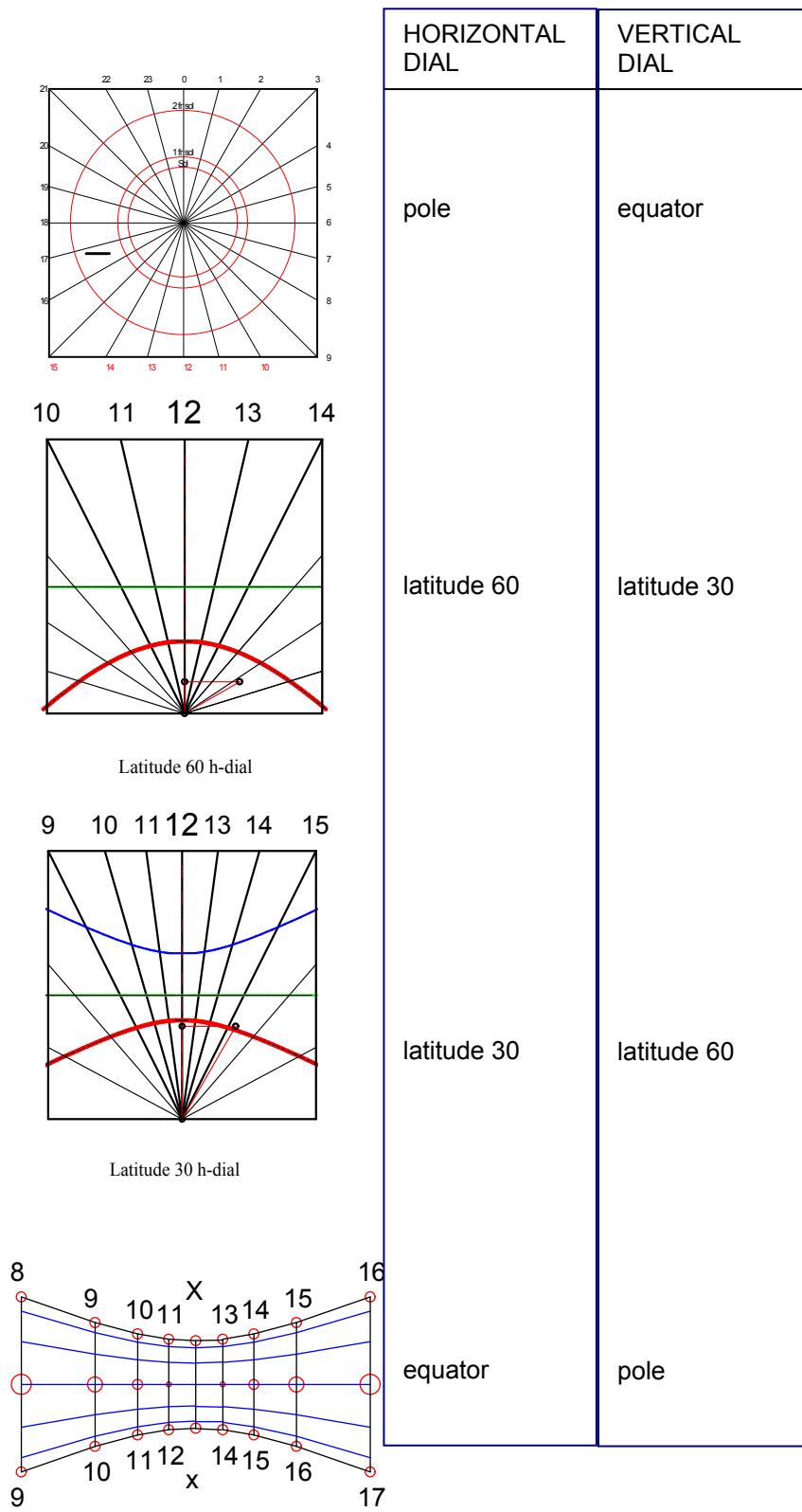
The picture to the left is one based on the work of Albrecht Durer in 1525, and shows a horizontal dial joined to a vertical dial, and an equatorial dial meets them at their junction.



The picture to the right is the same as the one above, however, the vertical dial is rotated, or declined.

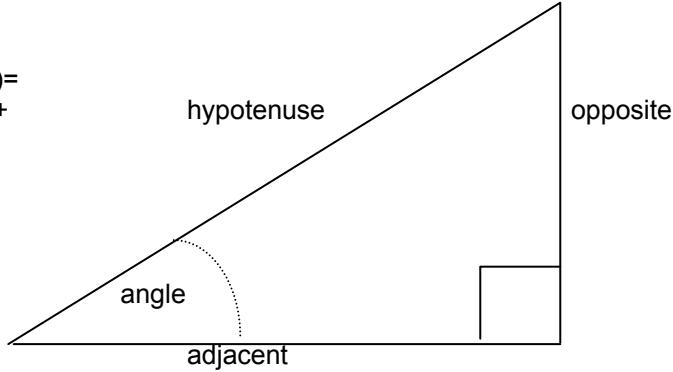
These provide a pictorial overview of most dial types, and from these are derived the geometrical methods, and from those, the mathematical formulae for sundial design.

Relationship between horizontal dials and vertical dials, except that as the shadows rotate in opposite directions, longitude correction if used, is reversed between horizontal and vertical dials.



**Trigonometry** has a set of functions that are defined that make working with angles and lines easy using simple mathematics. The pictorials at the start of this appendix allow dial types to visualize, geometric methods derived, and from those, trigonometric of pure mathematical formula to be derived. There are certain truisms about the geometry and trigonometry, or trigonometric functions.

Pythagoras's Theorem  
 $\text{sqrt}(\text{hypotenuse}) = \text{sqrt}(\text{adjacent}^2 + \text{opposite}^2)$



A right angles triangle has one angle that is 90 degrees, and the side opposite that angle is called the hypotenuse. For an angle other than the right angle, there are defined sides called the opposite side and the adjacent side.

And there are some predefined functions, and tables that give you their values. And spreadsheets provide this service also, however a spreadsheet usually uses "radians" rather than the more common degrees.

definitions:

$\tan(\text{angle})$	=	opposite / adjacent	[tangent]
$\sin(\text{angle})$	=	opposite / hypotenuse	[sine]
$\cos(\text{angle})$	=	adjacent / hypotenuse	[cosine]

The word "co" means complement of. So the cosine is the complement of the sine.

$$\cos(\text{angle}) = \sin(90 - \text{angle})$$

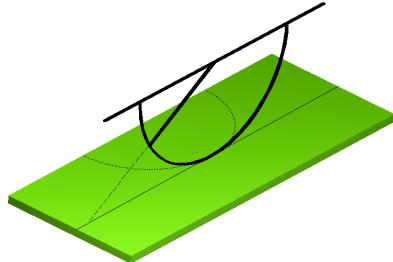
And for the tangent some extra things are true...

$$\begin{aligned} \cotan(\text{angle}) &= \tan(90 - \text{angle}) \\ &= 1 / \tan(\text{angle}) \text{ is easily proved} \\ &\text{because } 90\text{-angle is the opposite angle} \\ &\text{and thus its tan is the old adjacent} \\ &\text{divided by the old opposite.} \end{aligned}$$

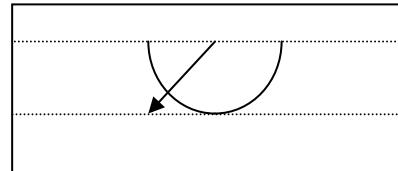
$$\text{Also you can deduce: } \tan(\text{angle}) = \sin(\text{angle}) / \cos(\text{angle})$$

The angle whose sine is  $x$  is referred to as  $\arcsin(x)$ , similarly for  $\arccos$  and  $\arctan$ , or  $\sin^{-1}$ ,  $\cos^{-1}$ , and  $\tan^{-1}$ . Sometimes the symbol  $\sin^{-1}(\text{value})$ ,  $\cos^{-1}$ , and  $\tan^{-1}$  are used.

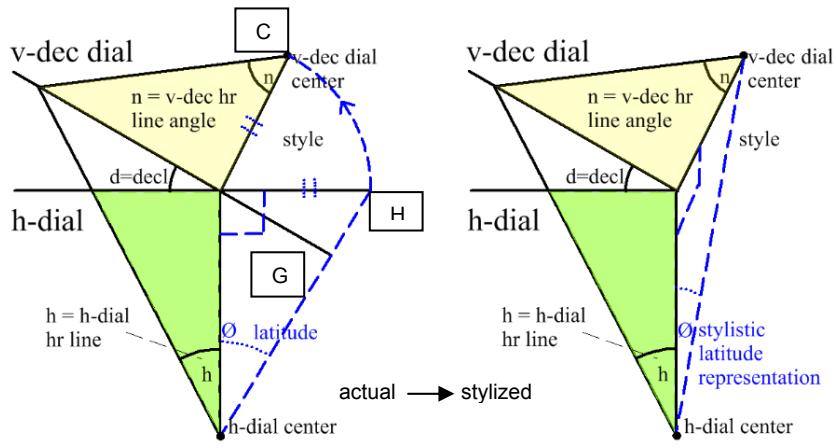
**Geometry** is the drawing of lines, arcs, circles and the like. It relates to trigonometry, however it can be used independently. Geometry can also use techniques such as rotating something on its side, such as a protractor on a gnomon, whereas doing that rotation in trigonometry would be much harder since it would involve three dimensions.



For example, to the left we see a protractor perpendicular to a plane and it has some angle marked. With geometry we can fold the protractor down to be in line with the plane which makes things much easier.



And these projections are very helpful in working out sundial problems. Where geometric methods were used, they were nothing more than rotations of planes by sometimes 90 degrees, sometimes by less.

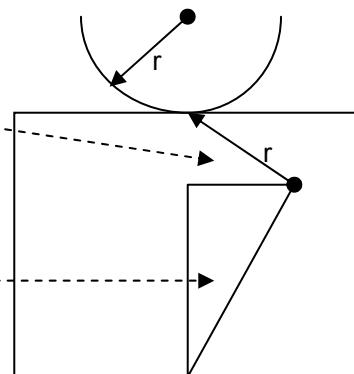


**CAUTION:** The left diagram is the correct geometric model where a gnomon "G" is rotated flat about its sub-style then the nodus linear height "H" is rotated to create what becomes a new dial center "C" for the vertical dial. The diagram to the right is a stylized version of the same thing. This book uses stylized depictions.

**CAUTION:** Some projections are not just 90 degree rotations. For example, in the horizontal dial, the gnomon is rotated 90 degrees first, and then a 90 degree line drawn from the nodus to the plate, and that is then rotated more than 90 degrees in effect, because the protractor's radius is the 90 degree line from the nodus and not the vertical dropped from the nodus to the dial plate.

The protractor's radius comes from rotating the 90 degree projection of the style by 180 degrees - latitude

But first the gnomon is rotated only 90 degrees.

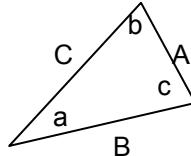


## TRIANGLES AND THINGS

All normal (i.e. non spherical) triangles have angles that add up to 360 degrees, a right angled triangle has one angle of 90 degrees, and an isosoles triangles has two of the angles equal and thus less than 90 degrees, and an equilateral triangle has all three angles equal and they are 60 degrees each.

LAW OF SINES ~ for any triangle:

$$\frac{A}{\sin(a)} = \frac{B}{\sin(b)} = \frac{C}{\sin(c)}$$



## CIRCLES AND THINGS

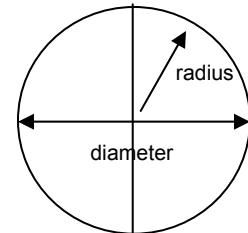
A circle has a center, and the perimeter of the circle is a constant distance from the center, that distance is called the radius.

The diameter is the distance between two points on the perimeter , or circumference, passing through the center, and is twice the length of the radius.

The length of the perimeter, its circumference, is:-

$$\text{circumference} = 2 * 3.1416 * \text{radius}$$

$$\text{circumference} = 3.1416 * \text{diameter}$$



circumference is the length of the perimeter.

The number 3.1416 is called pi, symbol  $\pi$ , and occurs in many circular mathematical expressions.

The circumference is divided up into degrees, degrees of arc, and there are 360 degrees in a circle, or 180 degrees in a semi-circle. Other measures are used, the next most common, especially when using spreadsheets, is the radian. There are  $2 * \pi$  , i.e.  $2 * 3.1416$ , or 6.2832 radians in a circle, so one radian is 57.296 degrees. That strange number actually makes sense when it is found that the trigonometric functions of sin, cos, and tan are derived from series, and readily employ the radian.

$$\begin{aligned}\sin &= x - (x^{**3})/3! + (x^{**5})/5! - (x^{**7})/7! + \dots \\ \cos &= 1 - (x^{**2})/2! + (x^{**4})/4! - (x^{**6})/6! + \dots \\ \tan &= x + (x^{**3})/3 + 2*(x^{**5})/15 + 17*x^{**7}/315 \dots \\ \text{asin} &= \text{atan}(x/\sqrt{1-x^2}) \\ \text{acos} &= (3.1416/2) - \text{asin}(x) \\ \text{atan} &= (x/(1+x^2)) * (1 + (2/3)*(x^2/(1+x^2))) + ((2^4)/(3*5))*(x^2/(1+x^2))*((x^2/(1+x^2))) \dots\end{aligned}$$

is an ATAN series good for x of any value. See the book Supplemental Shadows for programming this and other series.

While the series shown above may not be used much now, they were what the author used back in the IBM 360 and early 370 days when developing font rotation logic using the floating point feature of the computer, such code then was in BAL (basic assembler language).

The radius, diameter, and circumference formulae are helpful in armillary and equatorial dials. The symbol "!" is called a factorial, and is simply a string of multiplications from the number in question down to 1, thus 5! would be  $5*4*3*2*1$  or 120 if my arithmetic serves me well today.

## TRIGONOMETRIC FUNCTIONS

**A1.1**

degrees	radians	sin	cos	tan	cotan	degrees	radians	sin	cos	tan	cotan
1	0.017	0.0175	0.9998	0.0175	57.2900	46	0.803	0.7193	0.6947	1.0355	0.9657
2	0.035	0.0349	0.9994	0.0349	28.6363	47	0.820	0.7314	0.6820	1.0724	0.9325
3	0.052	0.0523	0.9986	0.0524	19.0811	48	0.838	0.7431	0.6691	1.1106	0.9004
4	0.070	0.0698	0.9976	0.0699	14.3007	49	0.855	0.7547	0.6561	1.1504	0.8693
5	0.087	0.0872	0.9962	0.0875	11.4301	50	0.873	0.7660	0.6428	1.1918	0.8391
6	0.105	0.1045	0.9945	0.1051	9.5144	51	0.890	0.7771	0.6293	1.2349	0.8098
7	0.122	0.1219	0.9925	0.1228	8.1443	52	0.908	0.7880	0.6157	1.2799	0.7813
8	0.140	0.1392	0.9903	0.1405	7.1154	53	0.925	0.7986	0.6018	1.3270	0.7536
9	0.157	0.1564	0.9877	0.1584	6.3138	54	0.942	0.8090	0.5878	1.3764	0.7265
10	0.175	0.1736	0.9848	0.1763	5.6713	55	0.960	0.8192	0.5736	1.4281	0.7002
11	0.192	0.1908	0.9816	0.1944	5.1446	56	0.977	0.8290	0.5592	1.4826	0.6745
12	0.209	0.2079	0.9781	0.2126	4.7046	57	0.995	0.8387	0.5446	1.5399	0.6494
13	0.227	0.2250	0.9744	0.2309	4.3315	58	1.012	0.8480	0.5299	1.6003	0.6249
14	0.244	0.2419	0.9703	0.2493	4.0108	59	1.030	0.8572	0.5150	1.6643	0.6009
15	0.262	0.2588	0.9659	0.2679	3.7321	60	1.047	0.8660	0.5000	1.7321	0.5774
16	0.279	0.2756	0.9613	0.2867	3.4874	61	1.065	0.8746	0.4848	1.8040	0.5543
17	0.297	0.2924	0.9563	0.3057	3.2709	62	1.082	0.8829	0.4695	1.8807	0.5317
18	0.314	0.3090	0.9511	0.3249	3.0777	63	1.100	0.8910	0.4540	1.9626	0.5095
19	0.332	0.3256	0.9455	0.3443	2.9042	64	1.117	0.8988	0.4384	2.0503	0.4877
20	0.349	0.3420	0.9397	0.3640	2.7475	65	1.134	0.9063	0.4226	2.1445	0.4663
21	0.367	0.3584	0.9336	0.3839	2.6051	66	1.152	0.9135	0.4067	2.2460	0.4452
22	0.384	0.3746	0.9272	0.4040	2.4751	67	1.169	0.9205	0.3907	2.3559	0.4245
23	0.401	0.3907	0.9205	0.4245	2.3559	68	1.187	0.9272	0.3746	2.4751	0.4040
24	0.419	0.4067	0.9135	0.4452	2.2460	69	1.204	0.9336	0.3584	2.6051	0.3839
25	0.436	0.4226	0.9063	0.4663	2.1445	70	1.222	0.9397	0.3420	2.7475	0.3640
26	0.454	0.4384	0.8998	0.4877	2.0503	71	1.239	0.9455	0.3256	2.9042	0.3443
27	0.471	0.4540	0.8910	0.5095	1.9626	72	1.257	0.9511	0.3090	3.0777	0.3249
28	0.489	0.4695	0.8829	0.5317	1.8807	73	1.274	0.9563	0.2924	3.2709	0.3057
29	0.506	0.4848	0.8746	0.5543	1.8040	74	1.292	0.9613	0.2756	3.4874	0.2867
30	0.524	0.5000	0.8660	0.5774	1.7321	75	1.309	0.9659	0.2588	3.7321	0.2679
31	0.541	0.5150	0.8572	0.6009	1.6643	76	1.326	0.9703	0.2419	4.0108	0.2493
32	0.559	0.5299	0.8480	0.6249	1.6003	77	1.344	0.9744	0.2250	4.3315	0.2309
33	0.576	0.5446	0.8387	0.6494	1.5399	78	1.361	0.9781	0.2079	4.7046	0.2126
34	0.593	0.5592	0.8290	0.6745	1.4826	79	1.379	0.9816	0.1908	5.1446	0.1944
35	0.611	0.5736	0.8192	0.7002	1.4281	80	1.396	0.9848	0.1736	5.6713	0.1763
36	0.628	0.5878	0.8090	0.7265	1.3764	81	1.414	0.9877	0.1564	6.3138	0.1584
37	0.646	0.6018	0.7986	0.7536	1.3270	82	1.431	0.9903	0.1392	7.1154	0.1405
38	0.663	0.6157	0.7880	0.7813	1.2799	83	1.449	0.9925	0.1219	8.1443	0.1228
39	0.681	0.6293	0.7771	0.8098	1.2349	84	1.466	0.9945	0.1045	9.5144	0.1051
40	0.698	0.6428	0.7660	0.8391	1.1918	85	1.484	0.9962	0.0872	11.4301	0.0875
41	0.716	0.6561	0.7547	0.8693	1.1504	86	1.501	0.9976	0.0698	14.301	0.0699
42	0.733	0.6691	0.7431	0.9004	1.1106	87	1.518	0.9986	0.0523	19.081	0.0524
43	0.750	0.6820	0.7314	0.9325	1.0724	88	1.536	0.9994	0.0349	28.636	0.0349
44	0.768	0.6947	0.7193	0.9657	1.0355	89	1.553	0.9998	0.0175	57.290	0.0175
45	0.785	0.7071	0.7071	1.0000	1.0000	90	1.571	1.0000	0.0000	inf	0.0000

## APPENDIX 2 - Tables independent of location

# JULIAN DAY OF YEAR

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
	0	31	59	90	120	151	181	212	243	273	304	334
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

For leap years, add one after day 59.

### ASTRONOMICAL JULIAN DAY:

in the following, Q3=yyyy, R3=mm, S3=dd then the astronomical Julian date is:-

```
=INT(365.25*(4716+(IF((IF(R3>2,1,0))=0,
Q3-1,Q3)))+INT(30.6001*((IF((IF(R3>2,1,0))=0,R3+12,R3))+1)))+S3-1524.5+(2-
INT(IF(IF(R3>2,1,0)=0,Q3-1,Q3)/100)+INT(INT(IF(IF(R3>2,1,0))=0, Q3-
1,Q3))/100)/4))
```

e.g. if yyyy=1957, mm=10, and dd=4.81 then the Julian date is:- 2436116.31

# SUN'S DECLINATION

## DECLINATION OF THE SUN BY THE DAY

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
1	-23.1	-17.3	-7.9	4.2	14.8	21.9	23.2	18.2	8.6	-2.9	-14.2	-21.7
2	-23.0	-17.1	-7.5	4.6	15.1	22.1	23.1	18.0	8.2	-3.3	-14.5	-21.8
3	-22.9	-16.8	-7.1	5.0	15.4	22.2	23.0	17.7	7.8	-3.6	-14.8	-22.0
4	-22.8	-16.5	-6.7	5.4	15.7	22.3	23.0	17.5	7.5	-4.0	-15.1	-22.1
5	-22.7	-16.2	-6.3	5.8	16.0	22.5	22.9	17.2	7.1	-4.4	-15.5	-22.3
6	-22.6	-15.9	-6.0	6.2	16.3	22.6	22.8	16.9	6.7	-4.8	-15.8	-22.4
7	-22.5	-15.6	-5.6	6.5	16.6	22.7	22.7	16.6	6.4	-5.2	-16.1	-22.5
8	-22.3	-15.3	-5.2	6.9	16.9	22.8	22.6	16.4	6.0	-5.6	-16.4	-22.6
9	-22.2	-14.9	-4.8	7.3	17.1	22.9	22.5	16.1	5.6	-6.0	-16.7	-22.7
10	-22.1	-14.6	-4.4	7.7	17.4	23.0	22.4	15.8	5.2	-6.3	-16.9	-22.8
11	-21.9	-14.3	-4.0	8.0	17.7	23.0	22.2	15.5	4.9	-6.7	-17.2	-22.9
12	-21.8	-14.0	-3.6	8.4	17.9	23.1	22.1	15.2	4.5	-7.1	-17.5	-23.0
13	-21.6	-13.6	-3.2	8.8	18.2	23.2	22.0	14.9	4.1	-7.5	-17.8	-23.1
14	-21.4	-13.3	-2.8	9.1	18.4	23.2	21.8	14.6	3.7	-7.8	-18.0	-23.2
15	-21.3	-13.0	-2.4	9.5	18.7	23.3	21.7	14.3	3.3	-8.2	-18.3	-23.2
16	-21.1	-12.6	-2.0	9.8	18.9	23.3	21.5	14.0	3.0	-8.6	-18.6	-23.3
17	-20.9	-12.3	-1.6	10.2	19.1	23.4	21.3	13.7	2.6	-9.0	-18.8	-23.3
18	-20.7	-11.9	-1.3	10.5	19.4	23.4	21.2	13.4	2.2	-9.3	-19.1	-23.4
19	-20.5	-11.6	-0.9	10.9	19.6	23.4	21.0	13.0	1.8	-9.7	-19.3	-23.4
20	-20.3	-11.2	-0.5	11.2	19.8	23.4	20.8	12.7	1.4	-10.1	-19.5	-23.4
21	-20.1	-10.8	-0.1	11.6	20.0	23.5	20.6	12.4	1.0	-10.4	-19.8	-23.4
22	-19.9	-10.5	0.3	11.9	20.2	23.5	20.4	12.0	0.6	-10.8	-20.0	-23.4
23	-19.6	-10.1	0.7	12.3	20.4	23.5	20.2	11.7	0.2	-11.1	-20.2	-23.4
24	-19.4	-9.7	1.1	12.6	20.6	23.4	20.0	11.4	-0.1	-11.5	-20.4	-23.4
25	-19.2	-9.4	1.5	12.9	20.8	23.4	19.8	11.0	-0.5	-11.8	-20.6	-23.4
26	-18.9	-9.0	1.9	13.3	21.0	23.4	19.6	10.7	-0.9	-12.2	-20.8	-23.4
27	-18.7	-8.6	2.3	13.6	21.2	23.4	19.4	10.3	-1.3	-12.5	-21.0	-23.3
28	-18.4	-8.3	2.7	13.9	21.3	23.3	19.2	10.0	-1.7	-12.9	-21.2	-23.3
29	-18.2		3.1	14.2	21.5	23.3	18.9	9.6	-2.1	-13.2	-21.4	-23.3
30	-17.9		3.5	14.5	21.7	23.2	18.7	9.3	-2.5	-13.5	-21.5	-23.2
31	-17.6		3.9	21.8		18.5		8.9		-13.9		-23.1

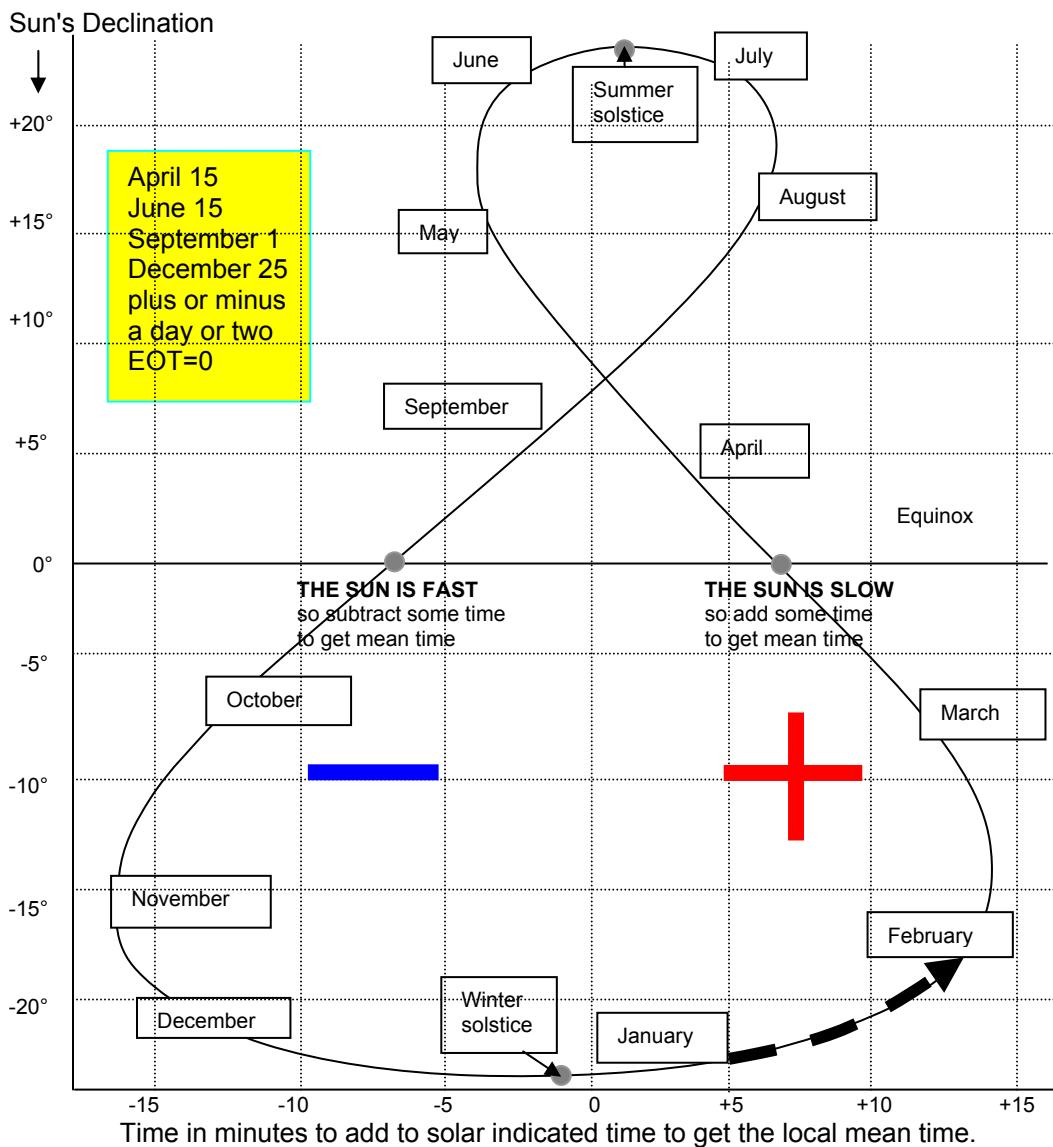
$\text{DEGREES}(0.006918 - 0.399912 \cdot \cos((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.070257 \cdot \sin((2 \cdot 3.1416 \cdot (jd-1)) / 365) - 0.006758 \cdot \cos(2 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.000907 \cdot \sin(2 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) - 0.002697 \cdot \cos(3 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.00148 \cdot \sin(3 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)))$

$\text{DEGREES} = (23.45 \cdot \sin(\text{radians}(0.9678 \cdot jd - 80)))$  alternative formula agrees within half a degree

Different declination charts may disagree, factors affecting them would be leap year approximations, and the formula employed. Many formulae are approximations.

# EOT ~ EQUATION OF TIME

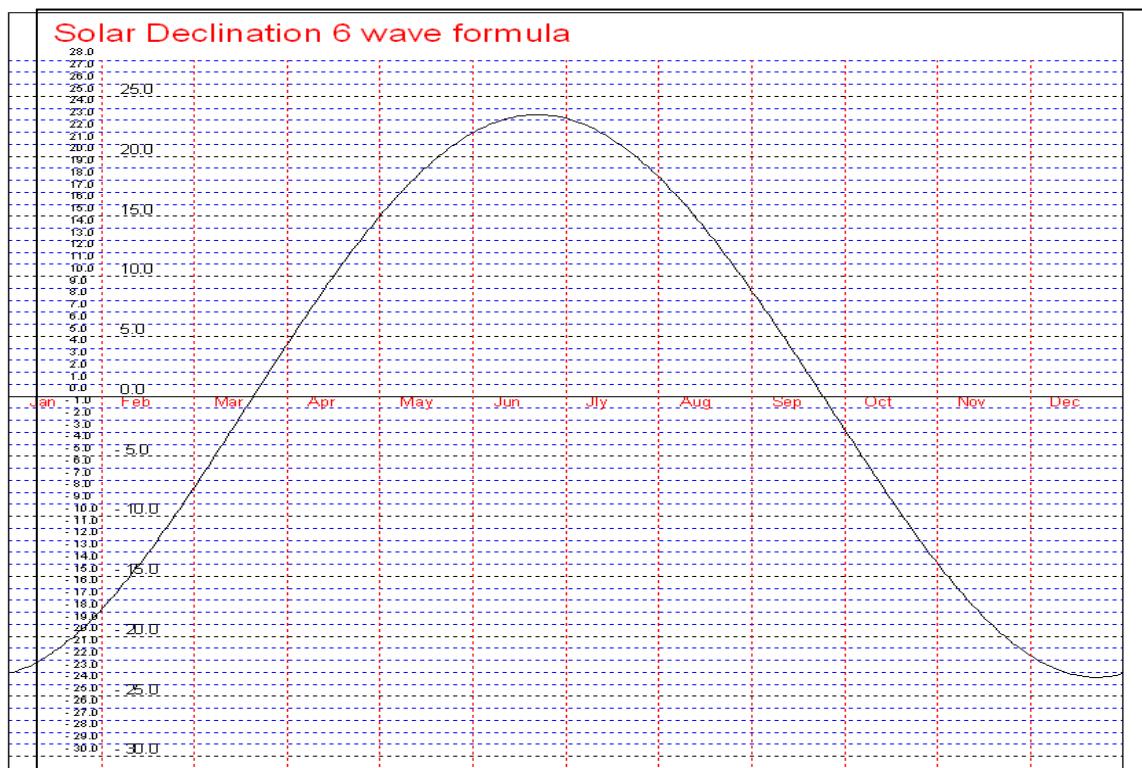
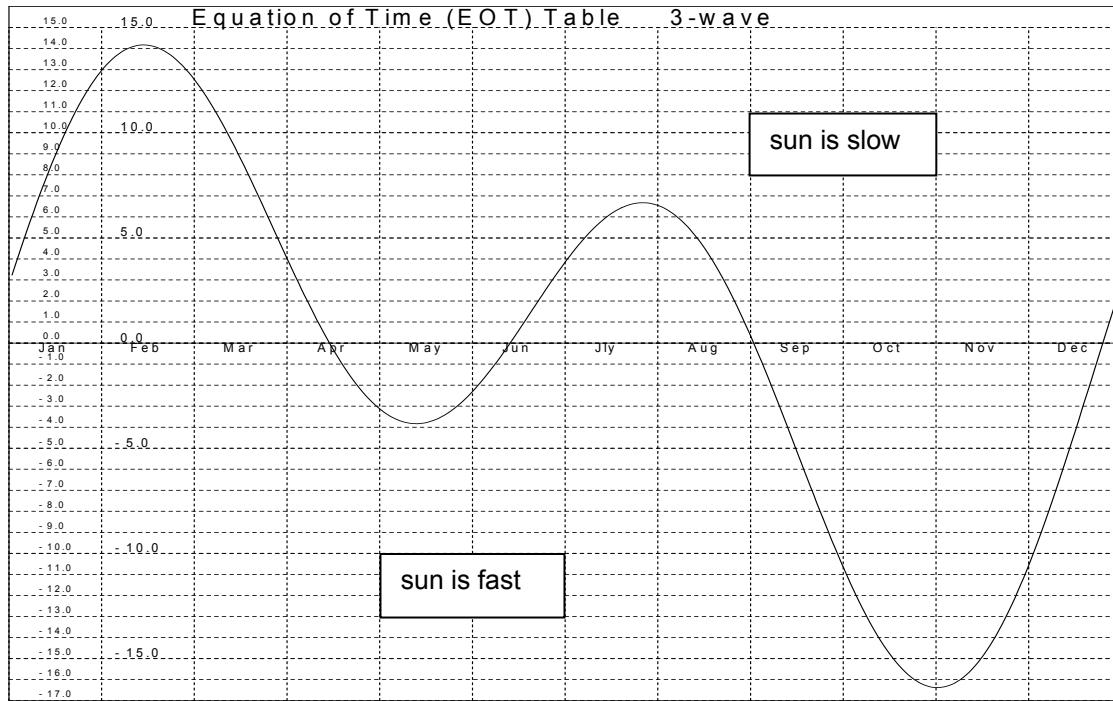
If "+" then add to solar time to get mean time as the sun is slow, if "–" then subtract. Some sources reverse signs. Formulae involving dates use approximations thus these tables may disagree with sources using other formulae.



Sundial indicates + EOT => Local mean time + longitude correction => Standard mean time  
 Local Apparent Time

Chapter 25 deals with analemmas and placing them on dial plates as well as gnomons.

## EQUATION OF TIME AND DECLINATION



ASTRONOMICAL Julian EOT, equinox & solstice data, peak EOT values for: **2016**  
also usable for **2020, 2024, 2028, 2032, ...**

**A2.1**

EQUATION OF TIME (EOT) FOR YEAR:				2016	LEAP YEAR		mm.ss					
1	2	3	4	5	6	7	8	9	10	11	12	
Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec	
1	3.05	13.27	12.20	3.52	-2.54	-2.10	3.52	6.21	0.02	-10.18	-16.27	-10.59
2	3.33	13.35	12.08	3.34	-3.01	-2.00	4.03	6.17	-0.17	-10.38	-16.29	-10.36
3	4.01	13.43	11.56	3.16	-3.07	-1.51	4.14	6.12	-0.37	-10.57	-16.29	-10.13
4	4.29	13.50	11.43	2.59	-3.13	-1.40	4.25	6.07	-0.56	-11.15	-16.28	-9.49
5	4.56	13.56	11.29	2.42	-3.18	-1.30	4.36	6.01	-1.16	-11.34	-16.27	-9.24
6	5.23	14.01	11.15	2.25	-3.22	-1.19	4.46	5.54	-1.36	-11.52	-16.25	-8.59
7	5.50	14.05	11.01	2.08	-3.27	-1.08	4.56	5.47	-1.57	-12.09	-16.22	-8.33
8	6.16	14.09	10.47	1.51	-3.30	-0.57	5.05	5.40	-2.17	-12.26	-16.18	-8.07
9	6.41	14.11	10.31	1.35	-3.33	-0.45	5.14	5.31	-2.38	-12.43	-16.13	-7.40
10	7.06	14.13	10.16	1.19	-3.35	-0.33	5.23	5.22	-2.59	-12.59	-16.08	-7.13
11	7.31	14.14	10.00	1.03	-3.37	-0.21	5.31	5.13	-3.20	-13.15	-16.01	-6.46
12	7.55	14.15	9.44	0.47	-3.38	-0.09	5.39	5.03	-3.41	-13.31	-15.54	-6.18
13	8.18	14.14	9.28	0.32	-3.39	0.04	5.46	4.53	-4.02	-13.45	-15.46	-5.50
14	8.41	14.13	9.12	0.17	-3.39	0.16	5.53	4.42	-4.24	-13.60	-15.37	-5.21
15	9.03	14.11	8.55	0.02	-3.39	0.29	5.59	4.30	-4.45	-14.14	-15.27	-4.53
16	9.24	14.08	8.38	-0.12	-3.38	0.42	6.05	4.18	-5.06	-14.27	-15.16	-4.24
17	9.45	14.05	8.20	-0.26	-3.36	0.55	6.10	4.05	-5.28	-14.39	-15.04	-3.55
18	10.05	14.01	8.03	-0.39	-3.34	1.08	6.15	3.52	-5.49	-14.51	-14.52	-3.25
19	10.24	13.56	7.46	-0.52	-3.31	1.21	6.19	3.38	-6.11	-15.03	-14.39	-2.56
20	10.43	13.50	7.28	-1.05	-3.28	1.34	6.23	3.24	-6.32	-15.14	-14.25	-2.26
21	11.01	13.44	7.10	-1.17	-3.24	1.47	6.26	3.10	-6.53	-15.24	-14.10	-1.56
22	11.18	13.37	6.52	-1.29	-3.20	2.00	6.28	2.55	-7.14	-15.33	-13.54	-1.27
23	11.35	13.30	6.34	-1.40	-3.15	2.13	6.30	2.39	-7.35	-15.42	-13.38	-0.57
24	11.50	13.22	6.16	-1.51	-3.10	2.26	6.31	2.23	-7.56	-15.50	-13.20	-0.27
25	12.05	13.13	5.58	-2.02	-3.04	2.39	6.32	2.07	-8.17	-15.57	-13.02	0.03
26	12.19	13.03	5.40	-2.12	-2.57	2.51	6.32	1.50	-8.38	-16.04	-12.43	0.32
27	12.33	12.54	5.22	-2.21	-2.51	3.04	6.32	1.33	-8.59	-16.10	-12.24	1.02
28	12.45	12.43	5.04	-2.30	-2.43	3.16	6.31	1.15	-9.19	-16.15	-12.04	1.31
29	12.57	12.32	4.45	-2.38	-2.36	3.28	6.29	0.57	-9.39	-16.19	-11.43	2.00
30	13.08		4.27	-2.46	-2.27	3.40	6.27	0.39	-9.59	-16.23	-11.21	2.29
31	13.18		4.10		-2.19		6.24	0.21		-16.26		2.58

EOT	2016		
MAJOR PEAKS	MINOR PEAKS		
MAX	MIN	MAX	MIN
14.15	-16.29	6.32	-3.39

Solstice and Equinox for this year			
yyyy	mm	dd	Astronomical Julian Day
2016	1	1	--> 2457388.5
2016			

Julian day and day of year for the event			
March equinox:	2457467.7	79.19	
June Solstice:	2457560.4	171.93	
September equinox:	2457654.1	265.59	
December solstice:	2457743.9	355.44	

ASTRONOMICAL Julian EOT, equinox & solstice data, peak EOT values for: **2017**  
also usable for **2021, 2025, 2029, 2033, ...**

EQUATION OF TIME (EOT) FOR YEAR:				2017	NON LEAP YEAR		mm.ss					
	1	2	3	4	5	6	7	8	9	10	11	12
	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
1	3.26	13.33	12.23	3.56	-2.52	-2.12	3.49	6.22	0.07	-10.14	-16.27	-11.05
2	3.54	13.41	12.11	3.38	-2.59	-2.03	4.01	6.18	-0.13	-10.33	-16.28	-10.42
3	4.22	13.48	11.59	3.21	-3.05	-1.53	4.12	6.13	-0.32	-10.52	-16.29	-10.18
4	4.50	13.54	11.46	3.03	-3.11	-1.43	4.23	6.08	-0.52	-11.11	-16.29	-9.55
5	5.17	13.60	11.33	2.46	-3.17	-1.32	4.33	6.02	-1.11	-11.29	-16.28	-9.30
6	5.43	14.04	11.19	2.29	-3.21	-1.22	4.44	5.56	-1.31	-11.47	-16.26	-9.05
7	6.09	14.08	11.05	2.12	-3.26	-1.11	4.54	5.49	-1.52	-12.05	-16.23	-8.39
8	6.35	14.11	10.50	1.55	-3.29	-0.59	5.03	5.42	-2.12	-12.22	-16.19	-8.13
9	7.00	14.13	10.35	1.39	-3.32	-0.48	5.12	5.33	-2.33	-12.39	-16.15	-7.47
10	7.25	14.14	10.20	1.23	-3.35	-0.36	5.21	5.25	-2.54	-12.56	-16.09	-7.20
11	7.49	14.15	10.04	1.07	-3.37	-0.24	5.29	5.15	-3.15	-13.11	-16.03	-6.53
12	8.12	14.14	9.48	0.51	-3.38	-0.12	5.37	5.06	-3.36	-13.27	-15.56	-6.25
13	8.35	14.13	9.32	0.36	-3.39	0.01	5.44	4.55	-3.57	-13.42	-15.48	-5.57
14	8.57	14.11	9.16	0.21	-3.39	0.13	5.51	4.44	-4.18	-13.56	-15.39	-5.28
15	9.19	14.09	8.59	0.06	-3.39	0.26	5.58	4.33	-4.40	-14.10	-15.29	-4.60
16	9.40	14.06	8.42	-0.08	-3.38	0.39	6.04	4.21	-5.01	-14.24	-15.19	-4.31
17	10.00	14.02	8.25	-0.22	-3.36	0.52	6.09	4.08	-5.22	-14.36	-15.07	-4.02
18	10.20	13.57	8.07	-0.36	-3.34	1.05	6.14	3.55	-5.44	-14.49	-14.55	-3.32
19	10.38	13.52	7.50	-0.49	-3.32	1.18	6.18	3.42	-6.05	-15.00	-14.42	-3.03
20	10.57	13.46	7.32	-1.02	-3.29	1.31	6.22	3.28	-6.27	-15.11	-14.28	-2.33
21	11.14	13.39	7.14	-1.14	-3.25	1.44	6.25	3.13	-6.48	-15.21	-14.14	-2.04
22	11.31	13.32	6.56	-1.26	-3.21	1.57	6.28	2.58	-7.09	-15.31	-13.58	-1.34
23	11.46	13.24	6.38	-1.38	-3.16	2.10	6.30	2.43	-7.30	-15.40	-13.42	-1.04
24	12.02	13.15	6.20	-1.49	-3.11	2.23	6.31	2.27	-7.51	-15.48	-13.25	-0.34
25	12.16	13.06	6.02	-1.59	-3.05	2.36	6.32	2.11	-8.12	-15.56	-13.07	-0.05
26	12.29	12.56	5.44	-2.09	-2.59	2.48	6.33	1.54	-8.33	-16.02	-12.48	0.25
27	12.42	12.46	5.26	-2.19	-2.52	3.01	6.32	1.37	-8.54	-16.08	-12.29	0.54
28	12.54	12.35	5.08	-2.28	-2.45	3.13	6.31	1.20	-9.14	-16.14	-12.09	1.24
29	13.05		4.50	-2.36	-2.37	3.26	6.30	1.02	-9.34	-16.18	-11.48	1.53
30	13.15		4.32	-2.44	-2.29	3.38	6.28	0.44	-9.54	-16.22	-11.27	2.22
31	13.25		4.14		-2.21		6.25	0.25		-16.25		2.51
												3.19

EOT 2017

MAJOR PEAKS		MINOR PEAKS	
MAX	MIN	MAX	MIN
14.15	-16.29	6.33	-3.39

Solstice and Equinox for this year

yyyy	mm	dd	Astronomical Julian Day
2017	1	1	--> 2457754.5
2017			

Julian day and day of year for the event

March equinox:	2457832.9	78.43
June Solstice:	2457925.7	171.18
September equinox:	2458019.3	264.83
December solstice:	2458109.2	354.69

ASTRONOMICAL Julian EOT, equinox & solstice data, peak EOT values for: **2018**  
also usable for **2022, 2026, 2030, 2034, ...**

EQUATION OF TIME (EOT) FOR YEAR:			2018	NON LEAP YEAR		<b>mm.ss</b>											
	1	2	3	4	5	6	7	8	9	10	11	12					
	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec					
1	3.19	13.31	12.26	4.00	-2.50	-2.14	3.47	6.23	0.11	-10.09	-16.27	-11.10					
2	3.47	13.39	12.14	3.43	-2.57	-2.05	3.58	6.19	-0.08	-10.28	-16.28	-10.48					
3	4.15	13.46	12.02	3.25	-3.04	-1.55	4.09	6.15	-0.27	-10.47	-16.29	-10.24					
4	4.43	13.53	11.49	3.07	-3.10	-1.45	4.20	6.10	-0.47	-11.06	-16.29	-10.01					
5	5.10	13.58	11.36	2.50	-3.15	-1.35	4.31	6.04	-1.06	-11.25	-16.28	-9.36					
6	5.37	14.03	11.22	2.33	-3.20	-1.24	4.41	5.58	-1.27	-11.43	-16.26	-9.11					
7	6.03	14.07	11.08	2.16	-3.24	-1.13	4.51	5.51	-1.47	-12.01	-16.24	-8.46					
8	6.29	14.10	10.54	1.59	-3.28	-1.02	5.01	5.44	-2.07	-12.18	-16.20	-8.20					
9	6.54	14.12	10.39	1.43	-3.31	-0.50	5.10	5.36	-2.28	-12.35	-16.16	-7.53					
10	7.19	14.14	10.23	1.26	-3.34	-0.39	5.19	5.27	-2.49	-12.52	-16.11	-7.27					
11	7.43	14.14	10.08	1.10	-3.36	-0.27	5.27	5.18	-3.10	-13.08	-16.05	-6.59					
12	8.06	14.14	9.52	0.55	-3.38	-0.14	5.35	5.08	-3.31	-13.23	-15.58	-6.32					
13	8.29	14.13	9.36	0.39	-3.39	-0.02	5.43	4.58	-3.52	-13.38	-15.50	-6.04					
14	8.52	14.12	9.19	0.24	-3.39	0.10	5.50	4.47	-4.13	-13.53	-15.41	-5.35					
15	9.14	14.10	9.03	0.10	-3.39	0.23	5.56	4.36	-4.34	-14.07	-15.32	-5.07					
16	9.35	14.06	8.46	-0.05	-3.38	0.36	6.02	4.24	-4.56	-14.20	-15.21	-4.38					
17	9.55	14.03	8.29	-0.19	-3.37	0.49	6.08	4.12	-5.17	-14.33	-15.10	-4.09					
18	10.15	13.58	8.11	-0.33	-3.35	1.02	6.13	3.59	-5.39	-14.46	-14.58	-3.40					
19	10.34	13.53	7.54	-0.46	-3.32	1.15	6.17	3.45	-6.00	-14.57	-14.45	-3.10					
20	10.52	13.47	7.36	-0.59	-3.29	1.28	6.21	3.31	-6.21	-15.08	-14.32	-2.41					
21	11.10	13.40	7.19	-1.11	-3.26	1.41	6.25	3.17	-6.43	-15.19	-14.17	-2.11					
22	11.26	13.33	7.01	-1.23	-3.22	1.54	6.27	3.02	-7.04	-15.29	-14.02	-1.41					
23	11.43	13.25	6.43	-1.35	-3.17	2.07	6.30	2.47	-7.25	-15.38	-13.46	-1.11					
24	11.58	13.17	6.25	-1.46	-3.12	2.20	6.31	2.31	-7.46	-15.46	-13.29	-0.42					
25	12.12	13.08	6.07	-1.57	-3.06	2.33	6.32	2.15	-8.07	-15.54	-13.11	-0.12					
26	12.26	12.58	5.48	-2.07	-3.00	2.46	6.33	1.58	-8.28	-16.01	-12.53	0.18					
27	12.39	12.48	5.30	-2.16	-2.54	2.58	6.33	1.41	-8.49	-16.07	-12.34	0.47					
28	12.51	12.37	5.12	-2.26	-2.47	3.11	6.32	1.24	-9.09	-16.13	-12.14	1.17					
29	13.02		4.54	-2.34	-2.39	3.23	6.30	1.06	-9.29	-16.17	-11.53	1.46					
30	13.13		4.36	-2.42	-2.31	3.35	6.28	0.48	-9.49	-16.21	-11.32	2.15					
31	13.22		4.18	-2.23			6.26	0.30		-16.24		2.44					
												3.12					

EOT	2018
MAJOR PEAKS	MINOR PEAKS
MAX	MIN
14.14	-16.29
6.33	-3.39

Solstice and Equinox for this year			
yyyy	mm	dd	Astronomical Julian Day
2018	1	1	--> 2458199.5
2018			

Julian day and day of year for the event			
March equinox:	2458198.2	78.67	
June Solstice:	2458290.9	171.42	
September equinox:	2458384.6	265.07	
December solstice:	2458474.4	354.93	

ASTRONOMICAL Julian EOT, equinox & solstice data, peak EOT values for: **2019**  
also usable for **2023, 2027, 2031, 2035, ...**

EQUATION OF TIME (EOT) FOR YEAR:				2019				NON LEAP YEAR				<b>mm.ss</b>			
	1	2	3	4	5	6	7	8	9	10	11	12			
	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec			
1	3.12	13.29	12.29	4.05	-2.48	-2.16	3.44	6.24	0.16	-10.04	-16.26	-11.16			
2	3.41	13.37	12.17	3.47	-2.55	-2.07	3.55	6.20	-0.03	-10.24	-16.28	-10.53			
3	4.08	13.44	12.05	3.29	-3.02	-1.57	4.07	6.16	-0.22	-10.43	-16.29	-10.30			
4	4.36	13.51	11.52	3.12	-3.08	-1.47	4.18	6.11	-0.42	-11.02	-16.29	-10.06			
5	5.03	13.57	11.39	2.54	-3.14	-1.37	4.29	6.05	-1.02	-11.20	-16.28	-9.42			
6	5.30	14.02	11.25	2.37	-3.19	-1.27	4.39	5.59	-1.22	-11.39	-16.27	-9.17			
7	5.56	14.06	11.11	2.20	-3.23	-1.16	4.49	5.53	-1.42	-11.56	-16.24	-8.52			
8	6.22	14.09	10.57	2.03	-3.27	-1.05	4.59	5.46	-2.02	-12.14	-16.21	-8.26			
9	6.48	14.12	10.42	1.47	-3.31	-0.53	5.08	5.38	-2.23	-12.31	-16.17	-8.00			
10	7.12	14.13	10.27	1.30	-3.33	-0.41	5.17	5.29	-2.44	-12.48	-16.12	-7.33			
11	7.37	14.14	10.12	1.14	-3.36	-0.29	5.26	5.20	-3.04	-13.04	-16.06	-7.06			
12	8.01	14.14	9.56	0.59	-3.37	-0.17	5.34	5.11	-3.26	-13.20	-15.59	-6.39			
13	8.24	14.14	9.40	0.43	-3.38	-0.05	5.41	5.01	-3.47	-13.35	-15.52	-6.11			
14	8.46	14.12	9.23	0.28	-3.39	0.08	5.48	4.50	-4.08	-13.49	-15.44	-5.42			
15	9.08	14.10	9.07	0.13	-3.39	0.20	5.55	4.39	-4.29	-14.04	-15.34	-5.14			
16	9.29	14.07	8.50	-0.01	-3.38	0.33	6.01	4.27	-4.51	-14.17	-15.24	-4.45			
17	9.50	14.03	8.33	-0.16	-3.37	0.46	6.07	4.15	-5.12	-14.30	-15.13	-4.16			
18	10.10	13.59	8.16	-0.29	-3.35	0.59	6.12	4.02	-5.33	-14.43	-15.01	-3.47			
19	10.29	13.54	7.58	-0.43	-3.33	1.12	6.16	3.49	-5.55	-14.55	-14.49	-3.18			
20	10.48	13.48	7.41	-0.56	-3.30	1.25	6.20	3.35	-6.16	-15.06	-14.35	-2.48			
21	11.05	13.42	7.23	-1.08	-3.27	1.38	6.24	3.21	-6.38	-15.16	-14.21	-2.18			
22	11.22	13.35	7.05	-1.20	-3.23	1.51	6.27	3.06	-6.59	-15.26	-14.06	-1.49			
23	11.39	13.27	6.47	-1.32	-3.18	2.04	6.29	2.51	-7.20	-15.36	-13.50	-1.19			
24	11.54	13.19	6.29	-1.43	-3.13	2.17	6.31	2.35	-7.41	-15.44	-13.33	-0.49			
25	12.09	13.10	6.11	-1.54	-3.08	2.30	6.32	2.19	-8.02	-15.52	-13.16	-0.19			
26	12.23	13.01	5.53	-2.04	-3.02	2.43	6.33	2.02	-8.23	-15.59	-12.57	0.10			
27	12.36	12.50	5.35	-2.14	-2.55	2.55	6.33	1.46	-8.44	-16.06	-12.39	0.40			
28	12.48	12.40	5.17	-2.23	-2.48	3.08	6.32	1.28	-9.04	-16.11	-12.19	1.09			
29	12.59		4.58	-2.32	-2.41	3.20	6.31	1.11	-9.24	-16.16	-11.58	1.39			
30	13.10		4.40	-2.40	-2.33	3.32	6.29	0.53	-9.44	-16.20	-11.37	2.08			
31	13.20		4.22		-2.25		6.27	0.34		-16.24		2.37			
												3.05			

EOT	2019		
MAJOR PEAKS	MINOR PEAKS		
MAX	MIN	MAX	MIN
14.14	-16.29	6.33	-3.39

Solstice and Equinox for this year			
yyyy	mm	dd	Astronomical Julian Day
2019	1	1	--> 2458484.5
2019			

Julian day and day of year for the event			
March equinox:	2458563.4	78.92	
June Solstice:	2458656.2	171.66	
September equinox:	2458749.8	265.32	
December solstice:	2458839.7	355.17	

Comparisons of mid month EOT values for five year spans in the last, this, and the next century.

Summarized annual comparison		20th century				
mm	dd	1904	1905	1906	1907	1908
1	15	9.01	9.17	9.12	9.06	9.01
2	15	14.26	14.24	14.24	14.25	14.25
3	15	9.15	9.19	9.23	9.27	9.15
4	15	0.13	0.17	0.20	0.24	0.13
5	15	-3.49	-3.49	-3.49	-3.49	-3.49
6	15	0.03	0.00	-0.03	-0.06	0.04
7	15	5.38	5.37	5.35	5.34	5.39
8	15	4.29	4.31	4.34	4.37	4.29
9	15	-4.32	-4.27	-4.22	-4.16	-4.32
10	15	-14.01	-13.57	-13.54	-13.51	-14.01
11	15	-15.27	-15.29	-15.32	-15.34	-15.27
12	15	-5.02	-5.09	-5.17	-5.24	-5.02

Summarized annual comparison		21st century				
mm	dd	2004	2005	2006	2007	2008
1	15	9.02	9.19	9.13	9.08	9.03
2	15	14.13	14.11	14.11	14.12	14.12
3	15	8.57	9.01	9.05	9.09	8.56
4	15	0.04	0.07	0.11	0.14	0.03
5	15	-3.40	-3.40	-3.40	-3.40	-3.39
6	15	0.26	0.23	0.20	0.17	0.27
7	15	5.57	5.55	5.54	5.53	5.58
8	15	4.30	4.33	4.36	4.39	4.30
9	15	-4.44	-4.38	-4.33	-4.28	-4.44
10	15	-14.12	-14.09	-14.06	-14.02	-14.13
11	15	-15.27	-15.29	-15.32	-15.34	-15.27
12	15	-4.54	-5.01	-5.08	-5.15	-4.53

Summarized annual comparison		22nd century				
mm	dd	2104	2105	2106	2107	2108
1	15	8.42	8.59	8.53	8.48	8.42
2	15	14.01	13.60	14.00	14.01	14.01
3	15	8.55	8.59	9.03	9.07	8.54
4	15	0.09	0.12	0.16	0.19	0.08
5	15	-3.31	-3.31	-3.31	-3.31	-3.31
6	15	0.37	0.34	0.31	0.28	0.38
7	15	6.09	6.08	6.07	6.05	6.10
8	15	4.43	4.46	4.49	4.51	4.43
9	15	-4.34	-4.28	-4.23	-4.18	-4.34
10	15	-14.10	-14.06	-14.03	-13.59	-14.10
11	15	-15.37	-15.39	-15.41	-15.44	-15.37
12	15	-5.14	-5.21	-5.28	-5.35	-5.13

## TIME AND SOLAR HOUR ANGLE

## A2.2: trigonometric

**A2.2**

Time to hour angle along with some trigonometric values, useful for formulae work.

Sun's apparent hour angle	LHA=local hour angle
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Time hhmm hh.hh	From midday	LHA	Radians	sin	cos	tan	cot	PM hh.hh	PM hhmm	
4.00	4.00	8.00	120.000	2.094	0.866	-0.500	-1.732	-0.577	8.00	8.00
4.15	4.25	7.75	116.250	2.029	0.897	-0.442	-2.028	-0.493	7.75	7.45
4.30	4.50	7.50	112.500	1.963	0.924	-0.383	-2.414	-0.414	7.50	7.30
4.45	4.75	7.25	108.750	1.898	0.947	-0.321	-2.946	-0.339	7.25	7.15
5.00	5.00	7.00	105.000	1.833	0.966	-0.259	-3.732	-0.268	7.00	7.00
5.15	5.25	6.75	101.250	1.767	0.981	-0.195	-5.027	-0.199	6.75	6.45
5.30	5.50	6.50	97.500	1.702	0.991	-0.131	-7.596	-0.132	6.50	6.30
5.45	5.75	6.25	93.750	1.636	0.998	-0.065	-15.257	-0.066	6.25	6.15
6.00	6.00	6.00	90.000	1.571	1.000	0.000	inf	0.000	6.00	6.00
6.15	6.25	5.75	86.250	1.505	0.998	0.065	15.257	0.066	5.75	5.45
6.30	6.50	5.50	82.500	1.440	0.991	0.131	7.596	0.132	5.50	5.30
6.45	6.75	5.25	78.750	1.374	0.981	0.195	5.027	0.199	5.25	5.15
7.00	7.00	5.00	75.000	1.309	0.966	0.259	3.732	0.268	5.00	5.00
7.15	7.25	4.75	71.250	1.244	0.947	0.321	2.946	0.339	4.75	4.45
7.30	7.50	4.50	67.500	1.178	0.924	0.383	2.414	0.414	4.50	4.30
7.45	7.75	4.25	63.750	1.113	0.897	0.442	2.028	0.493	4.25	4.15
8.00	8.00	4.00	60.000	1.047	0.866	0.500	1.732	0.577	4.00	4.00
8.15	8.25	3.75	56.250	0.982	0.831	0.556	1.497	0.668	3.75	3.45
8.30	8.50	3.50	52.500	0.916	0.793	0.609	1.303	0.767	3.50	3.30
8.45	8.75	3.25	48.750	0.851	0.752	0.659	1.140	0.877	3.25	3.15
9.00	9.00	3.00	45.000	0.785	0.707	0.707	1.000	1.000	3.00	3.00
9.15	9.25	2.75	41.250	0.720	0.659	0.752	0.877	1.140	2.75	2.45
9.30	9.50	2.50	37.500	0.654	0.609	0.793	0.767	1.303	2.50	2.30
9.45	9.75	2.25	33.750	0.589	0.556	0.831	0.668	1.497	2.25	2.15
10.00	10.00	2.00	30.000	0.524	0.500	0.866	0.577	1.732	2.00	2.00
10.15	10.25	1.75	26.250	0.458	0.442	0.897	0.493	2.028	1.75	1.45
10.30	10.50	1.50	22.500	0.393	0.383	0.924	0.414	2.414	1.50	1.30
10.45	10.75	1.25	18.750	0.327	0.321	0.947	0.339	2.946	1.25	1.15
11.00	11.00	1.00	15.000	0.262	0.259	0.966	0.268	3.732	1.00	1.00
11.15	11.25	0.75	11.250	0.196	0.195	0.981	0.199	5.027	0.75	0.45
11.30	11.50	0.50	7.500	0.131	0.131	0.991	0.132	7.596	0.50	0.30
11.45	11.75	0.25	3.750	0.065	0.065	0.998	0.066	15.257	0.25	0.15
12.00	12.00	0.00	0.000	0.000	0.000	1.000	0.000	inf	0.00	0.00

Standard time to hour angle for every degree (helpful for astro compass work)

Minutes	morning hours						noon		afternoon hours				
	6	7	8	9	10	11	12	13	14	15	16	17	
0	270	285	300	315	330	345	0	15	30	45	60	75	
4	271	286	301	316	331	346	1	16	31	46	61	76	
8	272	287	302	317	332	347	2	17	32	47	62	77	
12	273	288	303	318	333	348	3	18	33	48	63	78	
16	274	289	304	319	334	349	4	19	34	49	64	79	
20	275	290	305	320	335	350	5	20	35	50	65	80	
24	276	291	306	321	336	351	6	21	36	51	66	81	
28	277	292	307	322	337	352	7	22	37	52	67	82	
32	278	293	308	323	338	353	8	23	38	53	68	83	
36	279	294	309	324	339	354	9	24	39	54	69	84	
40	280	295	310	325	340	355	10	25	40	55	70	85	
44	281	296	311	326	341	356	11	26	41	56	71	86	
48	282	297	312	327	342	357	12	27	42	57	72	87	
52	283	298	313	328	343	358	13	28	43	58	73	88	
56	284	299	314	329	344	359	14	29	44	59	74	89	
60	285	300	315	330	345	360	15	30	45	60	75	90	

hour angle

Legal time to hour angle where 6 am is 270°, noon is 360° or 0°, and 6 pm is 90°.

For astro compass use, subtract (EOT + longitude correction) to get the hour angle for the legal time.

**A2.3**

For each degree of longitude, a conversion is made to hh.mm ~ hours and minutes

**LONGITUDE TO TIME**Degree ( $^{\circ}$ ) to Hours (h) and Minutes (m)

$^{\circ}$	hh.mm								
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0	0.00	30	2.00	60	4.00	90	6.00	120	8.00	150	10.00
1	0.04	31	2.04	61	4.04	91	6.04	121	8.04	151	10.04
2	0.08	32	2.08	62	4.08	92	6.08	122	8.08	152	10.08
3	0.12	33	2.12	63	4.12	93	6.12	123	8.12	153	10.12
4	0.16	34	2.16	64	4.16	94	6.16	124	8.16	154	10.16
5	0.20	35	2.20	65	4.20	95	6.20	125	8.20	155	10.20
6	0.24	36	2.24	66	4.24	96	6.24	126	8.24	156	10.24
7	0.28	37	2.28	67	4.28	97	6.28	127	8.28	157	10.28
8	0.32	38	2.32	68	4.32	98	6.32	128	8.32	158	10.32
9	0.36	39	2.36	69	4.36	99	6.36	129	8.36	159	10.36
10	0.40	40	2.40	70	4.40	100	6.40	130	8.40	160	10.40
11	0.44	41	2.44	71	4.44	101	6.44	131	8.44	161	10.44
12	0.48	42	2.48	72	4.48	102	6.48	132	8.48	162	10.48
13	0.52	43	2.52	73	4.52	103	6.52	133	8.52	163	10.52
14	0.56	44	2.56	74	4.56	104	6.56	134	8.56	164	10.56
15	1.00	45	3.00	75	5.00	105	7.00	135	9.00	165	11.00
16	1.04	46	3.04	76	5.04	106	7.04	136	9.04	166	11.04
17	1.08	47	3.08	77	5.08	107	7.08	137	9.08	167	11.08
18	1.12	48	3.12	78	5.12	108	7.12	138	9.12	168	11.12
19	1.16	49	3.16	79	5.16	109	7.16	139	9.16	169	11.16
20	1.20	50	3.20	80	5.20	110	7.20	140	9.20	170	11.20
21	1.24	51	3.24	81	5.24	111	7.24	141	9.24	171	11.24
22	1.28	52	3.28	82	5.28	112	7.28	142	9.28	172	11.28
23	1.32	53	3.32	83	5.32	113	7.32	143	9.32	173	11.32
24	1.36	54	3.36	84	5.36	114	7.36	144	9.36	174	11.36
25	1.40	55	3.40	85	5.40	115	7.40	145	9.40	175	11.40
26	1.44	56	3.44	86	5.44	116	7.44	146	9.44	176	11.44
27	1.48	57	3.48	87	5.48	117	7.48	147	9.48	177	11.48
28	1.52	58	3.52	88	5.52	118	7.52	148	9.52	178	11.52
29	1.56	59	3.56	89	5.56	119	7.56	149	9.56	179	11.56
										180	12.00

## CITY DATA WITH LATITUDE, LONGITUDE AND ITS CORRECTION

A2.5

City id	Country, State, City		Hemi-sphere	Latitude	Long	Mag dec/var	Time ref	Long corr
				+n -s	+w -e			
	UK	London	N +ve	51.5	0.5	1.5w	0	2
	UK	Weymouth	N +ve	50.6	2.5	2.1w	0	10
PHX	USA AZ	Phoenix	N +ve	33.5	112.1	10.8e	105	28.8
SDL	USA AZ	Scottsdale	N +ve	33.6	111.9	10.8e	105	27.6
SVC	USA NM	Silver City	N +ve	32.8	108.2	9.4e	105	12.8
LAS	USA NV	Las Vegas	N +ve	36.2	115.2	12.25e	120	-19.2
JFK	USA NY	New York	N +ve	40.7	73.8	13.2w	75	-4.8
OKC	USA OK	Oklahoma City	N +ve	35.3	97.5	4.25e	90	30
ELP	USA TX	El Paso	N +ve	31.8	106.5	8.75e	105	6

The magnetic declination or variation changes quite a bit. For example consider Phoenix for two different years:- July 2012 ~ 10.8° and July 2002 ~ 11.8°

The following websites may or may not work:-

<http://www.ngdc.noaa.gov/geomagmodels/struts/calcDeclination>  
<http://magnetic-declination.com/#>

## DAYLIGHT SAVING TIME AND TABLE OF TIME ZONES

A2.6

Zone	Name	Meridian	GMT+
	Newfoundland	52.5	3.5
4	Atlantic	60	4
5	Eastern EST	75	5
6	Central CST	90	6
7	Mountain MST	105	7
8	Pacific PST	120	8
9	Yukon	135	9
10	Alaska-Hawaii	150	10
11	Bering	165	11
	GMT Greenwich Mean Time		0
	BST British Summer Time		-1
	IST Irish Summer Time		-1
	WET Western Europe Time		0
	WEST Western Europe Summer Time		-1
	CET Central Europe Time		-1
	CEST Central Europe Summer Time		-2
	EET Eastern Europe Time		-2
	EEST Eastern Europe Summer Time		-3
	MSK Moscow Time		-3
	MSD Moscow Summer Time		-4

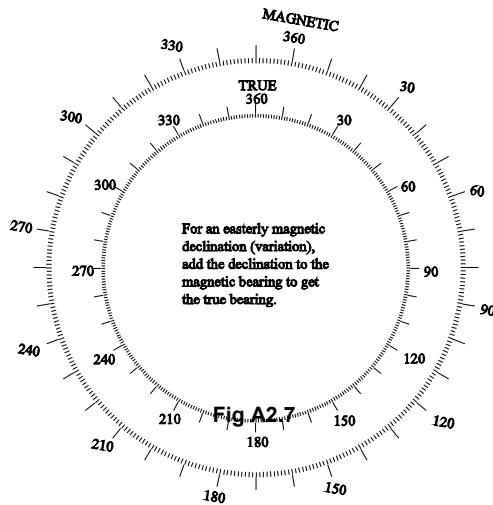
A useful web site for lat, long, and magnetic declination, and gives the time zone:

<http://www.airnav.com/airports/>

SUMMER TIME RULES: subject to variation based on various government's policies.

- USA: second Sunday in March to the first Sunday in November (some exceptions)
- EU: last Sunday in March to the last Sunday in October.
- The website: <http://webexhibits.org/daylightsaving/> has other useful information.

## MAGNETIC DECLINATION & VARIATION



It is easy to get confused about adding and subtracting magnetic variation (navigators) or magnetic declination (sundial people). This diagram simplifies the task by providing the following simple rules:-

For easterly variations/declinations (see A2.7):-

- TRUE = MAGNETIC + variation/declination
- just remember **TEMPE~**  
**True Equals Magnetic Plus Easterly**

For westerly variations/declinations:-

- TRUE = MAGNETIC - variation/declination

Consider:-

<http://www.magnetic-declination.com/>

## ASTRO COMPASS CHECKLIST

method one – time oriented

NOTE: LAT (upper case) means Local Apparent Time or L.A.T. and not latitude.

LONGITUDE CORRECTION:  
your location:  
reference location:  
difference:  
times 4°

EOT CORRECTION:  
chart A2.1  
NET CORRECTION  
standard time

so we use:

NOTE: we reverse the sign when we apply this to clock time to derive L.A.T.

	EXAMPLE	ACTUAL	
108.2			
105.0			
3.2			
+ 12m 48s			
Sept 26			
- 8m 41s			
+ 4m 7s			
<b>- 4m 7s</b>			

**Fig A2.8a**

Next: select a time about 5 minutes hence and calculate the LAT for that clock time: For September 26, longitude 108.2, the net correction is + 1 hour, 4 minutes, 7 seconds which is subtracted from the clock indicated summer time because we desire LAT.

Then: calculate the hour angle from noon for the LAT, this is 15° per hour, and 1° for four minutes (table A2.3).

Clock time for reading:  
clock to LAT correction:  
LAT (clock – correction):  
LAT from noon 12:00  
Degrees from noon:

9 : 30			
<b>- 4m 7s</b>			
9 : 26			
2h 34m			
38.5 °			

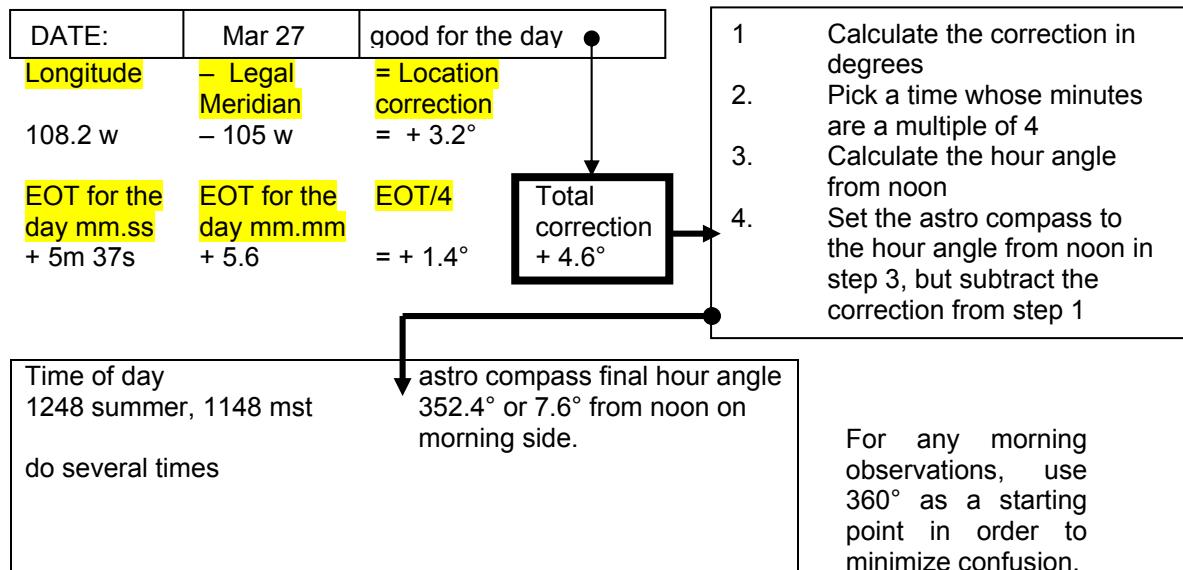
**Fig A2.8b**

HINT: The above uses legal standard time, not summer time. It saves a step.

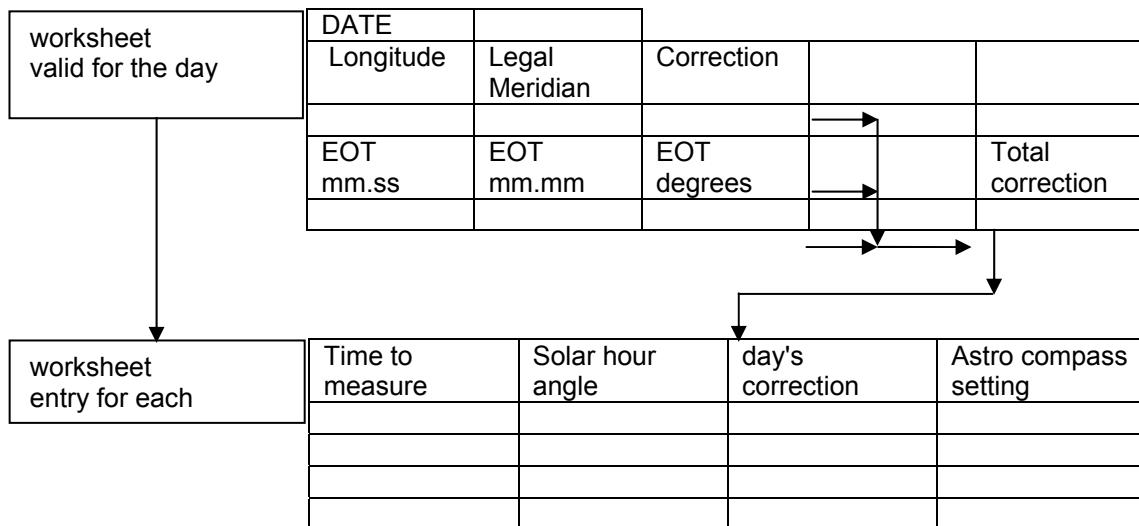
## ASTRO COMPASS CHECKLIST

method two – angle oriented (degree method)

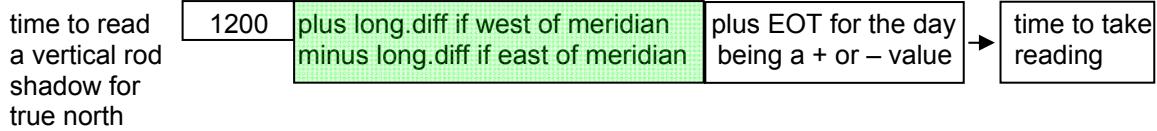
Table A2.2a may be helpful, and the master spreadsheet **illustratingShadows.xls**  
has an excellent time to degrees worksheet with longitude and EOT correction.



NOTE: always ensure that an astro-compass is correctly calibrated.



## NOON TRANSIT OF THE SUN ~ to determine true north

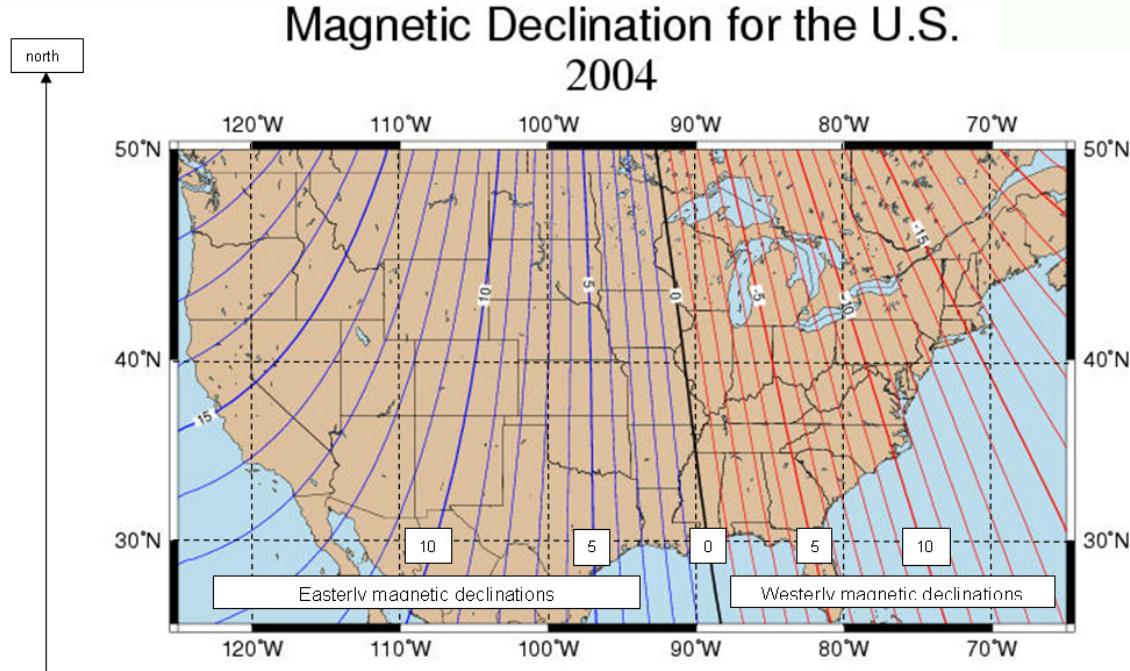


The following table is a midpoint of four consecutive years of the astronomically accurate EOT table, modified by adding noon to the EOT. You then add the minutes between your longitude and the legal meridian (if west) or subtract them (if east), at the resulting time, a shadow from a vertical rod will point to true north. The minutes added or subtracted are 4 times the longitude difference in degrees.

	Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
1	1203.3	1213.3	1212.3	1204.0	1157.5	1157.8	1203.4	1206.2	1200.1	1149.9	1143.7	1148.9
2	1203.8	1213.4	1212.2	1203.4	1157.4	1157.9	1203.6	1206.2	1159.9	1149.7	1143.7	1149.5
3	1204.2	1213.5	1212.0	1203.3	1157.0	1158.4	1204.1	1206.1	1159.7	1149.5	1143.7	1149.8
4	1204.7	1213.5	1211.5	1203.1	1156.9	1158.5	1204.2	1206.1	1159.5	1148.9	1143.7	1150.0
5	1205.2	1213.6	1211.4	1202.5	1156.8	1158.6	1204.3	1206.0	1158.9	1148.8	1143.7	1150.6
6	1205.6	1214.0	1211.2	1202.3	1156.8	1158.7	1204.4	1205.6	1158.7	1148.6	1143.7	1150.9
7	1206.0	1214.1	1211.1	1202.2	1156.8	1158.9	1204.5	1205.5	1158.5	1148.4	1143.8	1151.5
8	1206.5	1214.1	1210.6	1202.0	1156.7	1159.0	1204.6	1205.4	1157.9	1147.8	1143.8	1151.8
9	1206.9	1214.1	1210.4	1201.4	1156.7	1159.5	1205.1	1205.4	1157.7	1147.7	1143.8	1152.5
10	1207.3	1214.1	1210.2	1201.3	1156.7	1159.6	1205.2	1205.3	1157.5	1147.5	1143.9	1152.7
11	1207.7	1214.2	1210.1	1201.1	1156.6	1159.7	1205.3	1205.2	1156.9	1146.9	1144.0	1153.0
12	1208.1	1214.2	1209.5	1200.6	1156.6	1159.8	1205.3	1205.1	1156.7	1146.8	1144.4	1153.7
13	1208.5	1214.1	1209.4	1200.4	1156.6	1160.0	1205.4	1204.6	1156.5	1146.6	1144.5	1154.0
14	1208.9	1214.1	1209.2	1200.3	1156.6	1200.1	1205.5	1204.5	1155.9	1146.5	1144.6	1154.6
15	1209.2	1214.1	1209.0	1200.1	1156.6	1200.2	1205.5	1204.4	1155.7	1145.9	1144.7	1154.9
16	1209.6	1214.1	1208.5	1160.0	1156.6	1200.3	1206.0	1204.2	1155.5	1145.8	1144.8	1155.6
17	1209.9	1214.0	1208.3	1159.8	1156.6	1200.5	1206.1	1204.1	1154.8	1145.7	1144.9	1155.9
18	1210.2	1213.6	1208.1	1159.7	1156.6	1200.6	1206.1	1203.6	1154.6	1145.6	1145.4	1156.6
19	1210.6	1213.5	1207.6	1159.5	1156.7	1201.1	1206.2	1203.5	1154.4	1145.4	1145.5	1156.9
20	1210.9	1213.5	1207.4	1159.4	1156.7	1201.3	1206.2	1203.3	1153.8	1144.9	1145.7	1157.6
21	1211.2	1213.4	1207.2	1158.9	1156.7	1201.4	1206.2	1203.2	1153.6	1144.8	1145.8	1157.9
22	1211.4	1213.3	1207.0	1158.8	1156.8	1201.5	1206.3	1203.0	1153.0	1144.7	1146.0	1158.6
23	1211.7	1213.3	1206.4	1158.7	1156.8	1202.1	1206.3	1202.5	1152.8	1144.6	1146.5	1158.9
24	1212.0	1213.2	1206.3	1158.5	1156.9	1202.2	1206.3	1202.3	1152.5	1144.5	1146.7	1159.6
25	1212.2	1213.1	1206.1	1158.4	1156.9	1202.3	1206.3	1202.2	1151.9	1144.5	1146.9	1159.9
26	1212.4	1212.6	1205.5	1157.9	1157.0	1202.4	1206.3	1201.6	1151.7	1144.0	1147.5	1200.2
27	1212.7	1212.5	1205.3	1157.8	1157.4	1202.6	1206.3	1201.4	1151.5	1143.9	1147.7	1200.5
28	1212.9		1205.1	1157.7	1157.5	1203.1	1206.3	1201.2	1150.9	1143.9	1147.9	1201.2
29	1213.0		1204.6	1157.7	1157.6	1203.2	1206.3	1201.1	1150.7	1143.8	1148.5	1201.5
30	1213.2		1204.4	1157.7		1206.3	1200.5		1143.8			1202.1
BLACK	time is at or after legal noon standard time [no summer time correction]											If a leap year then after February
BLUE	before legal noon standard time [no summer time correction]											28 use the next day's value

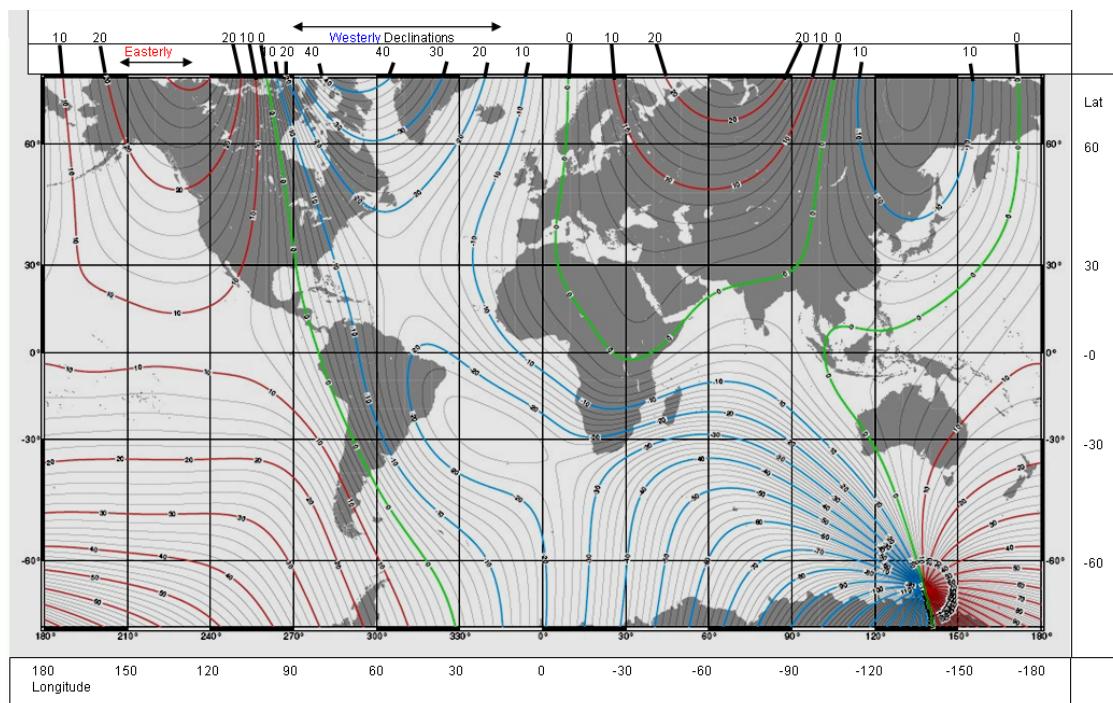
Map of the USA magnetic declination (variation) as of 2004 (fields vary with time)  
<http://www.ngdc.noaa.gov/seg/geomag/declination.shtml> →  
[http://www.ngdc.noaa.gov/geomag/icons/us\\_d\\_contour.jpg](http://www.ngdc.noaa.gov/geomag/icons/us_d_contour.jpg)

Fig A2.9a



Map of the world's magnetic declination (variation) as of 2005 (fields vary with time)  
<http://www.ngdc.noaa.gov/seg/geomag/declination.shtml> →  
<http://www.ngdc.noaa.gov/geomag/WMM/data/wmm-D05.pdf>

Fig A2.9b



Also, consider: [http://gsc.nrcan.gc.ca/geomag/field/magdec\\_e.php](http://gsc.nrcan.gc.ca/geomag/field/magdec_e.php)

**APPENDIX 3 – HORIZONTAL, VERTICAL, AND CEILING DIAL  
HOUR LINE ANGLES  
POLAR, MERIDIAN, AND EQUATORIAL DIAL VALUES**

A3.0

QUICK CROSS CHECK – horizontal dial hour line angles for hours from noon vs latitudes

latitude	h	v	hours from noon		no longitude correction						hour line angles for v & h dials								
			0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
0	90		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	89		0.0	0.1	0.3	0.4	0.6	0.8	1.0	1.3	1.7	2.4	3.7	7.6	90.0	97.6	93.7	92.4	91.7
2	88		0.0	0.3	0.5	0.8	1.2	1.5	2.0	2.6	3.5	4.8	7.4	14.8	90.0	104.8	97.4	94.8	93.5
3	87		0.0	0.4	0.8	1.2	1.7	2.3	3.0	3.9	5.2	7.2	11.1	21.7	90.0	111.7	101.1	97.2	95.2
4	86		0.0	0.5	1.1	1.7	2.3	3.1	4.0	5.2	6.9	9.6	14.6	27.9	90.0	117.9	104.6	99.6	96.9
5	85		0.0	0.7	1.3	2.1	2.9	3.8	5.0	6.5	8.6	11.9	18.0	33.5	90.0	123.5	108.0	101.9	98.6
6	84		0.0	0.8	1.6	2.5	3.5	4.6	6.0	7.8	10.3	14.2	21.3	38.4	90.0	128.4	111.3	104.2	100.3
7	83		0.0	0.9	1.9	2.9	4.0	5.3	6.9	9.0	11.9	16.4	24.5	42.8	90.0	132.8	114.5	106.4	101.9
8	82		0.0	1.0	2.1	3.3	4.6	6.1	7.9	10.3	13.6	18.6	27.4	46.6	90.0	136.6	117.4	108.6	103.6
9	81		0.0	1.2	2.4	3.7	5.2	6.8	8.9	11.5	15.2	20.7	30.3	49.9	90.0	139.9	120.3	110.7	105.2
10	80		0.0	1.3	2.7	4.1	5.7	7.6	9.9	12.8	16.7	22.7	32.9	52.8	90.0	142.8	122.9	112.7	106.7
11	79		0.0	1.4	2.9	4.5	6.3	8.3	10.8	14.0	18.3	24.7	35.5	55.4	90.0	145.4	125.5	114.7	108.3
12	78		0.0	1.6	3.2	4.9	6.8	9.1	11.7	15.2	19.8	26.7	37.8	57.7	90.0	147.7	127.8	116.7	109.8
13	77		0.0	1.7	3.4	5.3	7.4	9.8	12.7	16.3	21.3	28.5	40.0	59.7	90.0	149.7	130.0	118.5	111.3
14	76		0.0	1.8	3.7	5.7	8.0	10.5	13.6	17.5	22.7	30.3	42.1	61.4	90.0	151.4	132.1	120.3	112.7
15	75		0.0	2.0	4.0	6.1	8.5	11.2	14.5	18.6	24.1	32.0	44.0	63.0	90.0	153.0	134.0	122.0	114.1
16	74		0.0	2.1	4.2	6.5	9.0	11.9	15.4	19.8	25.5	33.6	45.8	64.5	90.0	154.5	135.8	123.6	115.5
17	73		0.0	2.2	4.5	6.9	9.6	12.6	16.3	20.9	26.9	35.2	47.5	65.8	90.0	155.8	137.5	125.2	116.9
18	72		0.0	2.3	4.7	7.3	10.1	13.3	17.2	21.9	28.2	36.7	49.1	66.9	90.0	156.9	139.1	126.7	118.2
19	71		0.0	2.5	5.0	7.7	10.6	14.0	18.0	23.0	29.4	38.2	50.5	68.0	90.0	158.0	140.5	128.2	119.4
20	70		0.0	2.6	5.2	8.1	11.2	14.7	18.9	24.0	30.6	39.5	51.9	68.9	90.0	158.9	141.9	129.5	120.6
21	69		0.0	2.7	5.5	8.4	11.7	15.4	19.7	25.0	31.8	40.9	53.2	69.8	90.0	159.8	143.2	130.9	121.8
22	68		0.0	2.8	5.7	8.8	12.2	16.0	20.5	26.0	33.0	42.1	54.4	70.6	90.0	160.6	144.4	132.1	123.0
23	67		0.0	2.9	6.0	9.2	12.7	16.7	21.3	27.0	34.1	43.3	55.6	71.4	90.0	161.4	145.6	133.3	124.1
24	66		0.0	3.1	6.2	9.6	13.2	17.3	22.1	27.9	35.2	44.5	56.6	72.1	90.0	162.1	146.6	134.5	125.2
25	65		0.0	3.2	6.5	9.9	13.7	18.0	22.9	28.8	36.2	45.6	57.6	72.7	90.0	162.7	147.6	135.6	126.2
26	64		0.0	3.3	6.7	10.3	14.2	18.6	23.7	29.7	37.2	46.6	58.6	73.3	90.0	163.3	148.6	136.6	127.2
27	63		0.0	3.4	6.9	10.7	14.7	19.2	24.4	30.6	38.2	47.6	59.5	73.8	90.0	163.8	149.5	137.6	128.2
28	62		0.0	3.5	7.2	11.0	15.2	19.8	25.1	31.5	39.1	48.6	60.3	74.3	90.0	164.3	150.3	138.6	129.1
29	61		0.0	3.7	7.4	11.4	15.6	20.4	25.9	32.3	40.0	49.5	61.1	74.8	90.0	164.8	151.1	139.5	130.0
30	60		0.0	3.8	7.6	11.7	16.1	21.0	26.6	33.1	40.9	50.4	61.8	75.2	90.0	165.2	151.8	140.4	130.9
35	55		0.0	4.3	8.7	13.4	18.3	23.8	29.8	36.8	44.8	54.2	65.0	77.1	90.0	167.1	155.0	144.2	134.8
40	50		0.0	4.8	9.8	14.9	20.4	26.3	32.7	40.0	48.1	57.2	67.4	78.4	90.0	168.4	157.4	147.2	138.1
45	45		0.0	5.3	10.7	16.3	22.2	28.5	35.3	42.7	50.8	59.6	69.2	79.5	90.0	169.5	159.2	149.6	140.8
50	40		0.0	5.8	11.6	17.6	23.9	30.4	37.5	45.0	53.0	61.6	70.7	80.2	90.0	170.2	160.7	151.6	143.0
55	35		0.0	6.2	12.4	18.7	25.3	32.2	39.3	46.9	54.8	63.2	71.9	80.9	90.0	170.9	161.9	153.2	144.8
60	30		0.0	6.5	13.1	19.7	26.6	33.6	40.9	48.5	56.3	64.4	72.8	81.4	90.0	171.4	162.8	154.4	146.3
65	25		0.0	6.8	13.6	20.6	27.6	34.8	42.2	49.7	57.5	65.4	73.5	81.7	90.0	171.7	163.5	155.4	147.5
70	20		0.0	7.1	14.1	21.3	28.5	35.8	43.2	50.8	58.4	66.2	74.1	82.0	90.0	172.0	164.1	156.2	148.4
75	15		0.0	7.2	14.5	21.8	29.1	36.5	44.0	51.5	59.1	66.8	74.5	82.2	90.0	172.2	164.5	156.8	149.1
80	10		0.0	7.4	14.8	22.2	29.6	37.1	44.6	52.1	59.6	67.2	74.8	82.4	90.0	172.4	164.8	157.2	149.6
85	5		0.0	7.5	14.9	22.4	29.9	37.4	44.9	52.4	59.9	67.4	74.9	82.5	90.0	172.5	164.9	157.4	149.9
90	0		0.0	7.5	15.0	22.5	30.0	37.5	45.0	52.5	60.0	67.5	75.0	82.5	90.0	172.5	165.0	157.5	150.0

vertical dial

horizontal dial

This chart shows h-dial hour line angles to 0.1° by the half hour by latitude. NOTE: latitude 0 is a polar dial and all hour lines are parallel (as they are for a polar). NOTE: latitude 90 has 15 degree angles, which is true for an equatorial dial, which it is. NO LONGITUDE CORRECTION.

**A3.1a**

HOUR LINE ANGLES				Horizontal or vertical dial							
------------------	--	--	--	-----------------------------	--	--	--	--	--	--	--

hour line angle      DEGREES(ATAN(TAN(RADIANS(15\*time))\*SIN(RADIANS(lat))))  
 hour line angle       $H = \text{atan}(\sin(\text{lat}) * \tan(\text{ha}))$

TIME	HORIZONTAL DIAL LATITUDE										
	30	31	32	33	34	35	36	37	38	39	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	noon
0.25	1.88	1.93	1.99	2.04	2.10	2.15	2.21	2.26	2.31	2.36	11.75
0.50	3.77	3.88	3.99	4.10	4.21	4.32	4.42	4.53	4.63	4.74	11.50
0.75	5.68	5.85	6.02	6.18	6.35	6.51	6.67	6.83	6.98	7.14	11.25
1	7.63	7.86	8.08	8.30	8.52	8.74	8.95	9.16	9.37	9.57	11
1.25	9.63	9.92	10.20	10.47	10.75	11.02	11.28	11.55	11.80	12.06	10.75
1.50	11.70	12.04	12.38	12.71	13.04	13.36	13.68	14.00	14.31	14.61	10.50
1.75	13.85	14.25	14.65	15.03	15.42	15.79	16.16	16.53	16.89	17.24	10.25
2	16.10	16.56	17.01	17.46	17.89	18.32	18.75	19.16	19.57	19.97	10
2.25	18.47	18.99	19.50	20.00	20.49	20.97	21.44	21.91	22.36	22.81	9.75
2.50	20.99	21.56	22.13	22.68	23.22	23.76	24.28	24.79	25.29	25.78	9.50
2.75	23.68	24.31	24.93	25.53	26.12	26.70	27.27	27.82	28.37	28.89	9.25
3	26.57	27.25	27.92	28.57	29.21	29.84	30.45	31.04	31.62	32.18	9
3.25	29.69	30.43	31.14	31.84	32.52	33.19	33.83	34.46	35.07	35.66	8.75
3.50	33.09	33.87	34.63	35.37	36.08	36.78	37.45	38.11	38.74	39.36	8.50
3.75	36.81	37.63	38.42	39.18	39.93	40.64	41.34	42.01	42.66	43.28	8.25
4	40.89	41.74	42.55	43.33	44.08	44.81	45.51	46.19	46.84	47.47	8
4.25	45.40	46.24	47.06	47.84	48.59	49.31	50.00	50.67	51.31	51.92	7.75
4.50	50.36	51.19	51.99	52.75	53.47	54.16	54.83	55.46	56.07	56.65	7.50
4.75	55.83	56.61	57.36	58.07	58.74	59.38	59.99	60.57	61.13	61.66	7.25
5	61.81	62.51	63.18	63.80	64.40	64.96	65.49	66.00	66.48	66.94	7
5.25	68.31	68.88	69.43	69.94	70.42	70.87	71.30	71.71	72.10	72.46	6.75
5.50	75.25	75.66	76.05	76.41	76.75	77.07	77.38	77.66	77.93	78.18	6.50
5.75	82.53	82.75	82.95	83.14	83.31	83.48	83.64	83.78	83.92	84.05	6.25
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	6
	60	59	58	57	56	55	54	53	52	51	
	VERTICAL DIAL LATITUDE										

NOTE: Values are degrees and tenths and hundredths of a degree. Thus latitude 39 at 1pm or 11am shows 9.57 degrees, not 9 degrees, 57 minutes. And 9.57 degrees converts to 9 degrees 34.2 minutes of arc, which is consistent with other publications.

**A3.1 b**

HOUR LINE ANGLES										
Horizontal or vertical dial										
TIME	HORIZONTAL DIAL LATITUDE									
TIME	40	41	42	43	44	45	46	47	48	49
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.25	2.41	2.46	2.51	2.56	2.61	2.65	2.70	2.74	2.79	2.83
0.50	4.84	4.94	5.03	5.13	5.23	5.32	5.41	5.50	5.59	5.67
0.75	7.29	7.43	7.58	7.73	7.87	8.01	8.14	8.28	8.41	8.54
1	9.77	9.97	10.16	10.36	10.54	10.73	10.91	11.09	11.26	11.43
1.25	12.31	12.56	12.80	13.03	13.27	13.50	13.72	13.94	14.16	14.37
1.50	14.91	15.20	15.49	15.77	16.05	16.32	16.59	16.85	17.11	17.36
1.75	17.59	17.93	18.26	18.59	18.91	19.22	19.53	19.83	20.13	20.41
2	20.36	20.75	21.12	21.49	21.85	22.21	22.55	22.89	23.22	23.54
2.25	23.24	23.67	24.09	24.50	24.90	25.29	25.67	26.04	26.41	26.76
2.50	26.25	26.72	27.18	27.62	28.06	28.48	28.90	29.30	29.69	30.08
2.75	29.41	29.91	30.40	30.88	31.35	31.80	32.25	32.68	33.09	33.50
3	32.73	33.27	33.79	34.29	34.79	35.26	35.73	36.18	36.62	37.04
3.25	36.24	36.80	37.34	37.87	38.38	38.88	39.36	39.83	40.28	40.71
3.50	39.95	40.53	41.09	41.63	42.15	42.66	43.15	43.62	44.08	44.53
3.75	43.89	44.48	45.04	45.59	46.11	46.62	47.11	47.58	48.04	48.48
4	48.07	48.65	49.21	49.75	50.27	50.77	51.25	51.71	52.16	52.58
4.25	52.50	53.07	53.61	54.13	54.63	55.11	55.57	56.01	56.43	56.84
4.50	57.20	57.73	58.24	58.73	59.19	59.64	60.07	60.47	60.87	61.24
4.75	62.16	62.64	63.10	63.54	63.96	64.36	64.74	65.10	65.45	65.78
5	67.37	67.78	68.18	68.55	68.91	69.25	69.57	69.88	70.17	70.45
5.25	72.81	73.13	73.44	73.74	74.02	74.29	74.54	74.78	75.02	75.23
5.50	78.43	78.65	78.87	79.07	79.27	79.45	79.63	79.80	79.95	80.10
5.75	84.18	84.29	84.41	84.51	84.61	84.70	84.79	84.88	84.96	85.04
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
	50	49	48	47	46	45	44	43	42	41
VERTICAL DIAL LATITUDE										

A3.1 c

HOUR LINE ANGLES | Horizontal or vertical dial

hour line angle DEGREES(ATAN(TAN(RADIANS(15\*time))\*SIN(RADIANS(lat))))  
 hour line angle  $H = \text{atan}(\sin(\text{lat}) * \tan(\text{ha}))$

TIME	HORIZONTAL DIAL LATITUDE										noon
	50	51	52	53	54	55	56	57	58	59	
12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	noon
0.25	2.87	2.92	2.96	3.00	3.04	3.07	3.11	3.15	3.18	3.22	11.75
0.50	5.76	5.84	5.92	6.00	6.08	6.16	6.23	6.30	6.37	6.44	11.50
0.75	8.66	8.79	8.91	9.03	9.14	9.25	9.36	9.47	9.57	9.68	11.25
1	11.60	11.76	11.92	12.08	12.23	12.38	12.52	12.67	12.80	12.94	11
1.25	14.58	14.78	14.98	15.17	15.36	15.54	15.72	15.89	16.06	16.22	10.75
1.50	17.60	17.84	18.08	18.30	18.53	18.74	18.95	19.16	19.35	19.55	10.50
1.75	20.70	20.97	21.24	21.50	21.75	22.00	22.24	22.47	22.70	22.91	10.25
2	23.86	24.17	24.46	24.75	25.04	25.31	25.58	25.84	26.09	26.33	10
2.25	27.11	27.44	27.77	28.09	28.39	28.69	28.98	29.27	29.54	29.80	9.75
2.50	30.45	30.81	31.16	31.50	31.83	32.15	32.46	32.76	33.05	33.33	9.50
2.75	33.89	34.28	34.65	35.01	35.36	35.69	36.02	36.33	36.64	36.93	9.25
3	37.45	37.85	38.24	38.61	38.97	39.32	39.66	39.99	40.30	40.60	9
3.25	41.14	41.55	41.94	42.32	42.69	43.05	43.39	43.72	44.04	44.35	8.75
3.50	44.95	45.36	45.76	46.15	46.51	46.87	47.21	47.54	47.86	48.17	8.50
3.75	48.90	49.31	49.70	50.08	50.45	50.80	51.13	51.46	51.77	52.06	8.25
4	53.00	53.39	53.77	54.14	54.49	54.82	55.15	55.46	55.75	56.04	8
4.25	57.23	57.60	57.96	58.31	58.64	58.95	59.25	59.54	59.82	60.09	7.75
4.50	61.60	61.94	62.27	62.59	62.89	63.18	63.45	63.72	63.97	64.21	7.50
4.75	66.10	66.40	66.69	66.97	67.24	67.49	67.73	67.96	68.18	68.40	7.25
5	70.72	70.98	71.22	71.45	71.67	71.89	72.09	72.28	72.47	72.64	7
5.25	75.44	75.64	75.83	76.01	76.19	76.35	76.51	76.66	76.80	76.94	6.75
5.50	80.25	80.39	80.52	80.64	80.76	80.87	80.98	81.08	81.18	81.27	6.50
5.75	85.11	85.18	85.25	85.31	85.37	85.43	85.48	85.53	85.58	85.63	6.25
6	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	6
	40	39	38	37	36	35	34	33	32	31	
	VERTICAL DIAL LATITUDE										

Horizontal and Vertical dial nomogram for hour line angles  
method 1 – three parallel lines nomogram

Hour around noon

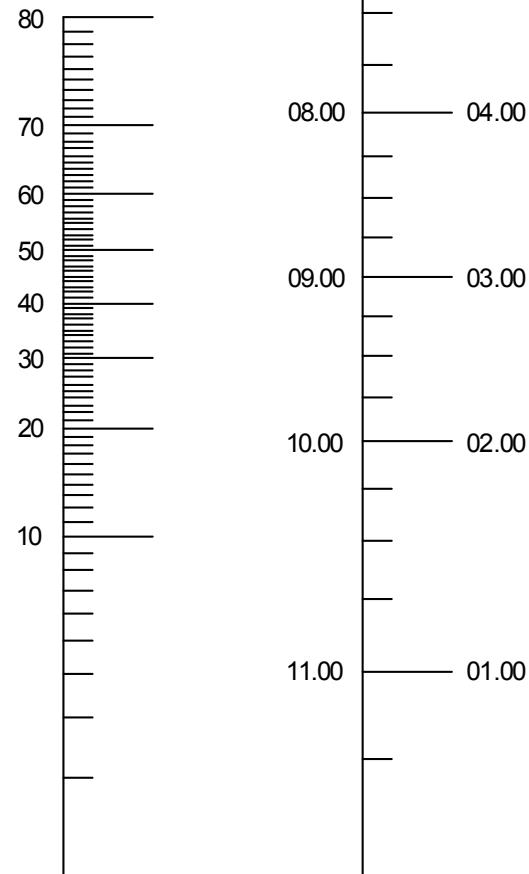
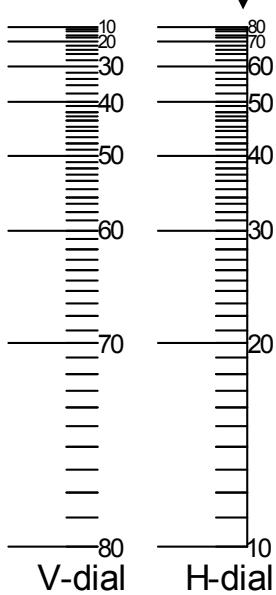
AM PM

First, mark latitude on the left vertical line  
Next, mark the desired time on the right vertical line  
Then, read hour line angle on the center vertical line

Dial Plate's hour line angle

Use the h-dial line for both v and h dial marking

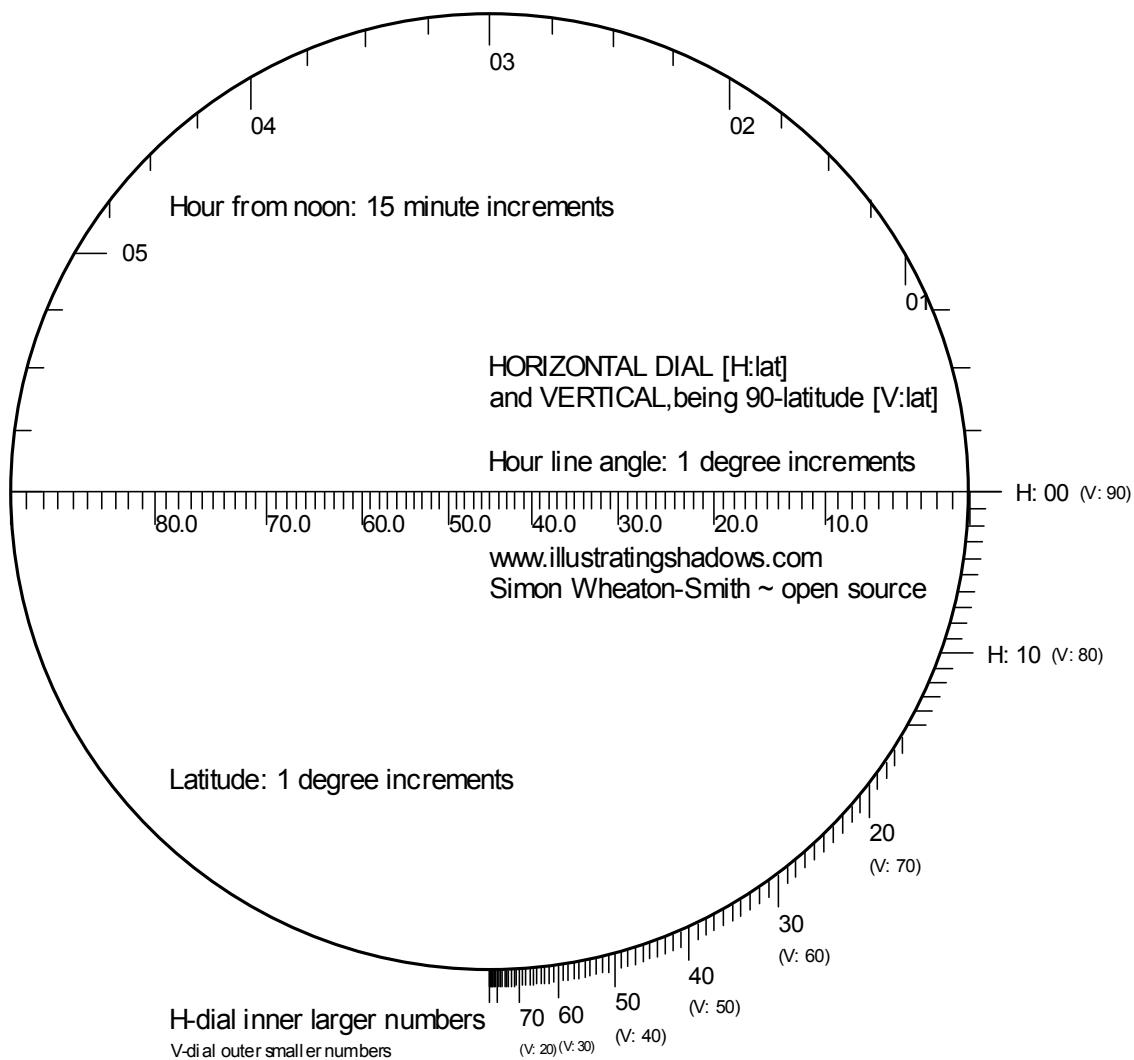
Latitude



Horizontal and Vertical dial plate  
[www.illustratingshadows.com](http://www.illustratingshadows.com)

latitude is  $\log(\sin(lat))$  so scale decreases vertically  
longitude is  $\log(\tan(long))$  so scale decreases first then increases  
hour line angle is  $\log(\tan(hla))$  so scale decreases first then increases

Horizontal and Vertical dial nomogram for hour line angles – method 3 – circular nomogram



The trigonometric and graphical logic for this nomogram is discussed in chapter 32, and the code that produced it is in “nomogram.bas” (for DeltaCAD), or “nomogram.pas: (for Lazarus/Pascal).

Of interest, this nomogram is based on the 1880 work of Philbert d’Ocagne, and notes from a book by Allcock and Jones published in the mid 1930s. However, substantially the same nomogram appears in the 1638 edition of “The Art of Dialling” by Samuel Foster who also developed the “dialing scales”.

## REFLECTING OR CEILING DIAL INFORMATION.

### Point above mirror to equinox point on meridian line

Distance from a point above a mirror, along the meridian line, to the equinox line, which you use by multiplying by the distance from the mirror to the ceiling. Also shown is the distance to the dial center from the equinox point on the meridian line.

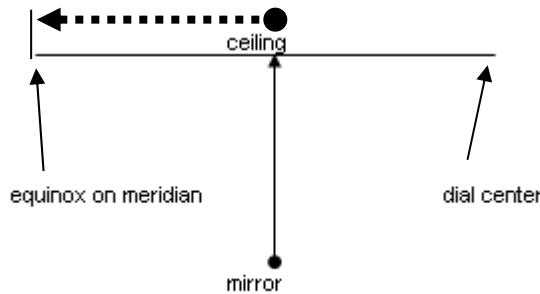
Distance from point on ceiling immediately above the mirror to the equinox,  
and the dial center assuming a mirror:ceiling height: 1

A3.2a

$$nb = mb * \tan(lat)$$

$$nc = mb * (\tan(lat) + 1/\tan(lat))$$

Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian to dial center	Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center
30	0.5774	2.309	46	1.0355	2.0012
31	0.6009	2.265	47	1.0724	2.0049
32	0.6249	2.225	48	1.1106	2.0110
33	0.6494	2.189	49	1.1504	2.0197
34	0.6745	2.157	50	1.1918	2.0309
35	0.7002	2.128	51	1.2349	2.0447
36	0.7265	2.103	52	1.2799	2.0612
37	0.7536	2.081	53	1.3270	2.0806
38	0.7813	2.061	54	1.3764	2.1029
39	0.8098	2.045	55	1.4281	2.1284
40	0.8391	2.031	56	1.4826	2.1571
41	0.8693	2.020	57	1.5399	2.1893
42	0.9004	2.011	58	1.6003	2.2252
43	0.9325	2.005	59	1.6643	2.2651
44	0.9657	2.001	60	1.7321	2.3094
45	1.0000	2.000	61	1.8040	2.3584



NOTE: these tables and this concept can be used for large horizontal dial layout if you reverse the hours.

## Reflecting or ceiling dial information

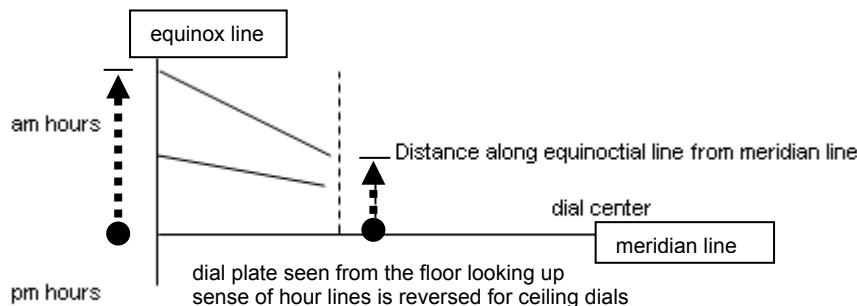
A3.2b

### Hour line points from equinox point (on meridian line) along equinox line

assumes a mirror:ceiling height:

1

Latitude	Distance from dial center to equinox line	Hours from noon		0900 or 1500		0500 or 1700	
		1	2	3	4	5	6
30	2.309	0.309	0.667	1.155	2.000	4.309	inf
31	2.265	0.313	0.674	1.167	2.021	4.354	inf
32	2.225	0.316	0.681	1.179	2.042	4.401	inf
33	2.189	0.319	0.688	1.192	2.065	4.450	inf
34	2.157	0.323	0.696	1.206	2.089	4.502	inf
35	2.128	0.327	0.705	1.221	2.114	4.556	inf
36	2.103	0.331	0.714	1.236	2.141	4.613	inf
37	2.081	0.336	0.723	1.252	2.169	4.673	inf
38	2.061	0.340	0.733	1.269	2.198	4.736	inf
39	2.045	0.345	0.743	1.287	2.229	4.802	inf
40	2.031	0.350	0.754	1.305	2.261	4.872	inf
41	2.020	0.355	0.765	1.325	2.295	4.945	inf
42	2.011	0.361	0.777	1.346	2.331	5.022	inf
43	2.005	0.366	0.789	1.367	2.368	5.103	inf
44	2.001	0.372	0.803	1.390	2.408	5.188	inf
45	2.000	0.379	0.816	1.414	2.449	5.278	inf
46	2.001	0.386	0.831	1.440	2.493	5.372	inf
47	2.005	0.393	0.847	1.466	2.540	5.472	inf
48	2.011	0.400	0.863	1.494	2.589	5.577	inf
49	2.020	0.408	0.880	1.524	2.640	5.689	inf
50	2.031	0.417	0.898	1.556	2.695	5.806	inf
51	2.045	0.426	0.917	1.589	2.752	5.930	inf
52	2.061	0.435	0.938	1.624	2.813	6.062	inf
53	2.081	0.445	0.959	1.662	2.878	6.201	inf
54	2.103	0.456	0.982	1.701	2.947	6.349	inf
55	2.128	0.467	1.007	1.743	3.020	6.507	inf
56	2.157	0.479	1.032	1.788	3.097	6.674	inf
57	2.189	0.492	1.060	1.836	3.180	6.852	inf
58	2.225	0.506	1.090	1.887	3.269	7.043	inf
59	2.265	0.520	1.121	1.942	3.363	7.246	inf
60	2.309	0.536	1.155	2.000	3.464	7.464	inf



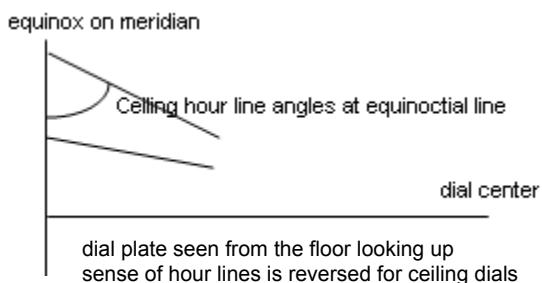
Reflecting or ceiling dial information.

**Hour line angles but along equinox line ( $90^\circ - \text{hour line angle from dial center}$ )**

A3.2c

Hour line angles at an hour point on the equinoctial line which is perpendicular to the meridian line

Latitude	Hours from noon		0900 or 1500		0500 or 1700		
	0	1	2	3	4	5	6
30	90.00	82.37	73.90	63.43	49.11	28.19	0.00
31	90.00	82.14	73.44	62.75	48.26	27.49	0.00
32	90.00	81.92	72.99	62.08	47.45	26.82	0.00
33	90.00	81.70	72.54	61.43	46.67	26.20	0.00
34	90.00	81.48	72.11	60.79	45.92	25.60	0.00
35	90.00	81.26	71.68	60.16	45.19	25.04	0.00
36	90.00	81.05	71.25	59.55	44.49	24.51	0.00
37	90.00	80.84	70.84	58.96	43.81	24.00	0.00
38	90.00	80.63	70.43	58.38	43.16	23.52	0.00
39	90.00	80.43	70.03	57.82	42.53	23.06	0.00
40	90.00	80.23	69.64	57.27	41.93	22.63	0.00
41	90.00	80.03	69.25	56.73	41.35	22.22	0.00
42	90.00	79.84	68.88	56.21	40.79	21.82	0.00
43	90.00	79.64	68.51	55.71	40.25	21.45	0.00
44	90.00	79.46	68.15	55.21	39.73	21.09	0.00
45	90.00	79.27	67.79	54.74	39.23	20.75	0.00
46	90.00	79.09	67.45	54.27	38.75	20.43	0.00
47	90.00	78.91	67.11	53.82	38.29	20.12	0.00
48	90.00	78.74	66.78	53.38	37.84	19.83	0.00
49	90.00	78.57	66.46	52.96	37.42	19.55	0.00
50	90.00	78.40	66.14	52.55	37.00	19.28	0.00
51	90.00	78.24	65.83	52.15	36.61	19.02	0.00
52	90.00	78.08	65.54	51.76	36.23	18.78	0.00
53	90.00	77.92	65.25	51.39	35.86	18.55	0.00
54	90.00	77.77	64.96	51.03	35.51	18.33	0.00
55	90.00	77.62	64.69	50.68	35.18	18.11	0.00
56	90.00	77.48	64.42	50.34	34.85	17.91	0.00
57	90.00	77.33	64.16	50.01	34.54	17.72	0.00
58	90.00	77.20	63.91	49.70	34.25	17.53	0.00
59	90.00	77.06	63.67	49.40	33.96	17.36	0.00
60	90.00	76.94	63.43	49.11	33.69	17.19	0.00



## POLAR DIAL AND MERIDIAN DIAL HOUR LINE DISTANCE

A3.3

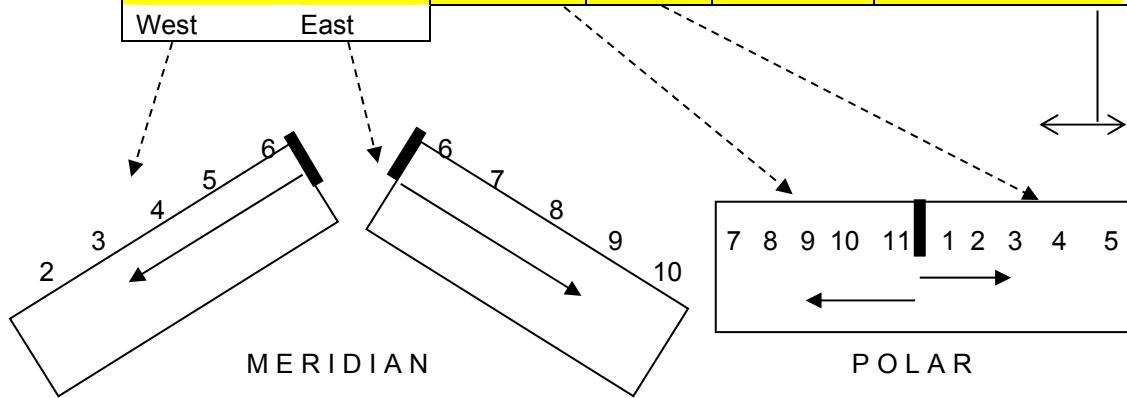
Polar and meridian hour line distances relative to **STYLE LINEAR HEIGHT**

A2.10 \* Hour baseline is from noon if polar, and 6 o'clock (am or pm) if meridian.

\* Hour relative distances are based on the linear style height.

1

MERIDIAN	POLAR		HOUR ANGLE	Distance from sub-style
1800	600	1200	1200	0.00
1745	615	1145	1215	3.75
1730	630	1130	1230	7.50
1715	645	1115	1245	11.25
1700	700	1100	1300	15.00
1645	715	1045	1315	18.75
1630	730	1030	1330	22.50
1615	745	1015	1345	26.25
1600	800	1000	1400	30.00
1545	815	945	1415	33.75
1530	830	930	1430	37.50
1515	845	915	1445	41.25
1500	900	900	1500	45.00
1445	915	845	1515	48.75
1430	930	830	1530	52.50
1415	945	815	1545	56.25
1400	1000	800	1600	60.00
1345	1015	745	1615	63.75
1330	1030	730	1630	67.50
1315	1045	715	1645	71.25
1300	1100	700	1700	75.00
1245	1115	645	1715	78.75
1230	1130	630	1730	82.50
1215	1145	615	1745	86.25
1200	1200	600	1800	90.00
				INF



Note: 3 pm on a west facing meridian dial, 9 am on an east facing meridian dial, and the 9 am and 3 pm hour lines of a polar dial are 45 degrees from the style, and as the tangent of 45 is 1 then their hour line distance from the sub-style is equal to the style linear height. In this book, the reference hours for calculations are: MERIDIAN DIALS: from 6 o'clock. (am or pm), and for POLAR DIALS: from noon. Other books use other conventions.

## POLAR DIAL AND MERIDIAN DIAL DECLINATION DISTANCE

A3.3a

### Polar and meridian declination distances relative to **STYLE LINEAR HEIGHT**

A2.10a

\* Hour and calendar relative distances are based on the linear style height.

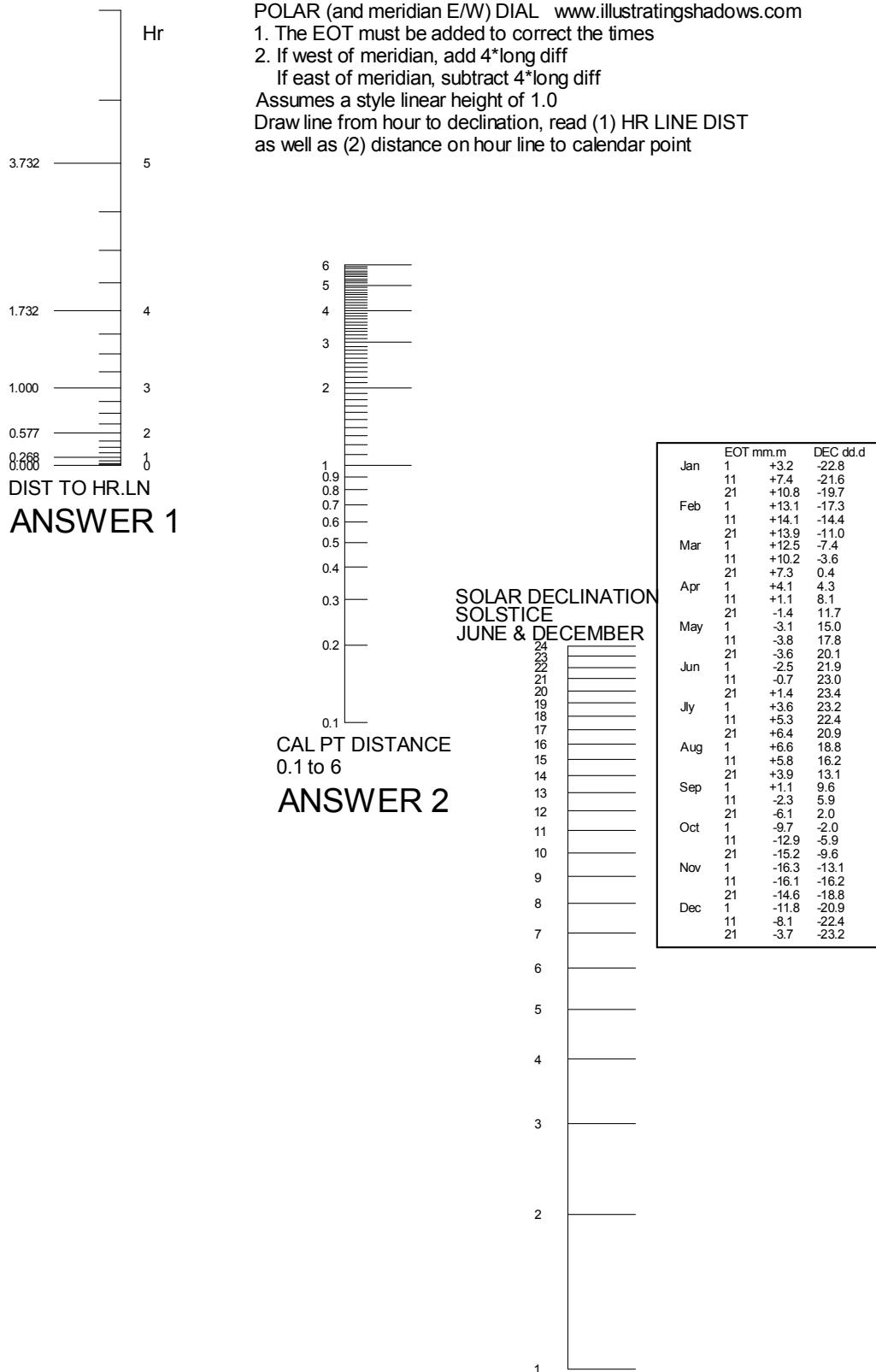
\* Hour baseline is from noon if polar, 6 o'clock if meridian.

			Meridian	6pm	5pm	4pm	3pm	2pm	1pm	1:30pm
			Polar	6am	7am	8am	9am	10am	11am	11:30am
			Polar	noon	11am	10am	9am	8am	7am	6:30am
			Polar	noon	1pm	2pm	3pm	4pm	5pm	5:30pm
Hour line from sub-style:				0.000	0.268	0.577	1.000	1.732	3.732	7.596
				0	1	2	3	4	5	5.5
Date	Julian	decl	Relative distance on hour line to calendar point							
1/1	1	-23.1	-0.426	-0.441	-0.492	-0.602	-0.851	-1.645	-3.261	
1/10	10	-22.1	-0.406	-0.420	-0.468	-0.574	-0.811	-1.567	-3.107	
<b>1/20</b>	<b>20</b>	<b>-20.3</b>	<b>-0.370</b>	<b>-0.383</b>	<b>-0.427</b>	<b>-0.523</b>	<b>-0.740</b>	<b>-1.429</b>	<b>-2.834</b>	
2/1	32	-17.3	-0.312	-0.323	-0.360	-0.441	-0.624	-1.206	-2.391	
2/10	41	-14.6	-0.261	-0.270	-0.301	-0.369	-0.521	-1.007	-1.998	
<b>2/20</b>	<b>51</b>	<b>-11.2</b>	<b>-0.198</b>	<b>-0.205</b>	<b>-0.229</b>	<b>-0.280</b>	<b>-0.396</b>	<b>-0.765</b>	<b>-1.517</b>	
3/1	60	-7.9	-0.138	-0.143	-0.160	-0.196	-0.277	-0.535	-1.060	
3/10	69	-4.4	-0.077	-0.080	-0.089	-0.109	-0.154	-0.298	-0.590	
<b>3/20</b>	<b>79</b>	<b>-0.5</b>	<b>-0.008</b>	<b>-0.008</b>	<b>-0.009</b>	<b>-0.011</b>	<b>-0.016</b>	<b>-0.031</b>	<b>-0.062</b>	
4/1	91	4.2	0.074	0.077	0.086	0.105	0.148	0.287	0.568	
4/10	100	7.7	0.134	0.139	0.155	0.190	0.269	0.519	1.030	
<b>4/20</b>	<b>110</b>	<b>11.2</b>	<b>0.199</b>	<b>0.206</b>	<b>0.230</b>	<b>0.281</b>	<b>0.398</b>	<b>0.768</b>	<b>1.523</b>	
5/1	121	14.8	0.265	0.274	0.306	0.374	0.530	1.023	2.028	
5/10	130	17.4	0.314	0.325	0.362	0.443	0.627	1.211	2.402	
<b>5/20</b>	<b>140</b>	<b>19.8</b>	<b>0.360</b>	<b>0.373</b>	<b>0.416</b>	<b>0.509</b>	<b>0.720</b>	<b>1.392</b>	<b>2.760</b>	
6/1	152	21.9	0.403	0.417	0.465	0.570	0.806	1.557	3.087	
6/10	161	23.0	0.424	0.439	0.489	0.599	0.847	1.637	3.246	
<b>6/20</b>	<b>171</b>	<b>23.4</b>	<b>0.434</b>	<b>0.449</b>	<b>0.501</b>	<b>0.613</b>	<b>0.867</b>	<b>1.675</b>	<b>3.322</b>	
7/1	182	23.2	0.428	0.443	0.494	0.605	0.856	1.654	3.280	
7/10	191	22.4	0.411	0.426	0.475	0.581	0.822	1.589	3.150	
<b>7/20</b>	<b>201</b>	<b>20.8</b>	<b>0.380</b>	<b>0.394</b>	<b>0.439</b>	<b>0.538</b>	<b>0.761</b>	<b>1.469</b>	<b>2.914</b>	
8/1	213	18.2	0.329	0.341	0.380	0.466	0.658	1.272	2.522	
8/10	222	15.8	0.283	0.293	0.327	0.400	0.566	1.093	2.168	
<b>8/20</b>	<b>232</b>	<b>12.7</b>	<b>0.226</b>	<b>0.233</b>	<b>0.260</b>	<b>0.319</b>	<b>0.451</b>	<b>0.871</b>	<b>1.728</b>	
9/1	244	8.6	0.151	0.156	0.174	0.213	0.301	0.582	1.155	
9/10	253	5.2	0.092	0.095	0.106	0.130	0.184	0.355	0.703	
<b>9/20</b>	<b>263</b>	<b>1.4</b>	<b>0.025</b>	<b>0.026</b>	<b>0.028</b>	<b>0.035</b>	<b>0.049</b>	<b>0.095</b>	<b>0.189</b>	
10/1	274	-2.9	-0.050	-0.052	-0.058	-0.071	-0.100	-0.193	-0.384	
10/10	283	-6.3	-0.111	-0.115	-0.128	-0.157	-0.222	-0.429	-0.850	
<b>10/20</b>	<b>293</b>	<b>-10.1</b>	<b>-0.177</b>	<b>-0.184</b>	<b>-0.205</b>	<b>-0.251</b>	<b>-0.355</b>	<b>-0.685</b>	<b>-1.359</b>	
11/1	305	-14.2	-0.253	-0.262	-0.292	-0.358	-0.506	-0.977	-1.937	
11/10	314	-16.9	-0.305	-0.315	-0.352	-0.431	-0.609	-1.177	-2.334	
<b>11/20</b>	<b>324</b>	<b>-19.5</b>	<b>-0.355</b>	<b>-0.367</b>	<b>-0.410</b>	<b>-0.502</b>	<b>-0.709</b>	<b>-1.371</b>	<b>-2.718</b>	
12/1	335	-21.7	-0.398	-0.412	-0.459	-0.563	-0.796	-1.537	-3.047	
12/10	344	-22.8	-0.421	-0.436	-0.486	-0.596	-0.842	-1.627	-3.227	
<b>12/20</b>	<b>354</b>	<b>-23.4</b>	<b>-0.433</b>	<b>-0.448</b>	<b>-0.500</b>	<b>-0.612</b>	<b>-0.866</b>	<b>-1.672</b>	<b>-3.316</b>	

## POLAR DIAL AND MERIDIAN DIAL DECLINATION DISTANCE

A3.3b

LOCAL TIME FROM TRANSIT



	EOT mm.m	DEC dd.d
Jan	+3.2	-22.8
	+7.4	-21.6
	+10.8	-19.7
Feb	+13.1	-17.3
	+14.1	-14.4
	+13.9	-11.0
Mar	+12.5	-7.4
	+10.2	-3.6
	+7.3	0.4
Apr	+4.1	4.3
	+1.1	8.1
	-1.4	11.7
May	-3.1	15.0
	-3.8	17.8
	-3.6	20.1
Jun	-2.5	21.9
	-0.7	23.0
	+1.4	23.4
Jly	+3.6	23.2
	+5.3	22.4
	+6.4	20.9
Aug	+6.6	18.8
	+5.8	16.2
	+3.9	13.1
Sep	+1.1	9.6
	-2.3	5.9
	-6.1	2.0
Oct	-9.7	-2.0
	-12.9	-5.9
	-15.2	-9.6
Nov	-16.3	-13.1
	-16.1	-16.2
	-14.6	-18.8
Dec	-11.8	-20.9
	-8.1	-22.4
	-3.7	-23.2

## POLAR DIAL AND MERIDIAN DIAL DECLINATION DISTANCE

method 2 – N or Z nomogram

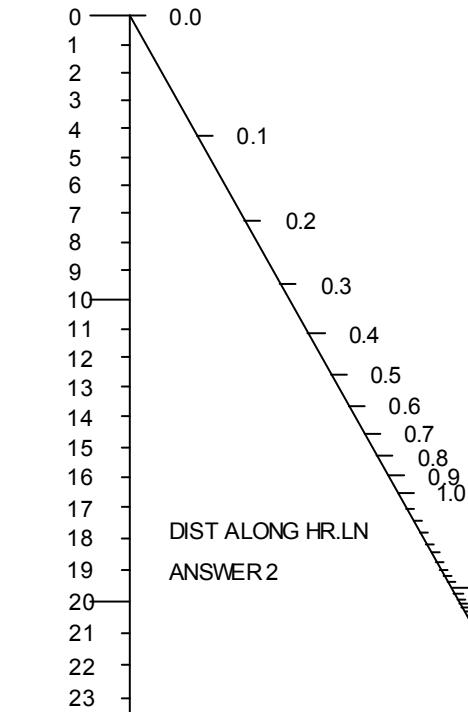
A3.3c

Diagram A3.3b used the three vertical line form of nomogram. As discussed in chapter 32, that has a raw data base line of 0, and since  $\log(0)$  is unusable, the nomogram appears to start with 1 in the case of the declination. Another nomogram form, the N or Z nomogram, is used directly for divisions, and thus logs are not used, and thus the declination scale starts with 0.

POLAR (and meridian E/W) DIAL www.illustratingshadows.com

1. The EOT must be added to correct the times
2. If west of meridian, add 4° long diff  
If east of meridian, subtract 4° long diff
- Assumes a style linear height of 1.0
- Draw line from hour to declination, read (1) HR LINE DIST as well as (2) distance on hour line to calendar point

MARCH & SEPTEMBER  
EQUINOX



HOURS FROM TRANSIT  
Hr tan  
cos  
1.000 0 0.000  
0.966 1 0.268

0.866 2 0.577  
0.707 3 1.000  
0.500 4 1.732  
0.259 5 3.732  
0.000 6

	EOT mm.m	DEC dd.d
Jan	1 +3.2	-22.8
	11 +7.4	-21.6
	21 +10.8	-19.7
	1 +13.1	-17.3
	11 +14.1	-14.4
	21 +13.9	-11.0
Mar	1 +12.5	-7.4
	11 +10.2	-3.6
	21 +7.3	0.4
Apr	1 +4.1	4.3
	11 +1.1	8.1
	21 -1.4	11.7
May	1 -3.1	15.0
	11 -3.8	17.8
	21 -3.6	20.1
Jun	1 -2.5	21.9
	11 -0.7	23.0
	21 +1.4	23.4
Jly	1 +3.6	23.2
	11 +5.3	22.4
	21 +6.4	20.9
Aug	1 +6.6	18.8
	11 +5.8	16.2
	21 +3.9	13.1
Sep	1 +1.1	9.6
	11 -2.3	5.9
	21 -6.1	2.0
Oct	1 -9.7	-2.0
	11 -12.9	-5.9
	21 -15.2	-9.6
Nov	1 -16.3	-13.1
	11 -16.1	-16.2
	21 -14.6	-18.8
Dec	1 -11.8	-20.9
	11 -8.1	-22.4
	21 -3.7	-23.2

**RADIUS OF CALENDAR LINES FOR EQUATORIAL DIALS  
AND DISTANCE UP (summer) OR DOWN (winter)  
FOR THE HORIZONTAL LINE SHOWING SUNRISE/SUNSET**

A3.4

SOLAR DECLINATION FOR EVERY TEN DAYS & EQUATORIAL CALENDAR RADII															
Date	decl	RELATIVE SIZE OF GNOMON									Julian	Approx	SUNSET LINE		
		0.1	0.2	0.25	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9	LAT	tan	
1/1	-23.1	0.23	0.47	0.59	0.70	0.94	1.17	1.41	1.64	1.76	1.88	2.11	1	3.24	20 0.36
1/10	-22.1	0.25	0.49	0.62	0.74	0.99	1.23	1.48	1.73	1.85	1.97	2.22	10	7.11	22 0.40
1/20	-20.3	0.27	0.54	0.68	0.81	1.08	1.35	1.62	1.89	2.03	2.16	2.43	20	10.38	24 0.45
2/1	-17.3	0.32	0.64	0.80	0.96	1.28	1.60	1.92	2.24	2.40	2.56	2.88	32	13.22	26 0.49
2/10	-14.6	0.38	0.77	0.96	1.15	1.53	1.92	2.30	2.68	2.88	3.07	3.45	41	14.13	28 0.53
2/20	-11.2	0.50	1.01	1.26	1.51	2.02	2.52	3.03	3.53	3.79	4.04	4.54	51	13.57	30 0.58
3/1	-7.9	0.72	1.45	1.81	2.17	2.89	3.61	4.34	5.06	5.42	5.78	6.50	60	12.43	31 0.60
3/10	-4.4	1.30	2.60	3.25	3.90	5.19	6.49	7.79	9.09	9.74	---	---	69	10.43	32 0.62
3/20	-0.5	---	---	---	---	---	---	---	---	---	---	---	79	7.51	33 0.65
4/1	4.2	1.35	2.70	3.37	4.04	5.39	6.74	8.09	9.44	---	---	---	91	4.03	34 0.67
4/10	7.7	0.74	1.49	1.86	2.23	2.98	3.72	4.46	5.21	5.58	5.95	6.70	100	1.20	35 0.70
4/20	11.2	0.50	1.01	1.26	1.51	2.01	2.52	3.02	3.52	3.77	4.03	4.53	110	-1.15	36 0.73
5/1	14.8	0.38	0.76	0.94	1.13	1.51	1.89	2.27	2.64	2.83	3.02	3.40	121	-3.11	37 0.75
5/10	17.4	0.32	0.64	0.80	0.96	1.28	1.59	1.91	2.23	2.39	2.55	2.87	130	-3.57	38 0.78
5/20	19.8	0.28	0.56	0.69	0.83	1.11	1.39	1.67	1.94	2.08	2.22	2.50	140	-3.52	39 0.81
6/1	21.9	0.25	0.50	0.62	0.74	0.99	1.24	1.49	1.74	1.86	1.99	2.23	152	-2.38	40 0.84
6/10	23.0	0.24	0.47	0.59	0.71	0.94	1.18	1.42	1.65	1.77	1.89	2.12	161	-1.06	41 0.87
6/20	23.4	0.23	0.46	0.58	0.69	0.92	1.15	1.38	1.61	1.73	1.85	2.08	171	0.56	42 0.90
7/1	23.2	0.23	0.47	0.58	0.70	0.93	1.17	1.40	1.64	1.75	1.87	2.10	182	3.10	43 0.93
7/10	22.4	0.24	0.49	0.61	0.73	0.97	1.22	1.46	1.70	1.82	1.95	2.19	191	4.39	44 0.97
7/20	20.8	0.26	0.53	0.66	0.79	1.05	1.31	1.58	1.84	1.97	2.10	2.37	201	5.40	45 1.00
8/1	18.2	0.30	0.61	0.76	0.91	1.21	1.52	1.82	2.13	2.28	2.43	2.73	213	5.41	46 1.04
8/10	15.8	0.35	0.71	0.88	1.06	1.41	1.77	2.12	2.47	2.65	2.83	3.18	222	4.44	47 1.07
8/20	12.7	0.44	0.89	1.11	1.33	1.77	2.22	2.66	3.10	3.33	3.55	3.99	232	2.43	48 1.11
9/1	8.6	0.66	1.33	1.66	1.99	2.65	3.32	3.98	4.64	4.98	5.31	5.97	244	-0.48	49 1.15
9/10	5.2	1.09	2.18	2.72	3.27	4.36	5.45	6.53	7.62	8.17	8.71	9.80	253	-3.57	50 1.19
9/20	1.4	---	---	---	---	---	---	---	---	---	---	---	263	-7.39	51 1.23
10/1	-2.9	2.00	3.99	4.99	5.99	7.99	9.99	---	---	---	---	---	274	-11.27	52 1.28
10/10	-6.3	0.90	1.80	2.25	2.70	3.60	4.50	5.41	6.31	6.76	7.21	8.11	283	-14.01	53 1.33
10/20	-10.1	0.56	1.13	1.41	1.69	2.26	2.82	3.38	3.95	4.23	4.51	5.08	293	-15.58	54 1.38
11/1	-14.2	0.40	0.79	0.99	1.19	1.58	1.98	2.37	2.77	2.97	3.16	3.56	305	-16.43	55 1.43
11/10	-16.9	0.33	0.66	0.82	0.98	1.31	1.64	1.97	2.30	2.46	2.63	2.95	314	-16.00	56 1.48
11/20	-19.5	0.28	0.56	0.70	0.85	1.13	1.41	1.69	1.97	2.11	2.26	2.54	324	-13.58	57 1.54
12/1	-21.7	0.25	0.50	0.63	0.75	1.01	1.26	1.51	1.76	1.89	2.01	2.26	335	-10.25	58 1.60
12/10	-22.8	0.24	0.47	0.59	0.71	0.95	1.19	1.42	1.66	1.78	1.90	2.14	344	-6.43	59 1.66
12/20	-23.4	0.23	0.46	0.58	0.69	0.92	1.16	1.39	1.62	1.73	1.85	2.08	354	-2.09	60 1.73

decl Radii of calendar circle. Equinox or >10 shows as --- Julian mm.ss

Calendar line radius is equal to the gnomon linear height divided by the tan of the declination, and the vertical distance for the horizontal sunrise/set line is equal to the gnomon linear height times the tan of the latitude. Radius of calendar lines is shown for tenths and quarters of a relative gnomon linear length. The sunrise and sunset line is horizontal and is above (summer) or below (winter) the gnomon's base by a distance = tan(latitude), and latitudes 20 through 60 are shown. If an equinox or if 10 or more then "—" is displayed.

## APPENDIX 4 ~ALTITUDE, AZIMUTH & DECLINATION DATA

LATITUDE 30		Altitude								A4.1a	
SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE											
LATTITUDE:	30	Shepherd gnomon:chart ratio:								0.11	8.70
Date	decl	am	altitude								Approx EOT
		600	700	800	900	1000	1100	1200	Julian		
1/1	-23.1			0.6	11.7	21.6	29.6	35.0	36.9	1	3.24
1/10	-22.1			1.1	12.3	22.3	30.5	36.0	37.9	10	7.11
1/20	-20.3			2.1	13.5	23.6	32.0	37.7	39.7	20	10.38
2/1	-17.3			3.7	15.3	25.8	34.5	40.5	42.7	32	13.22
2/10	-14.6			5.2	17.0	27.8	36.8	43.1	45.4	41	14.13
2/20	-11.2			7.0	19.1	30.2	39.7	46.3	48.8	51	13.57
3/1	-7.9			8.8	21.1	32.6	42.4	49.5	52.1	60	12.43
3/10	-4.4			10.7	23.2	34.9	45.2	52.7	55.6	69	10.43
3/20	-0.5			12.7	25.4	37.5	48.2	56.4	59.5	79	7.51
4/1	4.2	2.1	15.1	28.0	40.4	51.7	60.6	64.2	91		4.03
4/10	7.7	3.8	16.8	29.7	42.3	54.1	63.6	67.7	100		1.20
4/20	11.2	5.6	18.5	31.5	44.3	56.4	66.6	71.2	110		-1.15
5/1	14.8	7.4	20.2	33.1	46.1	58.5	69.5	74.8	121		-3.11
5/10	17.4	8.6	21.3	34.2	47.2	59.9	71.4	77.4	130		-3.57
5/20	19.8	9.8	22.4	35.2	48.2	61.1	73.0	79.8	140		-3.52
6/1	21.9	10.8	23.3	36.1	49.0	61.9	74.3	81.9	152		-2.38
6/10	23.0	11.2	23.7	36.4	49.4	62.3	74.9	83.0	161		-1.06
6/20	23.4	11.5	23.9	36.6	49.5	62.5	75.1	83.4	171		0.56
7/1	23.2	11.3	23.8	36.5	49.4	62.4	75.0	83.2	182		3.10
7/10	22.4	11.0	23.4	36.2	49.2	62.1	74.5	82.4	191		4.39
7/20	20.8	10.2	22.8	35.6	48.6	61.5	73.7	80.8	201		5.40
8/1	18.2	9.0	21.7	34.6	47.6	60.3	72.0	78.2	213		5.41
8/10	15.8	7.8	20.6	33.6	46.5	59.1	70.2	75.8	222		4.44
8/20	12.7	6.3	19.2	32.2	45.0	57.3	67.8	72.7	232		2.43
9/1	8.6	4.3	17.2	30.2	42.8	54.7	64.4	68.6	244		-0.48
9/10	5.2	2.6	15.6	28.5	41.0	52.4	61.5	65.2	253		-3.57
9/20	1.4	0.7	13.7	26.4	38.6	49.7	58.1	61.4	263		-7.39
10/1	-2.9		11.5	24.0	35.9	46.4	54.1	57.1	274		-11.27
10/10	-6.3		9.6	22.0	33.6	43.7	50.9	53.7	283		-14.01
10/20	-10.1		7.7	19.8	31.0	40.6	47.4	49.9	293		-15.58
11/1	-14.2		5.4	17.3	28.1	37.2	43.5	45.8	305		-16.43
11/10	-16.9		3.9	15.6	26.1	34.9	40.9	43.1	314		-16.00
11/20	-19.5		2.5	13.9	24.2	32.7	38.4	40.5	324		-13.58
12/1	-21.7		1.3	12.6	22.6	30.8	36.3	38.3	335		-10.25
12/10	-22.8		0.7	11.8	21.7	29.8	35.2	37.2	344		-6.43
12/20	-23.4		0.4	11.5	21.3	29.3	34.7	36.6	354		-2.09
decl	1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss		
pm	altitude										

$$\text{altitude} = \text{asin}(\sin(\text{decl}) * \sin(\text{lat}) + \cos(\text{decl}) * \cos(\text{lat}) * \cos(\text{time}))$$

LATITUDE 32		Altitude								A4.1 b			
SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE													
LATTITUDE:		32	Shepherd gnomon:chart ratio:		0.15	6.64							
Date	decl	am	altitude		600	700	800	900	1000	1100	1200	Julian	Approx EOT
1/1	-23.1		10.5	20.1	27.9	33.1	34.9					1	3.24
1/10	-22.1		0.2	11.2	20.9	28.8	34.1	35.9				10	7.11
1/20	-20.3		1.3	12.3	22.2	30.3	35.8	37.7				20	10.38
2/1	-17.3		3.0	14.3	24.5	32.9	38.6	40.7				32	13.22
2/10	-14.6		4.5	16.1	26.5	35.2	41.2	43.4				41	14.13
2/20	-11.2		6.5	18.2	29.0	38.1	44.5	46.8				51	13.57
3/1	-7.9		8.3	20.3	31.4	40.9	47.6	50.1				60	12.43
3/10	-4.4		10.3	22.5	33.9	43.8	50.9	53.6				69	10.43
3/20	-0.5		12.4	24.8	36.5	46.9	54.6	57.5				79	7.51
4/1	4.2	2.2	15.0	27.5	39.6	50.5	58.9	62.2				91	4.03
4/10	7.7	4.0	16.7	29.4	41.7	53.0	61.9	65.7				100	1.20
4/20	11.2	5.9	18.6	31.3	43.7	55.5	65.1	69.2				110	-1.15
5/1	14.8	7.8	20.4	33.1	45.7	57.7	68.0	72.8				121	-3.11
5/10	17.4	9.1	21.6	34.3	46.9	59.2	70.1	75.4				130	-3.57
5/20	19.8	10.3	22.7	35.3	48.1	60.5	71.9	77.8				140	-3.52
6/1	21.9	11.4	23.7	36.3	49.0	61.6	73.3	79.9				152	-2.38
6/10	23.0	11.9	24.1	36.7	49.4	62.0	73.9	81.0				161	-1.06
6/20	23.4	12.2	24.3	36.9	49.6	62.2	74.2	81.4				171	0.56
7/1	23.2	12.0	24.2	36.8	49.4	62.1	74.1	81.2				182	3.10
7/10	22.4	11.6	23.9	36.4	49.1	61.7	73.6	80.4				191	4.39
7/20	20.8	10.9	23.2	35.8	48.5	61.0	72.6	78.8				201	5.40
8/1	18.2	9.5	22.0	34.6	47.3	59.7	70.7	76.2				213	5.41
8/10	15.8	8.3	20.8	33.5	46.2	58.3	68.8	73.8				222	4.44
8/20	12.7	6.7	19.3	32.0	44.6	56.4	66.3	70.7				232	2.43
9/1	8.6	4.5	17.2	29.9	42.2	53.6	62.7	66.6				244	-0.48
9/10	5.2	2.8	15.5	28.1	40.2	51.2	59.8	63.2				253	-3.57
9/20	1.4	0.7	13.4	25.9	37.8	48.4	56.3	59.4				263	-7.39
10/1	-2.9		11.1	23.4	34.9	45.0	52.3	55.1				274	-11.27
10/10	-6.3		9.2	21.3	32.5	42.2	49.1	51.7				283	-14.01
10/20	-10.1		7.1	19.0	29.9	39.1	45.6	47.9				293	-15.58
11/1	-14.2		4.8	16.3	26.8	35.6	41.6	43.8				305	-16.43
11/10	-16.9		3.2	14.5	24.8	33.2	39.0	41.1				314	-16.00
11/20	-19.5		1.7	12.9	22.8	31.0	36.5	38.5				324	-13.58
12/1	-21.7		0.5	11.4	21.2	29.1	34.4	36.3				335	-10.25
12/10	-22.8			10.7	20.3	28.1	33.3	35.2				344	-6.43
12/20	-23.4			10.3	19.9	27.6	32.8	34.6				354	-2.09
decl		1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss			
	pm	altitude											

Blocked out numbers are those whose altitude is below the horizon. **Formulae involving dates** use approximations thus these tables may disagree with other sources using other formulae. This and other publications have figures that are well within drafting tolerances.

## LATITUDE 34

## Altitude

A4.1 c

SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE

LATTITUDE:	34	Shepherd gnomon:chart ratio:	0.19	5.37
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Date	decl	altitude								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1			9.3	18.7	26.2	31.2	32.9	1	3.24	
1/10	-22.1			10.0	19.5	27.1	32.1	33.9	10	7.11	
1/20	-20.3		0.4	11.2	20.8	28.6	33.9	35.7	20	10.38	
2/1	-17.3		2.2	13.2	23.1	31.2	36.7	38.7	32	13.22	
2/10	-14.6		3.8	15.1	25.2	33.6	39.3	41.4	41	14.13	
2/20	-11.2		5.8	17.3	27.8	36.6	42.6	44.8	51	13.57	
3/1	-7.9		7.8	19.5	30.3	39.4	45.8	48.1	60	12.43	
3/10	-4.4		9.8	21.7	32.8	42.3	49.1	51.6	69	10.43	
3/20	-0.5		12.1	24.2	35.6	45.5	52.8	55.5	79	7.51	
4/1	4.2	2.4	14.8	27.0	38.8	49.2	57.1	60.2	91	4.03	
4/10	7.7	4.3	16.7	29.0	41.0	51.8	60.2	63.7	100	1.20	
4/20	11.2	6.3	18.6	31.0	43.2	54.4	63.4	67.2	110	-1.15	
5/1	14.8	8.2	20.5	32.9	45.2	56.8	66.5	70.8	121	-3.11	
5/10	17.4	9.6	21.8	34.3	46.6	58.5	68.7	73.4	130	-3.57	
5/20	19.8	10.9	23.0	35.4	47.8	59.9	70.5	75.8	140	-3.52	
6/1	21.9	12.1	24.1	36.4	48.8	61.0	72.1	77.9	152	-2.38	
6/10	23.0	12.6	24.6	36.9	49.3	61.5	72.8	79.0	161	-1.06	
6/20	23.4	12.9	24.8	37.1	49.5	61.8	73.2	79.4	171	0.56	
7/1	23.2	12.7	24.7	37.0	49.4	61.7	73.0	79.2	182	3.10	
7/10	22.4	12.3	24.3	36.6	49.0	61.2	72.4	78.4	191	4.39	
7/20	20.8	11.5	23.5	35.9	48.3	60.4	71.3	76.8	201	5.40	
8/1	18.2	10.1	22.3	34.7	47.0	59.0	69.3	74.2	213	5.41	
8/10	15.8	8.8	21.0	33.4	45.8	57.5	67.3	71.8	222	4.44	
8/20	12.7	7.1	19.4	31.8	44.0	55.4	64.7	68.7	232	2.43	
9/1	8.6	4.8	17.2	29.6	41.5	52.5	61.1	64.6	244	-0.48	
9/10	5.2	2.9	15.4	27.6	39.4	50.0	58.1	61.2	253	-3.57	
9/20	1.4	0.8	13.2	25.4	36.9	47.0	54.5	57.4	263	-7.39	
10/1	-2.9		10.7	22.7	33.9	43.6	50.5	53.1	274	-11.27	
10/10	-6.3		8.7	20.5	31.4	40.7	47.2	49.7	283	-14.01	
10/20	-10.1		6.5	18.1	28.7	37.5	43.7	45.9	293	-15.58	
11/1	-14.2		4.1	15.4	25.5	34.0	39.7	41.8	305	-16.43	
11/10	-16.9		2.4	13.5	23.4	31.6	37.1	39.1	314	-16.00	
11/20	-19.5		0.9	11.8	21.4	29.3	34.6	36.5	324	-13.58	
12/1	-21.7			10.3	19.8	27.4	32.5	34.3	335	-10.25	
12/10	-22.8			9.5	18.9	26.4	31.4	33.2	344	-6.43	
12/20	-23.4			9.1	18.4	25.9	30.8	32.6	354	-2.09	

The gnomon:chart ratio on the top right of the chart is the ratio and reciprocal of the ratio of the gnomon's style length to the distance from the top of a chart of altitude curves, to the bottom on the lowest hour shown, typically 6/20 noon time. Used in shepherd's dials.

## LATITUDE 36

## Altitude

A4.1 d

SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE

LATTITUDE:

36

Shepherd gnomon:chart ratio:

0.22 4.49

Date	decl	altitude								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1			8.2	17.2	24.5	29.3	30.9	1	3.24	
1/10	-22.1			8.9	18.0	25.4	30.2	31.9	10	7.11	
1/20	-20.3			10.1	19.4	26.9	31.9	33.7	20	10.38	
2/1	-17.3		1.4	12.2	21.8	29.6	34.8	36.7	32	13.22	
2/10	-14.6		3.1	14.1	23.9	32.0	37.4	39.4	41	14.13	
2/20	-11.2		5.2	16.4	26.5	35.0	40.7	42.8	51	13.57	
3/1	-7.9		7.3	18.7	29.1	37.8	43.9	46.1	60	12.43	
3/10	-4.4		9.4	21.0	31.7	40.8	47.2	49.6	69	10.43	
3/20	-0.5		11.8	23.6	34.6	44.1	51.0	53.5	79	7.51	
4/1	4.2	2.5	14.6	26.5	37.9	47.9	55.4	58.2	91	4.03	
4/10	7.7	4.5	16.6	28.6	40.2	50.6	58.5	61.7	100	1.20	
4/20	11.2	6.6	18.7	30.8	42.5	53.3	61.8	65.2	110	-1.15	
5/1	14.8	8.7	20.7	32.8	44.7	55.9	64.9	68.8	121	-3.11	
5/10	17.4	10.1	22.1	34.2	46.2	57.6	67.1	71.4	130	-3.57	
5/20	19.8	11.5	23.3	35.4	47.5	59.1	69.1	73.8	140	-3.52	
6/1	21.9	12.7	24.5	36.5	48.6	60.4	70.8	75.9	152	-2.38	
6/10	23.0	13.3	25.0	37.0	49.1	61.0	71.6	77.0	161	-1.06	
6/20	23.4	13.5	25.2	37.2	49.3	61.2	71.9	77.4	171	0.56	
7/1	23.2	13.4	25.1	37.1	49.2	61.1	71.8	77.2	182	3.10	
7/10	22.4	12.9	24.7	36.7	48.8	60.6	71.1	76.4	191	4.39	
7/20	20.8	12.1	23.9	35.9	48.0	59.7	69.9	74.8	201	5.40	
8/1	18.2	10.6	22.5	34.6	46.7	58.1	67.8	72.2	213	5.41	
8/10	15.8	9.2	21.2	33.3	45.3	56.5	65.8	69.8	222	4.44	
8/20	12.7	7.4	19.5	31.6	43.4	54.4	63.1	66.7	232	2.43	
9/1	8.6	5.0	17.1	29.2	40.8	51.3	59.4	62.6	244	-0.48	
9/10	5.2	3.1	15.2	27.2	38.6	48.7	56.3	59.2	253	-3.57	
9/20	1.4	0.8	12.9	24.8	35.9	45.6	52.7	55.4	263	-7.39	
10/1	-2.9		10.4	22.0	32.8	42.1	48.7	51.1	274	-11.27	
10/10	-6.3		8.2	19.7	30.2	39.2	45.4	47.7	283	-14.01	
10/20	-10.1		5.9	17.2	27.4	36.0	41.8	43.9	293	-15.58	
11/1	-14.2		3.4	14.4	24.2	32.4	37.8	39.8	305	-16.43	
11/10	-16.9		1.7	12.5	22.1	29.9	35.2	37.1	314	-16.00	
11/20	-19.5		0.0	10.6	20.0	27.6	32.7	34.5	324	-13.58	
12/1	-21.7			9.1	18.3	25.7	30.6	32.3	335	-10.25	
12/10	-22.8			8.3	17.4	24.7	29.5	31.2	344	-6.43	
12/20	-23.4			7.9	16.9	24.2	28.9	30.6	354	-2.09	

decl	1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss
pm	altitude								

Note: the sun's altitude at solar noon is equal to the co-latitude ( $90 - \text{latitude}$ ) plus the declination. E.G. consider March 1 for latitude 36. The co-latitude is  $90 - 36$  or 54, which minus 7.9 (March 1 declination) provides 46.1, the noon altitude for March 1. This is useful for empirically building a Capuchin dial by direct solar measurement as it provides the month declination angles which are above and below the latitude line.

LATITUDE 38		Altitude								A4.1 e
SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE										
LATTITUDE:	38	Shepherd gnomon:chart ratio:		0.26	3.85					
Date	decl	am altitude								Approx EOT
		600	700	800	900	1000	1100	1200	Julian	
1/1	-23.1			7.0	15.8	22.8	27.3	28.9	1	3.24
1/10	-22.1			7.7	16.6	23.6	28.3	29.9	10	7.11
1/20	-20.3			9.0	18.0	25.2	30.0	31.7	20	10.38
2/1	-17.3		0.6	11.1	20.4	27.9	32.9	34.7	32	13.22
2/10	-14.6		2.4	13.1	22.6	30.3	35.5	37.4	41	14.13
2/20	-11.2		4.6	15.5	25.3	33.4	38.8	40.8	51	13.57
3/1	-7.9		6.8	17.8	27.9	36.3	42.0	44.1	60	12.43
3/10	-4.4		9.0	20.2	30.6	39.3	45.4	47.6	69	10.43
3/20	-0.5		11.5	22.9	33.5	42.6	49.1	51.5	79	7.51
4/1	4.2	2.6	14.4	26.0	37.0	46.6	53.6	56.2	91	4.03
4/10	7.7	4.7	16.5	28.2	39.4	49.3	56.8	59.7	100	1.20
4/20	11.2	6.9	18.7	30.4	41.8	52.1	60.1	63.2	110	-1.15
5/1	14.8	9.1	20.8	32.6	44.1	54.8	63.3	66.8	121	-3.11
5/10	17.4	10.6	22.3	34.1	45.7	56.7	65.6	69.4	130	-3.57
5/20	19.8	12.0	23.6	35.4	47.1	58.3	67.6	71.8	140	-3.52
6/1	21.9	13.3	24.8	36.6	48.3	59.7	69.4	73.9	152	-2.38
6/10	23.0	13.9	25.3	37.1	48.9	60.3	70.2	75.0	161	-1.06
6/20	23.4	14.2	25.6	37.3	49.1	60.6	70.6	75.4	171	0.56
7/1	23.2	14.0	25.5	37.2	49.0	60.4	70.4	75.2	182	3.10
7/10	22.4	13.5	25.0	36.8	48.5	59.9	69.7	74.4	191	4.39
7/20	20.8	12.6	24.2	36.0	47.7	59.0	68.5	72.8	201	5.40
8/1	18.2	11.1	22.7	34.5	46.2	57.2	66.3	70.2	213	5.41
8/10	15.8	9.6	21.3	33.1	44.7	55.5	64.2	67.8	222	4.44
8/20	12.7	7.8	19.5	31.3	42.8	53.2	61.4	64.7	232	2.43
9/1	8.6	5.3	17.1	28.8	40.0	50.0	57.6	60.6	244	-0.48
9/10	5.2	3.2	15.0	26.7	37.7	47.4	54.5	57.2	253	-3.57
9/20	1.4	0.9	12.7	24.1	34.9	44.2	50.9	53.4	263	-7.39
10/1	-2.9		10.0	21.3	31.7	40.6	46.8	49.1	274	-11.27
10/10	-6.3		7.7	18.9	29.1	37.6	43.5	45.7	283	-14.01
10/20	-10.1		5.4	16.3	26.2	34.4	39.9	41.9	293	-15.58
11/1	-14.2		2.7	13.4	22.9	30.7	35.9	37.8	305	-16.43
11/10	-16.9		0.9	11.4	20.7	28.3	33.3	35.1	314	-16.00
11/20	-19.5			9.5	18.6	25.9	30.8	32.5	324	-13.58
12/1	-21.7			8.0	16.9	24.0	28.7	30.3	335	-10.25
12/10	-22.8			7.1	15.9	23.0	27.5	29.2	344	-6.43
12/20	-23.4			6.7	15.5	22.4	27.0	28.6	354	-2.09

LATITUDE 40		Altitude								A4.1 f	
SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE											
LATTITUDE:	40	Shepherd gnomon:chart ratio:		0.30	3.36						
Date	decl	am	altitude	600	700	800	900	1000	1100	1200	
1/1	-23.1			5.8	14.3	21.0	25.4	26.9	1	3.24	
1/10	-22.1			6.5	15.1	21.9	26.4	27.9	10	7.11	
1/20	-20.3			7.8	16.6	23.5	28.1	29.7	20	10.38	
2/1	-17.3			10.0	19.0	26.2	31.0	32.7	32	13.22	
2/10	-14.6			1.7	12.0	21.2	28.7	33.6	41	14.13	
2/20	-11.2			4.0	14.5	24.0	31.7	36.9	51	13.57	
3/1	-7.9			6.2	16.9	26.6	34.7	40.2	60	12.43	
3/10	-4.4			8.5	19.4	29.4	37.7	43.5	69	10.43	
3/20	-0.5			11.1	22.2	32.4	41.2	47.3	79	7.51	
4/1	4.2			2.7	14.2	25.4	36.0	45.2	91	4.03	
4/10	7.7			4.9	16.4	27.7	38.5	48.0	100	1.20	
4/20	11.2			7.2	18.6	30.1	41.0	50.9	110	-1.15	
5/1	14.8			9.5	20.9	32.3	43.5	53.7	121	-3.11	
5/10	17.4			11.1	22.4	33.9	45.2	55.6	130	-3.57	
5/20	19.8			12.6	23.9	35.3	46.7	57.4	140	-3.52	
6/1	21.9			13.9	25.1	36.5	48.0	58.8	152	-2.38	
6/10	23.0			14.5	25.7	37.1	48.5	59.5	161	-1.06	
6/20	23.4			14.8	26.0	37.4	48.8	59.8	171	0.56	
7/1	23.2			14.7	25.8	37.2	48.7	59.6	182	3.10	
7/10	22.4			14.1	25.3	36.8	48.2	59.1	191	4.39	
7/20	20.8			13.2	24.4	35.9	47.3	58.1	201	5.40	
8/1	18.2			11.6	22.9	34.4	45.7	56.2	213	5.41	
8/10	15.8			10.1	21.5	32.9	44.1	54.4	222	4.44	
8/20	12.7			8.1	19.6	31.0	42.1	52.1	232	2.43	
9/1	8.6			5.5	17.0	28.3	39.2	48.7	244	-0.48	
9/10	5.2			3.4	14.8	26.1	36.7	46.0	253	-3.57	
9/20	1.4			0.9	12.4	23.5	33.9	42.8	263	-7.39	
10/1	-2.9			9.5	20.5	30.6	39.1	45.0	274	-11.27	
10/10	-6.3			7.2	18.0	27.9	36.0	41.6	283	-14.01	
10/20	-10.1			4.8	15.4	24.9	32.8	38.0	293	-15.58	
11/1	-14.2			2.0	12.3	21.6	29.1	34.0	305	-16.43	
11/10	-16.9			0.1	10.3	19.3	26.6	31.4	314	-16.00	
11/20	-19.5				8.4	17.2	24.2	28.8	30.5	324	-13.58
12/1	-21.7				6.8	15.4	22.3	26.7	28.3	335	-10.25
12/10	-22.8				5.9	14.5	21.2	25.6	27.2	344	-6.43
12/20	-23.4				5.5	14.0	20.7	25.1	26.6	354	-2.09
decl	1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss		
pm	altitude										

## LATITUDE 45

## Altitude

A4.1 g

SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE

LATTITUDE: 45 Shepherd gnomon:chart ratio: 0.40 2.53

Date	decl	am altitude							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1			2.8	10.6	16.6	20.6	21.9	1	3.24
1/10	-22.1			3.5	11.4	17.6	21.5	22.9	10	7.11
1/20	-20.3			4.9	12.9	19.2	23.3	24.7	20	10.38
2/1	-17.3			7.3	15.5	22.0	26.2	27.7	32	13.22
2/10	-14.6			9.4	17.8	24.5	28.8	30.4	41	14.13
2/20	-11.2		2.4	12.1	20.7	27.6	32.2	33.8	51	13.57
3/1	-7.9		4.8	14.7	23.5	30.6	35.4	37.1	60	12.43
3/10	-4.4		7.4	17.4	26.4	33.8	38.8	40.6	69	10.43
3/20	-0.5		10.2	20.4	29.6	37.3	42.6	44.5	79	7.51
4/1	4.2	3.0	13.6	23.9	33.4	41.5	47.2	49.2	91	4.03
4/10	7.7	5.4	16.0	26.4	36.1	44.5	50.5	52.7	100	1.20
4/20	11.2	7.9	18.5	29.0	38.9	47.6	53.9	56.2	110	-1.15
5/1	14.8	10.4	21.0	31.5	41.6	50.6	57.3	59.8	121	-3.11
5/10	17.4	12.2	22.7	33.3	43.5	52.7	59.7	62.4	130	-3.57
5/20	19.8	13.9	24.3	34.9	45.2	54.7	61.9	64.8	140	-3.52
6/1	21.9	15.3	25.7	36.3	46.7	56.3	63.9	66.9	152	-2.38
6/10	23.0	16.0	26.4	37.0	47.4	57.1	64.8	68.0	161	-1.06
6/20	23.4	16.3	26.7	37.3	47.7	57.5	65.2	68.4	171	0.56
7/1	23.2	16.2	26.5	37.1	47.6	57.3	65.0	68.2	182	3.10
7/10	22.4	15.6	26.0	36.6	47.0	56.6	64.2	67.4	191	4.39
7/20	20.8	14.6	25.0	35.6	45.9	55.5	62.8	65.8	201	5.40
8/1	18.2	12.8	23.3	33.8	44.1	53.4	60.4	63.2	213	5.41
8/10	15.8	11.1	21.6	32.2	42.3	51.4	58.2	60.8	222	4.44
8/20	12.7	8.9	19.5	30.0	40.0	48.8	55.3	57.7	232	2.43
9/1	8.6	6.0	16.6	27.1	36.9	45.3	51.3	53.6	244	-0.48
9/10	5.2	3.7	14.3	24.6	34.2	42.4	48.1	50.2	253	-3.57
9/20	1.4	1.0	11.6	21.8	31.2	39.0	44.4	46.4	263	-7.39
10/1	-2.9		8.5	18.5	27.6	35.2	40.3	42.1	274	-11.27
10/10	-6.3		6.0	15.9	24.8	32.0	36.9	38.7	283	-14.01
10/20	-10.1		3.3	13.0	21.6	28.7	33.3	34.9	293	-15.58
11/1	-14.2		0.2	9.8	18.1	24.9	29.3	30.8	305	-16.43
11/10	-16.9			7.6	15.8	22.3	26.6	28.1	314	-16.00
11/20	-19.5			5.6	13.6	19.9	24.0	25.5	324	-13.58
12/1	-21.7			3.9	11.7	17.9	21.9	23.3	335	-10.25
12/10	-22.8			2.9	10.7	16.9	20.8	22.2	344	-6.43
12/20	-23.4			2.5	10.3	16.3	20.2	21.6	354	-2.09

<b>LATITUDE 50</b>		<b>Altitude</b>							<b>A4.1 h</b>	
		SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE								
LATTITUDE:	50	Shepherd gnomon:chart ratio:		0.50	2.00					
Date	decl	am	altitude						Approx EOT	
		600	700	800	900	1000	1100	1200	Julian	
1/1	-23.1				6.8	12.2	15.7	16.9	1	3.24
1/10	-22.1			0.6	7.7	13.2	16.7	17.9	10	7.11
1/20	-20.3			2.0	9.2	14.9	18.5	19.7	20	10.38
2/1	-17.3			4.5	11.9	17.6	21.4	22.7	32	13.22
2/10	-14.6			6.8	14.3	20.2	24.0	25.4	41	14.13
2/20	-11.2		0.8	9.6	17.3	23.4	27.4	28.8	51	13.57
3/1	-7.9		3.4	12.3	20.2	26.5	30.7	32.1	60	12.43
3/10	-4.4		6.1	15.2	23.2	29.7	34.1	35.6	69	10.43
3/20	-0.5		9.2	18.4	26.6	33.4	37.9	39.5	79	7.51
4/1	4.2	3.2	12.9	22.2	30.7	37.7	42.5	44.2	91	4.03
4/10	7.7	5.9	15.5	24.9	33.5	40.8	45.8	47.7	100	1.20
4/20	11.2	8.6	18.2	27.7	36.5	44.1	49.3	51.2	110	-1.15
5/1	14.8	11.3	20.9	30.4	39.5	47.2	52.8	54.8	121	-3.11
5/10	17.4	13.2	22.8	32.4	41.5	49.5	55.2	57.4	130	-3.57
5/20	19.8	15.0	24.6	34.2	43.4	51.6	57.5	59.8	140	-3.52
6/1	21.9	16.6	26.1	35.8	45.1	53.4	59.6	61.9	152	-2.38
6/10	23.0	17.4	26.9	36.5	45.8	54.2	60.5	63.0	161	-1.06
6/20	23.4	17.7	27.2	36.8	46.2	54.6	61.0	63.4	171	0.56
7/1	23.2	17.5	27.0	36.7	46.0	54.4	60.7	63.2	182	3.10
7/10	22.4	16.9	26.4	36.1	45.4	53.7	59.9	62.4	191	4.39
7/20	20.8	15.8	25.3	34.9	44.2	52.4	58.5	60.8	201	5.40
8/1	18.2	13.9	23.4	33.0	42.2	50.2	56.0	58.2	213	5.41
8/10	15.8	12.0	21.6	31.2	40.2	48.1	53.7	55.8	222	4.44
8/20	12.7	9.7	19.3	28.8	37.7	45.4	50.7	52.7	232	2.43
9/1	8.6	6.6	16.2	25.6	34.3	41.7	46.7	48.6	244	-0.48
9/10	5.2	4.0	13.6	23.0	31.5	38.6	43.5	45.2	253	-3.57
9/20	1.4	1.1	10.7	19.9	28.2	35.1	39.8	41.4	263	-7.39
10/1	-2.9		7.3	16.4	24.6	31.2	35.6	37.1	274	-11.27
10/10	-6.3		4.6	13.6	21.5	28.0	32.2	33.7	283	-14.01
10/20	-10.1		1.7	10.5	18.3	24.5	28.5	29.9	293	-15.58
11/1	-14.2		7.1	14.6	20.6	24.5	25.8	305	-16.43	
11/10	-16.9		4.8	12.2	18.0	21.8	23.1	314	-16.00	
11/20	-19.5		2.7	9.9	15.6	19.2	20.5	324	-13.58	
12/1	-21.7		0.9	8.0	13.5	17.1	18.3	335	-10.25	
12/10	-22.8		0.9	7.0	12.5	16.0	17.2	344	-6.43	
12/20	-23.4			6.5	11.9	15.4	16.6	354	-2.09	
decl	1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss	
pm	altitude									

## LATITUDE 55

## Altitude

A4.1 i

SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE

LATTITUDE:	55	Shepherd gnomon:chart ratio:	0.61	1.63
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Date	decl	600	700	800	900	1000	1100	1200	Julian	Approx EOT
1/1	-23.1				3.0	7.8	10.9	11.9	1	3.24
1/10	-22.1				3.9	8.8	11.9	12.9	10	7.11
1/20	-20.3				5.5	10.5	13.6	14.7	20	10.38
2/1	-17.3			1.7	8.2	13.3	16.5	17.7	32	13.22
2/10	-14.6			4.1	10.7	15.9	19.2	20.4	41	14.13
2/20	-11.2			7.0	13.8	19.2	22.6	23.8	51	13.57
3/1	-7.9	2.0	9.9	16.8	22.3	25.9	27.1	60	12.43	
3/10	-4.4	4.9	12.9	20.0	25.6	29.3	30.6	69	10.43	
3/20	-0.5	8.2	16.3	23.5	29.3	33.2	34.5	79	7.51	
4/1	4.2	3.5	12.0	20.3	27.7	33.8	37.8	39.2	91	4.03
4/10	7.7	6.3	14.8	23.2	30.7	37.0	41.2	42.7	100	1.20
4/20	11.2	9.2	17.8	26.2	33.9	40.3	44.7	46.2	110	-1.15
5/1	14.8	12.1	20.7	29.1	37.0	43.6	48.2	49.8	121	-3.11
5/10	17.4	14.2	22.7	31.2	39.2	46.0	50.7	52.4	130	-3.57
5/20	19.8	16.1	24.7	33.2	41.2	48.2	53.0	54.8	140	-3.52
6/1	21.9	17.8	26.4	34.9	43.0	50.1	55.1	56.9	152	-2.38
6/10	23.0	18.6	27.1	35.7	43.9	51.0	56.1	58.0	161	-1.06
6/20	23.4	19.0	27.5	36.1	44.3	51.4	56.5	58.4	171	0.56
7/1	23.2	18.8	27.3	35.9	44.0	51.2	56.3	58.2	182	3.10
7/10	22.4	18.1	26.7	35.2	43.4	50.4	55.5	57.4	191	4.39
7/20	20.8	16.9	25.5	34.0	42.1	49.1	54.0	55.8	201	5.40
8/1	18.2	14.8	23.4	31.9	39.9	46.7	51.5	53.2	213	5.41
8/10	15.8	12.9	21.5	29.9	37.8	44.5	49.1	50.8	222	4.44
8/20	12.7	10.4	19.0	27.4	35.2	41.7	46.1	47.7	232	2.43
9/1	8.6	7.0	15.6	23.9	31.5	37.8	42.1	43.6	244	-0.48
9/10	5.2	4.3	12.9	21.1	28.6	34.7	38.8	40.2	253	-3.57
9/20	1.4	1.2	9.7	17.9	25.2	31.1	35.0	36.4	263	-7.39
10/1	-2.9		6.2	14.2	21.4	27.1	30.8	32.1	274	-11.27
10/10	-6.3		3.3	11.2	18.2	23.8	27.4	28.7	283	-14.01
10/20	-10.1		0.2	8.0	14.9	20.2	23.7	24.9	293	-15.58
11/1	-14.2		4.4	11.1	16.3	19.7	20.8	305	-16.43	
11/10	-16.9		2.0	8.6	13.7	16.9	18.1	314	-16.00	
11/20	-19.5			6.2	11.2	14.4	15.5	324	-13.58	
12/1	-21.7			4.2	9.1	12.2	13.3	335	-10.25	
12/10	-22.8			3.2	8.0	11.1	12.2	344	-6.43	
12/20	-23.4			2.7	7.5	10.5	11.6	354	-2.09	

decl	1800	1700	1600	1500	1400	1300	1200	Julian	mm.ss
pm	altitude								

## LATITUDE 60

## Altitude

A4.1 j

SOLAR DECLINATION FOR EVERY TEN DAYS & HOURLY SOLAR ALTITUDE

LATTITUDE: 60 Shepherd gnomon:chart ratio: 0.74 1.35

Date	decl	600	700	800	900	1000	1100	1200	Julian	Approx EOT
1/1	-23.1					3.4	6.0	6.9	1	3.24
1/10	-22.1				0.1	4.3	7.0	7.9	10	7.11
1/20	-20.3				1.8	6.1	8.8	9.7	20	10.38
2/1	-17.3			4.6	8.9	11.7	12.7	32	13.22	
2/10	-14.6		1.3	7.1	11.6	14.4	15.4	41	14.13	
2/20	-11.2		4.4	10.3	14.9	17.8	18.8	51	13.57	
3/1	-7.9	0.5	7.4	13.4	18.1	21.1	22.1	60	12.43	
3/10	-4.4	3.6	10.5	16.6	21.4	24.5	25.6	69	10.43	
3/20	-0.5	7.0	14.1	20.3	25.2	28.4	29.5	79	7.51	
4/1	4.2	3.7	11.1	18.3	24.6	29.7	33.1	91	4.03	
4/10	7.7	6.6	14.1	21.3	27.8	33.0	36.4	100	1.20	
4/20	11.2	9.7	17.2	24.5	31.0	36.4	40.0	110	-1.15	
5/1	14.8	12.8	20.3	27.6	34.3	39.8	43.5	121	-3.11	
5/10	17.4	15.0	22.5	29.8	36.6	42.2	46.0	130	-3.57	
5/20	19.8	17.1	24.5	31.9	38.8	44.5	48.4	140	-3.52	
6/1	21.9	18.9	26.3	33.8	40.7	46.5	50.5	152	-2.38	
6/10	23.0	19.7	27.2	34.6	41.6	47.4	51.5	161	-1.06	
6/20	23.4	20.2	27.6	35.0	42.0	47.9	52.0	171	0.56	
7/1	23.2	19.9	27.4	34.8	41.7	47.6	51.7	182	3.10	
7/10	22.4	19.2	26.7	34.1	41.0	46.9	50.9	191	4.39	
7/20	20.8	17.9	25.4	32.8	39.7	45.4	49.4	201	5.40	
8/1	18.2	15.7	23.2	30.5	37.3	43.0	46.8	213	5.41	
8/10	15.8	13.6	21.1	28.4	35.2	40.7	44.5	222	4.44	
8/20	12.7	11.0	18.5	25.7	32.4	37.8	41.4	232	2.43	
9/1	8.6	7.4	14.9	22.1	28.6	33.9	37.3	244	-0.48	
9/10	5.2	4.5	12.0	19.2	25.5	30.7	34.1	253	-3.57	
9/20	1.4	1.2	8.7	15.7	22.0	27.0	30.3	263	-7.39	
10/1	-2.9		4.9	11.9	18.0	22.9	26.0	274	-11.27	
10/10	-6.3		1.9	8.8	14.8	19.6	22.6	283	-14.01	
10/20	-10.1			5.4	11.4	16.0	18.9	293	-15.58	
11/1	-14.2			1.7	7.5	12.0	14.8	305	-16.43	
11/10	-16.9				4.9	9.3	12.1	314	-16.00	
11/20	-19.5				2.5	6.8	9.5	324	-13.58	
12/1	-21.7				0.5	4.7	7.4	335	-10.25	
12/10	-22.8					3.6	6.3	344	-6.43	
12/20	-23.4					3.1	5.7	354	-2.09	

decl 1800 1700 1600 1500 1400 1300 1200 Julian mm.ss  
pm altitude

## LATITUDE 30

## Azimuth

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

30

**A4.2 a**

Date	declination	am							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1	69.8	62.7	54.5	44.4	32.0	16.9	0.0	1	3.24
1/10	-22.1	70.6	63.5	55.2	45.1	32.5	17.2	0.0	10	7.11
1/20	-20.3	72.2	65.0	56.6	46.4	33.6	17.9	0.0	20	10.38
2/1	-17.3	74.9	67.5	59.0	48.6	35.4	19.0	0.0	32	13.22
2/10	-14.6	77.3	69.8	61.2	50.7	37.2	20.1	0.0	41	14.13
2/20	-11.2	80.3	72.7	64.0	53.4	39.6	21.6	0.0	51	13.57
3/1	-7.9	83.2	75.5	66.9	56.2	42.1	23.2	0.0	60	12.43
3/10	-4.4	86.2	78.5	69.9	59.3	45.0	25.2	0.0	69	10.43
3/20	-0.5	89.6	82.0	73.5	63.0	48.7	27.8	0.0	79	7.51
4/1	4.2	93.7	86.1	77.9	67.7	53.6	31.7	0.0	91	4.03
4/10	7.7	96.6	89.2	81.2	71.5	57.7	35.2	0.0	100	1.20
4/20	11.2	99.8	92.5	84.9	75.6	62.4	39.8	0.0	110	-1.15
5/1	14.8	102.9	95.9	88.6	80.0	67.8	45.6	0.0	121	-3.11
5/10	17.4	105.2	98.4	91.4	83.4	72.1	50.8	0.0	130	-3.57
5/20	19.8	107.3	100.7	94.1	86.6	76.4	56.5	0.0	140	-3.52
6/1	21.9	109.2	102.8	96.5	89.6	80.5	62.6	0.0	152	-2.38
6/10	23.0	110.1	103.8	97.7	91.1	82.5	65.8	0.0	161	-1.06
6/20	23.4	110.6	104.3	98.2	91.8	83.4	67.5	0.0	171	0.56
7/1	23.2	110.3	104.0	97.9	91.4	82.9	66.6	0.0	182	3.10
7/10	22.4	109.6	103.2	97.0	90.2	81.3	63.9	0.0	191	4.39
7/20	20.8	108.2	101.7	95.2	88.0	78.3	59.3	0.0	201	5.40
8/1	18.2	105.9	99.2	92.3	84.5	73.5	52.6	0.0	213	5.41
8/10	15.8	103.8	96.8	89.7	81.3	69.4	47.4	0.0	222	4.44
8/20	12.7	101.1	93.9	86.4	77.4	64.6	42.0	0.0	232	2.43
9/1	8.6	97.4	90.1	82.1	72.5	58.8	36.3	0.0	244	-0.48
9/10	5.2	94.5	87.0	78.9	68.8	54.7	32.7	0.0	253	-3.57
9/20	1.4	91.2	83.6	75.2	64.8	50.5	29.3	0.0	263	-7.39
10/1	-2.9	87.5	79.9	71.3	60.7	46.4	26.2	0.0	274	-11.27
10/10	-6.3	84.5	76.9	68.2	57.5	43.4	24.1	0.0	283	-14.01
10/20	-10.1	81.3	73.7	65.0	54.4	40.4	22.1	0.0	293	-15.58
11/1	-14.2	77.6	70.2	61.6	51.0	37.5	20.2	0.0	305	-16.43
11/10	-16.9	75.2	67.9	59.3	48.9	35.7	19.1	0.0	314	-16.00
11/20	-19.5	72.9	65.7	57.2	46.9	34.0	18.1	0.0	324	-13.58
12/1	-21.7	71.0	63.9	55.5	45.4	32.7	17.4	0.0	335	-10.25
12/10	-22.8	70.0	62.9	54.6	44.5	32.1	17.0	0.0	344	-6.43
12/20	-23.4	69.5	62.4	54.2	44.1	31.8	16.8	0.0	354	-2.09
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss

$$\text{azimuth} = \text{atan}(\sin(\text{ha}) / (\sin(\text{lat}) * \cos(\text{ha}) - \tan(\text{decl}) * \cos(\text{lat})) )$$

Formulae involving dates use approximations thus these tables may disagree with other sources using other formulae. This and other publications have figures that are well within drafting tolerances.

## LATITUDE 32

## Azimuth

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

32

**A4.2 b**

Date	declination	am							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1	70.2	62.7	54.1	43.9	31.4	16.5	0.0	1	3.24
1/10	-22.1	71.0	63.5	54.9	44.5	31.9	16.8	0.0	10	7.11
1/20	-20.3	72.6	65.0	56.2	45.8	32.9	17.4	0.0	20	10.38
2/1	-17.3	75.2	67.4	58.5	47.9	34.6	18.4	0.0	32	13.22
2/10	-14.6	77.5	69.6	60.7	49.9	36.3	19.4	0.0	41	14.13
2/20	-11.2	80.5	72.5	63.4	52.5	38.6	20.8	0.0	51	13.57
3/1	-7.9	83.3	75.2	66.2	55.2	40.9	22.4	0.0	60	12.43
3/10	-4.4	86.3	78.2	69.1	58.1	43.6	24.2	0.0	69	10.43
3/20	-0.5	89.6	81.5	72.6	61.7	47.0	26.5	0.0	79	7.51
4/1	4.2	93.6	85.6	76.9	66.2	51.6	30.0	0.0	91	4.03
4/10	7.7	96.5	88.6	80.1	69.8	55.4	33.0	0.0	100	1.20
4/20	11.2	99.6	91.9	83.6	73.7	59.9	37.0	0.0	110	-1.15
5/1	14.8	102.7	95.2	87.3	78.0	64.9	42.0	0.0	121	-3.11
5/10	17.4	104.9	97.6	90.1	81.2	68.9	46.5	0.0	130	-3.57
5/20	19.8	107.0	99.9	92.7	84.4	72.9	51.4	0.0	140	-3.52
6/1	21.9	108.9	102.0	95.1	87.3	76.8	56.7	0.0	152	-2.38
6/10	23.0	109.8	102.9	96.2	88.7	78.7	59.5	0.0	161	-1.06
6/20	23.4	110.2	103.4	96.8	89.4	79.7	60.9	0.0	171	0.56
7/1	23.2	110.0	103.2	96.5	89.1	79.1	60.1	0.0	182	3.10
7/10	22.4	109.2	102.4	95.5	87.9	77.6	57.8	0.0	191	4.39
7/20	20.8	107.9	100.9	93.8	85.8	74.7	53.8	0.0	201	5.40
8/1	18.2	105.6	98.4	90.9	82.3	70.2	48.0	0.0	213	5.41
8/10	15.8	103.5	96.1	88.3	79.2	66.3	43.6	0.0	222	4.44
8/20	12.7	100.8	93.2	85.1	75.5	61.8	38.9	0.0	232	2.43
9/1	8.6	97.3	89.4	81.0	70.8	56.5	34.0	0.0	244	-0.48
9/10	5.2	94.5	86.5	77.8	67.2	52.7	30.8	0.0	253	-3.57
9/20	1.4	91.2	83.1	74.3	63.4	48.8	27.8	0.0	263	-7.39
10/1	-2.9	87.6	79.5	70.5	59.5	44.9	25.0	0.0	274	-11.27
10/10	-6.3	84.6	76.5	67.5	56.5	42.1	23.1	0.0	283	-14.01
10/20	-10.1	81.4	73.4	64.4	53.4	39.4	21.3	0.0	293	-15.58
11/1	-14.2	77.9	70.0	61.0	50.2	36.6	19.6	0.0	305	-16.43
11/10	-16.9	75.5	67.7	58.9	48.2	34.9	18.6	0.0	314	-16.00
11/20	-19.5	73.3	65.6	56.8	46.3	33.3	17.7	0.0	324	-13.58
12/1	-21.7	71.4	63.8	55.2	44.8	32.1	17.0	0.0	335	-10.25
12/10	-22.8	70.3	62.9	54.3	44.0	31.5	16.6	0.0	344	-6.43
12/20	-23.4	69.8	62.4	53.9	43.6	31.2	16.4	0.0	354	-2.09
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss

note: There are a couple of different formulae in circulation for azimuth. They tend to agree except for 6 o'clock am and pm. This spreadsheet formula works for all hours.

**LATITUDE 34**
**Azimuth**

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

**A4.2 c**

THIS TABLE IS FOR LATITUDE:

**34**

Date	declination	am								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1	70.6	62.7	53.9	43.4	30.8	16.2	0.0	1	3.24	
1/10	-22.1	71.4	63.5	54.6	44.0	31.4	16.5	0.0	10	7.11	
1/20	-20.3	73.0	65.0	55.9	45.2	32.3	17.0	0.0	20	10.38	
2/1	-17.3	75.5	67.3	58.1	47.2	33.9	18.0	0.0	32	13.22	
2/10	-14.6	77.8	69.5	60.2	49.1	35.5	18.9	0.0	41	14.13	
2/20	-11.2	80.7	72.3	62.9	51.6	37.6	20.2	0.0	51	13.57	
3/1	-7.9	83.5	75.0	65.5	54.2	39.9	21.6	0.0	60	12.43	
3/10	-4.4	86.3	77.8	68.4	57.0	42.4	23.2	0.0	69	10.43	
3/20	-0.5	89.6	81.1	71.7	60.4	45.5	25.3	0.0	79	7.51	
4/1	4.2	93.5	85.1	75.9	64.7	49.8	28.4	0.0	91	4.03	
4/10	7.7	96.4	88.0	79.0	68.1	53.3	31.1	0.0	100	1.20	
4/20	11.2	99.4	91.2	82.4	71.9	57.4	34.6	0.0	110	-1.15	
5/1	14.8	102.4	94.4	86.0	76.0	62.1	38.9	0.0	121	-3.11	
5/10	17.4	104.6	96.8	88.7	79.2	65.8	42.7	0.0	130	-3.57	
5/20	19.8	106.6	99.1	91.3	82.2	69.6	47.0	0.0	140	-3.52	
6/1	21.9	108.5	101.1	93.6	85.0	73.3	51.5	0.0	152	-2.38	
6/10	23.0	109.4	102.1	94.7	86.4	75.1	53.9	0.0	161	-1.06	
6/20	23.4	109.8	102.5	95.3	87.1	76.0	55.1	0.0	171	0.56	
7/1	23.2	109.5	102.3	95.0	86.7	75.5	54.4	0.0	182	3.10	
7/10	22.4	108.8	101.5	94.0	85.6	74.0	52.4	0.0	191	4.39	
7/20	20.8	107.5	100.0	92.4	83.5	71.3	49.0	0.0	201	5.40	
8/1	18.2	105.3	97.6	89.6	80.2	67.1	44.1	0.0	213	5.41	
8/10	15.8	103.2	95.3	87.0	77.2	63.5	40.3	0.0	222	4.44	
8/20	12.7	100.6	92.5	83.9	73.6	59.3	36.2	0.0	232	2.43	
9/1	8.6	97.1	88.8	79.9	69.1	54.3	31.9	0.0	244	-0.48	
9/10	5.2	94.4	85.9	76.8	65.7	50.8	29.2	0.0	253	-3.57	
9/20	1.4	91.2	82.7	73.3	62.1	47.2	26.5	0.0	263	-7.39	
10/1	-2.9	87.6	79.1	69.7	58.3	43.6	24.0	0.0	274	-11.27	
10/10	-6.3	84.7	76.2	66.8	55.4	40.9	22.3	0.0	283	-14.01	
10/20	-10.1	81.6	73.2	63.8	52.5	38.4	20.6	0.0	293	-15.58	
11/1	-14.2	78.2	69.9	60.5	49.4	35.8	19.0	0.0	305	-16.43	
11/10	-16.9	75.8	67.6	58.4	47.5	34.2	18.1	0.0	314	-16.00	
11/20	-19.5	73.6	65.6	56.5	45.7	32.7	17.2	0.0	324	-13.58	
12/1	-21.7	71.7	63.8	54.9	44.3	31.6	16.6	0.0	335	-10.25	
12/10	-22.8	70.8	62.9	54.0	43.5	31.0	16.2	0.0	344	-6.43	
12/20	-23.4	70.3	62.5	53.6	43.2	30.7	16.1	0.0	354	-2.09	
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss	

## LATITUDE 36

## Azimuth

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

36

**A4.2 d**

Date	declination	am								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1	71.0	62.8	53.6	42.9	30.4	15.8	0.0	1	3.24	
1/10	-22.1	71.8	63.6	54.3	43.6	30.8	16.1	0.0	10	7.11	
1/20	-20.3	73.3	65.0	55.6	44.7	31.7	16.6	0.0	20	10.38	
2/1	-17.3	75.8	67.3	57.8	46.6	33.3	17.5	0.0	32	13.22	
2/10	-14.6	78.1	69.4	59.8	48.5	34.8	18.4	0.0	41	14.13	
2/20	-11.2	80.9	72.1	62.3	50.8	36.8	19.6	0.0	51	13.57	
3/1	-7.9	83.6	74.7	64.9	53.3	38.8	20.8	0.0	60	12.43	
3/10	-4.4	86.4	77.5	67.6	55.9	41.2	22.3	0.0	69	10.43	
3/20	-0.5	89.6	80.7	70.9	59.2	44.1	24.3	0.0	79	7.51	
4/1	4.2	93.4	84.6	74.9	63.3	48.1	27.0	0.0	91	4.03	
4/10	7.7	96.2	87.4	77.9	66.5	51.3	29.4	0.0	100	1.20	
4/20	11.2	99.1	90.5	81.3	70.2	55.1	32.5	0.0	110	-1.15	
5/1	14.8	102.1	93.7	84.7	74.1	59.5	36.2	0.0	121	-3.11	
5/10	17.4	104.2	96.0	87.3	77.1	62.9	39.5	0.0	130	-3.57	
5/20	19.8	106.2	98.2	89.8	80.0	66.5	43.1	0.0	140	-3.52	
6/1	21.9	108.1	100.2	92.1	82.8	69.9	47.0	0.0	152	-2.38	
6/10	23.0	108.9	101.2	93.2	84.1	71.6	49.0	0.0	161	-1.06	
6/20	23.4	109.3	101.6	93.8	84.8	72.4	50.0	0.0	171	0.56	
7/1	23.2	109.1	101.4	93.5	84.4	72.0	49.5	0.0	182	3.10	
7/10	22.4	108.4	100.6	92.6	83.3	70.6	47.7	0.0	191	4.39	
7/20	20.8	107.1	99.1	90.9	81.3	68.1	44.9	0.0	201	5.40	
8/1	18.2	104.9	96.7	88.2	78.1	64.1	40.7	0.0	213	5.41	
8/10	15.8	102.9	94.5	85.7	75.2	60.7	37.4	0.0	222	4.44	
8/20	12.7	100.3	91.8	82.7	71.7	56.8	33.9	0.0	232	2.43	
9/1	8.6	97.0	88.2	78.8	67.4	52.3	30.1	0.0	244	-0.48	
9/10	5.2	94.2	85.4	75.8	64.2	49.0	27.7	0.0	253	-3.57	
9/20	1.4	91.1	82.2	72.4	60.8	45.6	25.3	0.0	263	-7.39	
10/1	-2.9	87.7	78.7	68.9	57.2	42.3	23.1	0.0	274	-11.27	
10/10	-6.3	84.9	75.9	66.1	54.4	39.9	21.5	0.0	283	-14.01	
10/20	-10.1	81.8	73.0	63.2	51.7	37.5	20.0	0.0	293	-15.58	
11/1	-14.2	78.4	69.7	60.1	48.7	35.0	18.5	0.0	305	-16.43	
11/10	-16.9	76.2	67.6	58.0	46.9	33.5	17.6	0.0	314	-16.00	
11/20	-19.5	74.0	65.6	56.1	45.2	32.1	16.8	0.0	324	-13.58	
12/1	-21.7	72.2	63.9	54.6	43.8	31.0	16.2	0.0	335	-10.25	
12/10	-22.8	71.2	63.0	53.8	43.1	30.5	15.9	0.0	344	-6.43	
12/20	-23.4	70.7	62.5	53.4	42.7	30.2	15.7	0.0	354	-2.09	
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss	

**LATITUDE 38****Azimuth****A4.2 e**

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

38

Date	declination	am							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1	71.5	62.9	53.4	42.5	29.9	15.5	0.0	1	3.24
1/10	-22.1	72.3	63.6	54.1	43.1	30.4	15.8	0.0	10	7.11
1/20	-20.3	73.7	65.0	55.3	44.2	31.2	16.3	0.0	20	10.38
2/1	-17.3	76.2	67.2	57.4	46.1	32.7	17.1	0.0	32	13.22
2/10	-14.6	78.4	69.3	59.3	47.8	34.1	17.9	0.0	41	14.13
2/20	-11.2	81.1	71.9	61.8	50.1	36.0	19.0	0.0	51	13.57
3/1	-7.9	83.8	74.5	64.3	52.4	37.9	20.2	0.0	60	12.43
3/10	-4.4	86.5	77.2	66.9	55.0	40.1	21.6	0.0	69	10.43
3/20	-0.5	89.6	80.3	70.1	58.0	42.8	23.3	0.0	79	7.51
4/1	4.2	93.3	84.0	73.9	61.9	46.5	25.8	0.0	91	4.03
4/10	7.7	96.0	86.8	76.9	65.0	49.5	27.9	0.0	100	1.20
4/20	11.2	98.9	89.8	80.1	68.5	53.0	30.6	0.0	110	-1.15
5/1	14.8	101.8	92.9	83.5	72.2	57.0	33.8	0.0	121	-3.11
5/10	17.4	103.9	95.2	86.0	75.1	60.2	36.7	0.0	130	-3.57
5/20	19.8	105.8	97.3	88.4	77.9	63.5	39.8	0.0	140	-3.52
6/1	21.9	107.6	99.3	90.6	80.5	66.7	43.0	0.0	152	-2.38
6/10	23.0	108.5	100.2	91.7	81.8	68.3	44.8	0.0	161	-1.06
6/20	23.4	108.9	100.7	92.2	82.5	69.0	45.7	0.0	171	0.56
7/1	23.2	108.6	100.4	92.0	82.1	68.6	45.2	0.0	182	3.10
7/10	22.4	108.0	99.7	91.1	81.1	67.3	43.7	0.0	191	4.39
7/20	20.8	106.7	98.3	89.5	79.1	65.0	41.3	0.0	201	5.40
8/1	18.2	104.5	95.9	86.8	76.0	61.3	37.7	0.0	213	5.41
8/10	15.8	102.6	93.8	84.4	73.3	58.2	34.8	0.0	222	4.44
8/20	12.7	100.1	91.1	81.5	70.0	54.6	31.8	0.0	232	2.43
9/1	8.6	96.8	87.6	77.7	65.9	50.3	28.5	0.0	244	-0.48
9/10	5.2	94.1	84.9	74.8	62.8	47.3	26.4	0.0	253	-3.57
9/20	1.4	91.1	81.8	71.6	59.5	44.2	24.2	0.0	263	-7.39
10/1	-2.9	87.7	78.4	68.1	56.1	41.1	22.2	0.0	274	-11.27
10/10	-6.3	85.0	75.7	65.5	53.5	38.9	20.8	0.0	283	-14.01
10/20	-10.1	82.0	72.8	62.7	50.9	36.6	19.4	0.0	293	-15.58
11/1	-14.2	78.7	69.6	59.7	48.1	34.3	18.1	0.0	305	-16.43
11/10	-16.9	76.5	67.5	57.7	46.3	32.9	17.2	0.0	314	-16.00
11/20	-19.5	74.4	65.6	55.9	44.7	31.6	16.5	0.0	324	-13.58
12/1	-21.7	72.6	63.9	54.3	43.4	30.6	15.9	0.0	335	-10.25
12/10	-22.8	71.6	63.0	53.5	42.7	30.0	15.6	0.0	344	-6.43
12/20	-23.4	71.2	62.6	53.2	42.3	29.8	15.5	0.0	354	-2.09
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss

## LATITUDE 40

## Azimuth

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

40

**A4.2 f**

Date	declination	am								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1	71.9	63.0	53.2	42.2	29.5	15.3	0.0	1	3.24	
1/10	-22.1	72.7	63.7	53.9	42.7	30.0	15.5	0.0	10	7.11	
1/20	-20.3	74.2	65.0	55.1	43.8	30.8	16.0	0.0	20	10.38	
2/1	-17.3	76.6	67.2	57.1	45.6	32.1	16.7	0.0	32	13.22	
2/10	-14.6	78.7	69.2	59.0	47.2	33.5	17.5	0.0	41	14.13	
2/20	-11.2	81.4	71.8	61.4	49.4	35.2	18.5	0.0	51	13.57	
3/1	-7.9	83.9	74.3	63.7	51.6	37.0	19.6	0.0	60	12.43	
3/10	-4.4	86.6	76.9	66.3	54.0	39.1	20.8	0.0	69	10.43	
3/20	-0.5	89.6	79.9	69.3	56.9	41.6	22.4	0.0	79	7.51	
4/1	4.2	93.3	83.5	73.0	60.6	45.0	24.6	0.0	91	4.03	
4/10	7.7	95.9	86.2	75.8	63.6	47.8	26.6	0.0	100	1.20	
4/20	11.2	98.7	89.2	78.9	66.9	51.0	28.9	0.0	110	-1.15	
5/1	14.8	101.5	92.2	82.2	70.4	54.7	31.8	0.0	121	-3.11	
5/10	17.4	103.5	94.4	84.6	73.1	57.7	34.2	0.0	130	-3.57	
5/20	19.8	105.4	96.5	87.0	75.8	60.7	36.9	0.0	140	-3.52	
6/1	21.9	107.2	98.4	89.2	78.3	63.6	39.7	0.0	152	-2.38	
6/10	23.0	108.0	99.3	90.2	79.6	65.1	41.1	0.0	161	-1.06	
6/20	23.4	108.4	99.7	90.7	80.2	65.8	41.9	0.0	171	0.56	
7/1	23.2	108.2	99.5	90.4	79.9	65.4	41.5	0.0	182	3.10	
7/10	22.4	107.5	98.7	89.6	78.8	64.2	40.2	0.0	191	4.39	
7/20	20.8	106.2	97.4	88.0	77.0	62.0	38.1	0.0	201	5.40	
8/1	18.2	104.2	95.1	85.4	74.0	58.7	35.1	0.0	213	5.41	
8/10	15.8	102.2	93.0	83.1	71.4	55.8	32.6	0.0	222	4.44	
8/20	12.7	99.8	90.4	80.3	68.3	52.5	30.0	0.0	232	2.43	
9/1	8.6	96.6	87.0	76.6	64.4	48.6	27.1	0.0	244	-0.48	
9/10	5.2	94.0	84.3	73.8	61.5	45.8	25.2	0.0	253	-3.57	
9/20	1.4	91.1	81.3	70.7	58.4	42.9	23.3	0.0	263	-7.39	
10/1	-2.9	87.8	78.0	67.4	55.1	40.0	21.4	0.0	274	-11.27	
10/10	-6.3	85.1	75.4	64.9	52.7	37.9	20.1	0.0	283	-14.01	
10/20	-10.1	82.3	72.6	62.2	50.1	35.8	18.9	0.0	293	-15.58	
11/1	-14.2	79.0	69.6	59.3	47.5	33.7	17.6	0.0	305	-16.43	
11/10	-16.9	76.9	67.5	57.4	45.8	32.3	16.9	0.0	314	-16.00	
11/20	-19.5	74.8	65.6	55.6	44.2	31.1	16.2	0.0	324	-13.58	
12/1	-21.7	73.1	64.0	54.1	43.0	30.1	15.6	0.0	335	-10.25	
12/10	-22.8	72.1	63.1	53.4	42.3	29.6	15.3	0.0	344	-6.43	
12/20	-23.4	71.7	62.7	53.0	42.0	29.4	15.2	0.0	354	-2.09	
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss	

**LATITUDE 45**
**Azimuth**
**A4.2 g**

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

**45**

Date	declination	am								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1	73.2	63.4	52.9	41.4	28.7	14.7	0.0	1	3.24	
1/10	-22.1	74.0	64.1	53.5	41.9	29.1	14.9	0.0	10	7.11	
1/20	-20.3	75.3	65.3	54.6	42.9	29.8	15.3	0.0	20	10.38	
2/1	-17.3	77.6	67.3	56.5	44.5	31.0	16.0	0.0	32	13.22	
2/10	-14.6	79.6	69.2	58.2	45.9	32.1	16.6	0.0	41	14.13	
2/20	-11.2	82.0	71.5	60.3	47.8	33.6	17.5	0.0	51	13.57	
3/1	-7.9	84.4	73.8	62.5	49.8	35.1	18.3	0.0	60	12.43	
3/10	-4.4	86.9	76.2	64.8	51.9	36.9	19.3	0.0	69	10.43	
3/20	-0.5	89.7	78.9	67.5	54.4	39.0	20.6	0.0	79	7.51	
4/1	4.2	93.0	82.3	70.8	57.7	41.8	22.3	0.0	91	4.03	
4/10	7.7	95.4	84.8	73.4	60.2	44.0	23.8	0.0	100	1.20	
4/20	11.2	98.0	87.5	76.2	63.1	46.7	25.5	0.0	110	-1.15	
5/1	14.8	100.6	90.2	79.1	66.1	49.6	27.6	0.0	121	-3.11	
5/10	17.4	102.5	92.3	81.3	68.5	52.0	29.3	0.0	130	-3.57	
5/20	19.8	104.3	94.2	83.5	70.9	54.4	31.1	0.0	140	-3.52	
6/1	21.9	105.9	96.0	85.5	73.1	56.8	33.0	0.0	152	-2.38	
6/10	23.0	106.7	96.9	86.4	74.2	58.0	34.0	0.0	161	-1.06	
6/20	23.4	107.0	97.3	86.9	74.7	58.6	34.5	0.0	171	0.56	
7/1	23.2	106.8	97.1	86.6	74.4	58.2	34.2	0.0	182	3.10	
7/10	22.4	106.2	96.4	85.9	73.5	57.2	33.4	0.0	191	4.39	
7/20	20.8	105.1	95.1	84.4	71.9	55.5	32.0	0.0	201	5.40	
8/1	18.2	103.1	93.0	82.1	69.3	52.8	29.9	0.0	213	5.41	
8/10	15.8	101.3	91.0	79.9	67.0	50.5	28.2	0.0	222	4.44	
8/20	12.7	99.1	88.6	77.4	64.3	47.8	26.3	0.0	232	2.43	
9/1	8.6	96.1	85.5	74.1	60.9	44.7	24.2	0.0	244	-0.48	
9/10	5.2	93.7	83.0	71.6	58.4	42.4	22.7	0.0	253	-3.57	
9/20	1.4	91.0	80.3	68.8	55.7	40.0	21.2	0.0	263	-7.39	
10/1	-2.9	88.0	77.3	65.8	52.9	37.7	19.8	0.0	274	-11.27	
10/10	-6.3	85.5	74.9	63.5	50.7	35.9	18.8	0.0	283	-14.01	
10/20	-10.1	82.9	72.3	61.1	48.5	34.1	17.8	0.0	293	-15.58	
11/1	-14.2	79.9	69.5	58.4	46.2	32.3	16.7	0.0	305	-16.43	
11/10	-16.9	77.8	67.6	56.7	44.7	31.1	16.1	0.0	314	-16.00	
11/20	-19.5	75.9	65.8	55.1	43.3	30.1	15.5	0.0	324	-13.58	
12/1	-21.7	74.3	64.3	53.8	42.1	29.2	15.0	0.0	335	-10.25	
12/10	-22.8	73.4	63.5	53.1	41.6	28.8	14.8	0.0	344	-6.43	
12/20	-23.4	73.0	63.1	52.7	41.3	28.6	14.7	0.0	354	-2.09	
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss	

## LATITUDE 50

## Azimuth

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

50

A4.2 h

Date	declination	am								Julian	Approx EOT
		600	700	800	900	1000	1100	1200			
1/1	-23.1	74.7	64.0	52.8	40.9	28.1	14.3	0.0	1	3.24	
1/10	-22.1	75.4	64.6	53.4	41.4	28.4	14.5	0.0	10	7.11	
1/20	-20.3	76.6	65.7	54.4	42.2	29.0	14.8	0.0	20	10.38	
2/1	-17.3	78.7	67.6	56.0	43.6	30.1	15.4	0.0	32	13.22	
2/10	-14.6	80.5	69.3	57.6	44.9	31.0	15.9	0.0	41	14.13	
2/20	-11.2	82.7	71.4	59.5	46.6	32.3	16.6	0.0	51	13.57	
3/1	-7.9	84.9	73.4	61.4	48.3	33.6	17.3	0.0	60	12.43	
3/10	-4.4	87.2	75.6	63.5	50.1	35.0	18.2	0.0	69	10.43	
3/20	-0.5	89.7	78.1	65.9	52.3	36.8	19.2	0.0	79	7.51	
4/1	4.2	92.7	81.1	68.8	55.1	39.1	20.5	0.0	91	4.03	
4/10	7.7	94.9	83.4	71.1	57.2	40.9	21.6	0.0	100	1.20	
4/20	11.2	97.3	85.8	73.6	59.7	43.0	22.9	0.0	110	-1.15	
5/1	14.8	99.7	88.3	76.2	62.3	45.4	24.4	0.0	121	-3.11	
5/10	17.4	101.4	90.2	78.2	64.3	47.3	25.7	0.0	130	-3.57	
5/20	19.8	103.0	92.0	80.1	66.3	49.2	27.0	0.0	140	-3.52	
6/1	21.9	104.5	93.6	81.9	68.2	51.0	28.3	0.0	152	-2.38	
6/10	23.0	105.2	94.4	82.7	69.1	52.0	29.0	0.0	161	-1.06	
6/20	23.4	105.6	94.8	83.1	69.6	52.4	29.3	0.0	171	0.56	
7/1	23.2	105.4	94.6	82.9	69.4	52.2	29.1	0.0	182	3.10	
7/10	22.4	104.8	93.9	82.2	68.6	51.4	28.6	0.0	191	4.39	
7/20	20.8	103.7	92.7	80.9	67.2	50.0	27.6	0.0	201	5.40	
8/1	18.2	101.9	90.8	78.8	65.0	47.9	26.1	0.0	213	5.41	
8/10	15.8	100.3	89.0	76.9	63.0	46.1	24.9	0.0	222	4.44	
8/20	12.7	98.2	86.8	74.6	60.7	44.0	23.5	0.0	232	2.43	
9/1	8.6	95.5	84.0	71.7	57.8	41.4	21.9	0.0	244	-0.48	
9/10	5.2	93.4	81.8	69.5	55.7	39.6	20.8	0.0	253	-3.57	
9/20	1.4	90.9	79.3	67.0	53.4	37.7	19.7	0.0	263	-7.39	
10/1	-2.9	88.2	76.6	64.4	50.9	35.7	18.5	0.0	274	-11.27	
10/10	-6.3	85.9	74.4	62.3	49.1	34.2	17.7	0.0	283	-14.01	
10/20	-10.1	83.5	72.1	60.1	47.2	32.7	16.9	0.0	293	-15.58	
11/1	-14.2	80.8	69.5	57.8	45.1	31.2	16.0	0.0	305	-16.43	
11/10	-16.9	78.9	67.8	56.2	43.8	30.2	15.5	0.0	314	-16.00	
11/20	-19.5	77.2	66.2	54.8	42.6	29.3	15.0	0.0	324	-13.58	
12/1	-21.7	75.7	64.8	53.6	41.6	28.5	14.6	0.0	335	-10.25	
12/10	-22.8	74.9	64.1	53.0	41.0	28.2	14.4	0.0	344	-6.43	
12/20	-23.4	74.5	63.7	52.6	40.8	28.0	14.3	0.0	354	-2.09	
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss	

**LATITUDE 55****Azimuth****A4.2 i**

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

THIS TABLE IS FOR LATITUDE:

55

Date	declination	am							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1	76.3	64.7	53.0	40.7	27.7	14.0	0.0	1	3.24
1/10	-22.1	76.9	65.3	53.4	41.1	28.0	14.2	0.0	10	7.11
1/20	-20.3	78.0	66.3	54.3	41.8	28.5	14.5	0.0	20	10.38
2/1	-17.3	79.8	68.0	55.8	43.0	29.4	14.9	0.0	32	13.22
2/10	-14.6	81.5	69.5	57.2	44.1	30.2	15.4	0.0	41	14.13
2/20	-11.2	83.5	71.4	58.9	45.6	31.3	16.0	0.0	51	13.57
3/1	-7.9	85.5	73.2	60.6	47.0	32.4	16.6	0.0	60	12.43
3/10	-4.4	87.5	75.1	62.3	48.6	33.6	17.2	0.0	69	10.43
3/20	-0.5	89.7	77.4	64.4	50.5	35.0	18.0	0.0	79	7.51
4/1	4.2	92.4	80.0	67.0	52.8	36.9	19.1	0.0	91	4.03
4/10	7.7	94.4	82.0	69.0	54.6	38.3	19.9	0.0	100	1.20
4/20	11.2	96.5	84.2	71.2	56.7	40.0	20.9	0.0	110	-1.15
5/1	14.8	98.6	86.4	73.4	58.9	41.9	22.0	0.0	121	-3.11
5/10	17.4	100.2	88.1	75.1	60.5	43.4	22.9	0.0	130	-3.57
5/20	19.8	101.7	89.7	76.8	62.2	44.8	23.9	0.0	140	-3.52
6/1	21.9	103.0	91.1	78.4	63.8	46.3	24.8	0.0	152	-2.38
6/10	23.0	103.7	91.8	79.1	64.6	47.0	25.3	0.0	161	-1.06
6/20	23.4	104.0	92.2	79.5	64.9	47.3	25.5	0.0	171	0.56
7/1	23.2	103.8	92.0	79.3	64.7	47.1	25.4	0.0	182	3.10
7/10	22.4	103.3	91.4	78.7	64.1	46.6	25.0	0.0	191	4.39
7/20	20.8	102.3	90.4	77.5	62.9	45.5	24.3	0.0	201	5.40
8/1	18.2	100.7	88.6	75.7	61.1	43.8	23.3	0.0	213	5.41
8/10	15.8	99.2	87.1	74.1	59.5	42.4	22.4	0.0	222	4.44
8/20	12.7	97.4	85.1	72.1	57.5	40.8	21.4	0.0	232	2.43
9/1	8.6	94.9	82.6	69.5	55.1	38.8	20.2	0.0	244	-0.48
9/10	5.2	93.0	80.6	67.6	53.3	37.3	19.3	0.0	253	-3.57
9/20	1.4	90.8	78.4	65.5	51.4	35.7	18.4	0.0	263	-7.39
10/1	-2.9	88.4	76.0	63.2	49.3	34.1	17.5	0.0	274	-11.27
10/10	-6.3	86.4	74.1	61.3	47.7	32.9	16.8	0.0	283	-14.01
10/20	-10.1	84.2	72.0	59.4	46.1	31.7	16.2	0.0	293	-15.58
11/1	-14.2	81.7	69.7	57.4	44.3	30.3	15.5	0.0	305	-16.43
11/10	-16.9	80.1	68.2	56.0	43.2	29.5	15.0	0.0	314	-16.00
11/20	-19.5	78.5	66.7	54.7	42.1	28.7	14.6	0.0	324	-13.58
12/1	-21.7	77.1	65.5	53.6	41.2	28.1	14.2	0.0	335	-10.25
12/10	-22.8	76.4	64.8	53.1	40.7	27.7	14.1	0.0	344	-6.43
12/20	-23.4	76.1	64.5	52.8	40.5	27.6	14.0	0.0	354	-2.09
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss

**LATITUDE 60**
**Azimuth**

SOLAR DECLINATION FOR EVERY TEN DAYS AND HOURLY SOLAR AZIMUTH

**A4.2 j**

THIS TABLE IS FOR LATITUDE:

**60**

Date	declination	am							Julian	Approx EOT
		600	700	800	900	1000	1100	1200		
1/1	-23.1	78.0	65.7	53.3	40.6	27.4	13.9	0.0	1	3.24
1/10	-22.1	78.5	66.2	53.7	40.9	27.7	14.0	0.0	10	7.11
1/20	-20.3	79.5	67.0	54.5	41.6	28.1	14.2	0.0	20	10.38
2/1	-17.3	81.1	68.5	55.8	42.6	28.9	14.6	0.0	32	13.22
2/10	-14.6	82.6	69.8	57.0	43.6	29.6	15.0	0.0	41	14.13
2/20	-11.2	84.3	71.5	58.4	44.8	30.5	15.5	0.0	51	13.57
3/1	-7.9	86.0	73.1	59.9	46.1	31.4	15.9	0.0	60	12.43
3/10	-4.4	87.8	74.8	61.4	47.4	32.4	16.5	0.0	69	10.43
3/20	-0.5	89.8	76.7	63.2	48.9	33.5	17.1	0.0	79	7.51
4/1	4.2	92.1	79.0	65.4	50.9	35.0	17.9	0.0	91	4.03
4/10	7.7	93.8	80.8	67.1	52.4	36.2	18.6	0.0	100	1.20
4/20	11.2	95.7	82.6	68.9	54.0	37.5	19.3	0.0	110	-1.15
5/1	14.8	97.5	84.6	70.9	55.8	39.0	20.2	0.0	121	-3.11
5/10	17.4	98.9	86.0	72.3	57.2	40.1	20.8	0.0	130	-3.57
5/20	19.8	100.2	87.4	73.7	58.6	41.3	21.5	0.0	140	-3.52
6/1	21.9	101.4	88.7	75.0	59.8	42.4	22.2	0.0	152	-2.38
6/10	23.0	102.0	89.3	75.7	60.5	42.9	22.5	0.0	161	-1.06
6/20	23.4	102.2	89.6	76.0	60.8	43.2	22.7	0.0	171	0.56
7/1	23.2	102.1	89.4	75.8	60.6	43.0	22.6	0.0	182	3.10
7/10	22.4	101.6	88.9	75.3	60.1	42.6	22.3	0.0	191	4.39
7/20	20.8	100.8	88.0	74.3	59.2	41.8	21.8	0.0	201	5.40
8/1	18.2	99.3	86.5	72.8	57.7	40.5	21.1	0.0	213	5.41
8/10	15.8	98.1	85.1	71.4	56.3	39.4	20.4	0.0	222	4.44
8/20	12.7	96.4	83.4	69.7	54.8	38.1	19.7	0.0	232	2.43
9/1	8.6	94.3	81.2	67.6	52.8	36.5	18.8	0.0	244	-0.48
9/10	5.2	92.6	79.5	65.9	51.3	35.4	18.1	0.0	253	-3.57
9/20	1.4	90.7	77.6	64.1	49.7	34.1	17.4	0.0	263	-7.39
10/1	-2.9	88.6	75.5	62.1	48.0	32.8	16.7	0.0	274	-11.27
10/10	-6.3	86.8	73.9	60.6	46.6	31.8	16.2	0.0	283	-14.01
10/20	-10.1	84.9	72.1	58.9	45.2	30.8	15.6	0.0	293	-15.58
11/1	-14.2	82.8	70.1	57.1	43.7	29.7	15.0	0.0	305	-16.43
11/10	-16.9	81.3	68.7	55.9	42.8	29.0	14.7	0.0	314	-16.00
11/20	-19.5	79.9	67.4	54.8	41.8	28.3	14.3	0.0	324	-13.58
12/1	-21.7	78.8	66.3	53.9	41.1	27.8	14.0	0.0	335	-10.25
12/10	-22.8	78.1	65.8	53.4	40.7	27.5	13.9	0.0	344	-6.43
12/20	-23.4	77.8	65.5	53.1	40.5	27.4	13.8	0.0	354	-2.09
	decl	6pm	5pm	4pm	3pm	2pm	1pm	noon		mm.ss

## Analemmatic hour points and gnomon positions

A4.3

### AVERAGE MONTHLY DECLINATION (mid month) AND ANALEMMATIC VALUES

Major ellipse axis (east-west diameter)	2.00	Semi major ellipse axis (east-west radius) M =	1.00
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Avg		Latitude									
Date	decl	30	32	34	36	38	40	45	50	55	60
Jan	-20.3	-0.32	-0.31	-0.31	-0.30	-0.29	-0.28	-0.26	-0.24	-0.21	-0.18
Feb	-12.5	-0.19	-0.19	-0.18	-0.18	-0.17	-0.17	-0.16	-0.14	-0.13	-0.11
Mar	-1.6	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01
Apr	9.6	0.15	0.14	0.14	0.14	0.13	0.13	0.12	0.11	0.10	0.08
May	18.4	0.29	0.28	0.28	0.27	0.26	0.25	0.24	0.21	0.19	0.17
Jun	22.5	0.36	0.35	0.34	0.34	0.33	0.32	0.29	0.27	0.24	0.21
Jly	20.9	0.33	0.32	0.32	0.31	0.30	0.29	0.27	0.25	0.22	0.19
Aug	14.1	0.22	0.21	0.21	0.20	0.20	0.19	0.18	0.16	0.14	0.13
Sep	3.8	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.04	0.04	0.03
Oct	-7.4	-0.11	-0.11	-0.11	-0.11	-0.10	-0.10	-0.09	-0.08	-0.07	-0.06
Nov	-17	-0.26	-0.26	-0.25	-0.25	-0.24	-0.23	-0.22	-0.20	-0.18	-0.15
Dec	-22.1	-0.35	-0.34	-0.34	-0.33	-0.32	-0.31	-0.29	-0.26	-0.23	-0.20
semi minor axis "m":-		0.50	0.53	0.56	0.59	0.62	0.64	0.71	0.77	0.82	0.87

$m = M * \sin(\text{lat})$

"m" is the north south radius

		Angle from center to the ellipse for the hour point: $x = \arctan(\tan(\text{hour from noon} * 15) / \sin(\text{latitude}))$											
		ANGLE		Latitude									
HOUR	HOUR	am	pm	30	32	34	36	38	40	45	50	55	60
12	12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
11	13	28.2	26.8	25.6	24.5	23.5	22.6	20.8	19.3	18.1	17.2		
10	14	49.1	47.5	45.9	44.5	43.2	41.9	39.2	37.0	35.2	33.7		
9	15	63.4	62.1	60.8	59.6	58.4	57.3	54.7	52.5	50.7	49.1		
8	16	73.9	73.0	72.1	71.3	70.4	69.6	67.8	66.1	64.7	63.4		
7	17	82.4	81.9	81.5	81.0	80.6	80.2	79.3	78.4	77.6	76.9		
6	18	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0		
5	19	-82.4	-81.9	-81.5	-81.0	-80.6	-80.2	-79.3	-78.4	-77.6	-76.9		
4	20	-73.9	-73.0	-72.1	-71.3	-70.4	-69.6	-67.8	-66.1	-64.7	-63.4		

am pm negative angles are simply below the east west line.

The average declination for a month is an approximation.

**Formulae involving dates** use approximations thus these tables may disagree with other sources using other formulae. This and other publications have figures that are well within drafting tolerances.

## Analemmatic hour point X and Y coordinates: table

A4.3a

ANALEMMATIC DIAL by hour and latitude with X and Y values

m	latitude	Hour from noon		0		1		2		3		4		5		6	
		hla =	0	X =	Y =	X =	Y =	X =	Y =	X =	Y =	X =	Y =	X =	Y =	X =	Y =
0.174	10	0.00	0.17	0.26	0.17	0.50	0.15	0.71	0.12	0.87	0.09	0.97	0.04	1.00	0.00		
0.342	20	0.00	0.34	0.26	0.33	0.50	0.30	0.71	0.24	0.87	0.17	0.97	0.09	1.00	0.00		
0.500	30	0.00	0.50	0.26	0.48	0.50	0.43	0.71	0.35	0.87	0.25	0.97	0.13	1.00	0.00		
0.530	32	0.00	0.53	0.26	0.51	0.50	0.46	0.71	0.37	0.87	0.26	0.97	0.14	1.00	0.00		
0.559	34	0.00	0.56	0.26	0.54	0.50	0.48	0.71	0.40	0.87	0.28	0.97	0.14	1.00	0.00		
0.588	36	0.00	0.59	0.26	0.57	0.50	0.51	0.71	0.42	0.87	0.29	0.97	0.15	1.00	0.00		
0.616	38	0.00	0.62	0.26	0.59	0.50	0.53	0.71	0.44	0.87	0.31	0.97	0.16	1.00	0.00		
0.643	40	0.00	0.64	0.26	0.62	0.50	0.56	0.71	0.45	0.87	0.32	0.97	0.17	1.00	0.00		
0.669	42	0.00	0.67	0.26	0.65	0.50	0.58	0.71	0.47	0.87	0.33	0.97	0.17	1.00	0.00		
0.695	44	0.00	0.69	0.26	0.67	0.50	0.60	0.71	0.49	0.87	0.35	0.97	0.18	1.00	0.00		
0.719	46	0.00	0.72	0.26	0.69	0.50	0.62	0.71	0.51	0.87	0.36	0.97	0.19	1.00	0.00		
0.743	48	0.00	0.74	0.26	0.72	0.50	0.64	0.71	0.53	0.87	0.37	0.97	0.19	1.00	0.00		
0.766	50	0.00	0.77	0.26	0.74	0.50	0.66	0.71	0.54	0.87	0.38	0.97	0.20	1.00	0.00		
0.788	52	0.00	0.79	0.26	0.76	0.50	0.68	0.71	0.56	0.87	0.39	0.97	0.20	1.00	0.00		
0.809	54	0.00	0.81	0.26	0.78	0.50	0.70	0.71	0.57	0.87	0.40	0.97	0.21	1.00	0.00		
0.829	56	0.00	0.83	0.26	0.80	0.50	0.72	0.71	0.59	0.87	0.41	0.97	0.21	1.00	0.00		
0.848	58	0.00	0.85	0.26	0.82	0.50	0.73	0.71	0.60	0.87	0.42	0.97	0.22	1.00	0.00		
0.866	60	0.00	0.87	0.26	0.84	0.50	0.75	0.71	0.61	0.87	0.43	0.97	0.22	1.00	0.00		
0.883	62	0.00	0.88	0.26	0.85	0.50	0.76	0.71	0.62	0.87	0.44	0.97	0.23	1.00	0.00		
0.899	64	0.00	0.90	0.26	0.87	0.50	0.78	0.71	0.64	0.87	0.45	0.97	0.23	1.00	0.00		
0.914	66	0.00	0.91	0.26	0.88	0.50	0.79	0.71	0.65	0.87	0.46	0.97	0.24	1.00	0.00		
0.927	68	0.00	0.93	0.26	0.90	0.50	0.80	0.71	0.66	0.87	0.46	0.97	0.24	1.00	0.00		
0.940	70	0.00	0.94	0.26	0.91	0.50	0.81	0.71	0.66	0.87	0.47	0.97	0.24	1.00	0.00		
0.951	72	0.00	0.95	0.26	0.92	0.50	0.82	0.71	0.67	0.87	0.48	0.97	0.25	1.00	0.00		
0.961	74	0.00	0.96	0.26	0.93	0.50	0.83	0.71	0.68	0.87	0.48	0.97	0.25	1.00	0.00		
0.970	76	0.00	0.97	0.26	0.94	0.50	0.84	0.71	0.69	0.87	0.49	0.97	0.25	1.00	0.00		
0.978	78	0.00	0.98	0.26	0.94	0.50	0.85	0.71	0.69	0.87	0.49	0.97	0.25	1.00	0.00		
0.985	80	0.00	0.98	0.26	0.95	0.50	0.85	0.71	0.70	0.87	0.49	0.97	0.25	1.00	0.00		
0.990	82	0.00	0.99	0.26	0.96	0.50	0.86	0.71	0.70	0.87	0.50	0.97	0.26	1.00	0.00		
0.995	84	0.00	0.99	0.26	0.96	0.50	0.86	0.71	0.70	0.87	0.50	0.97	0.26	1.00	0.00		
0.998	86	0.00	1.00	0.26	0.96	0.50	0.86	0.71	0.71	0.87	0.50	0.97	0.26	1.00	0.00		
0.999	88	0.00	1.00	0.26	0.97	0.50	0.87	0.71	0.71	0.87	0.50	0.97	0.26	1.00	0.00		
1.000	90	0.00	1.00	0.26	0.97	0.50	0.87	0.71	0.71	0.87	0.50	0.97	0.26	1.00	0.00		

X is the distance from the N/S lii  $X = \sin(\text{local hour angle})$

Y is the distance from the e/w ex  $Y = \cos(\text{local hour angle}) * \sin(\text{lat})$

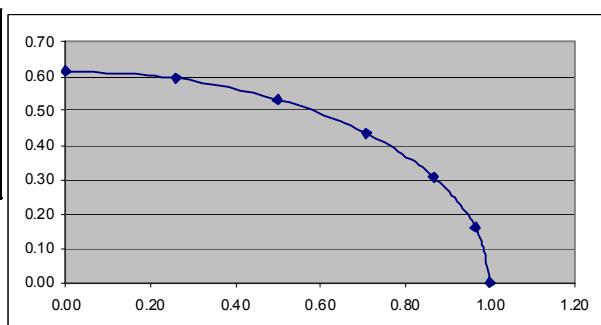
ASSUMES that M=1 being the east to center, or center to west radius

m is the north to equinox or equinox to south radius, (assumes M=1, the E or W radius)

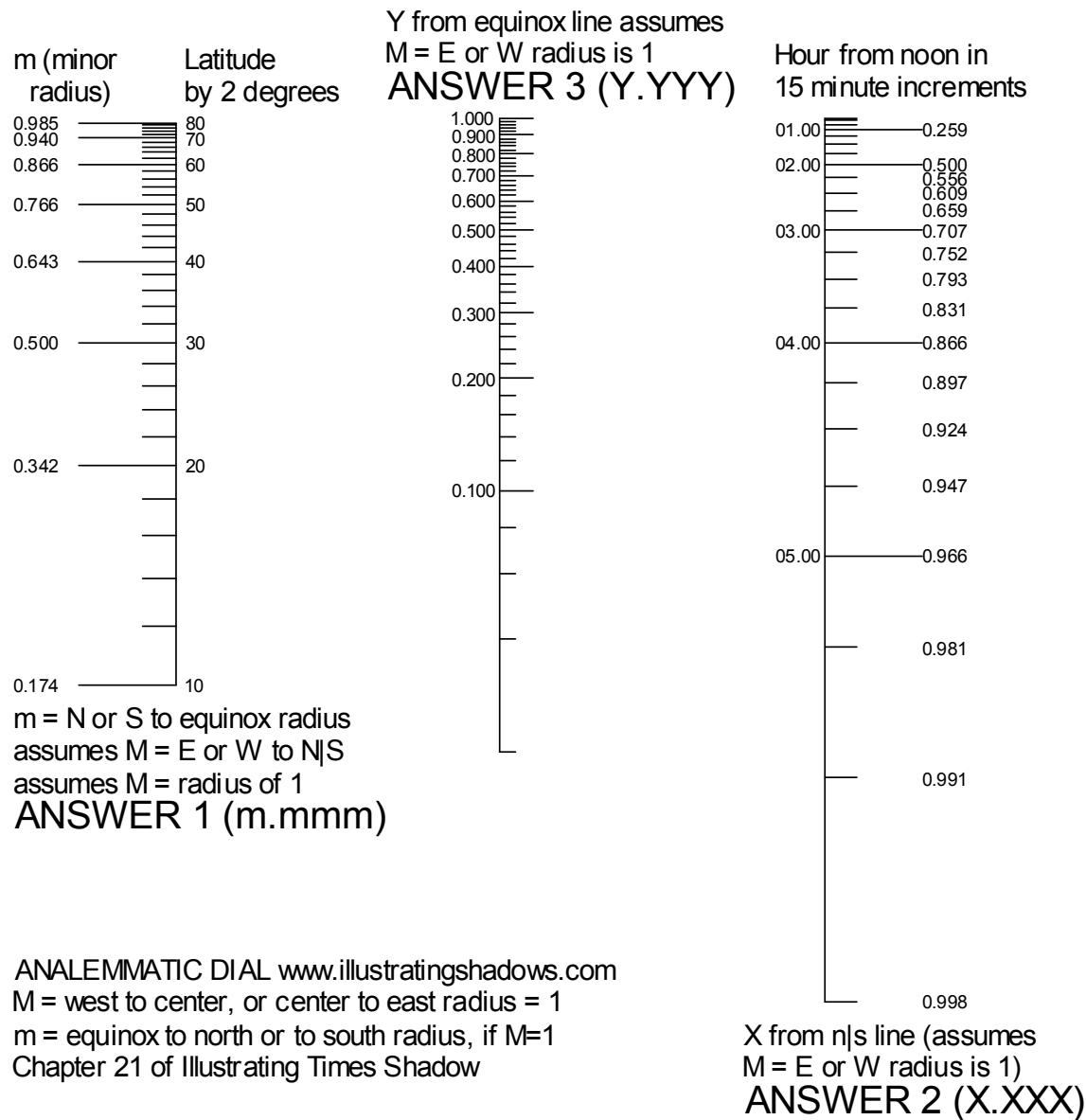
Chapter 21 of Illustrating Time's Shadow

SAMPLE GRAPH

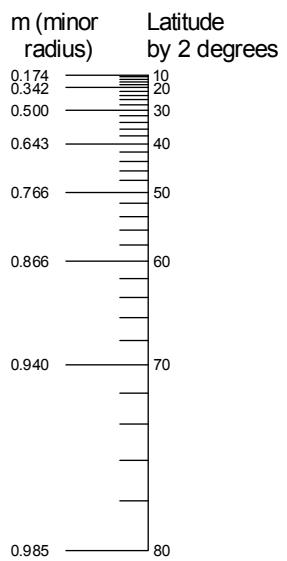
X	Y	lat 38
0.00	0.62	
0.26	0.59	
0.50	0.53	
0.71	0.44	
0.87	0.31	
0.97	0.16	
1.00	0.00	



## Analemmatic hour point X and Y coordinates, and "m": nomogram



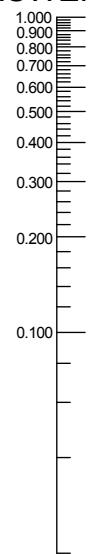
## Analemmatic calendar N|S coordinate, and "m": nomogram



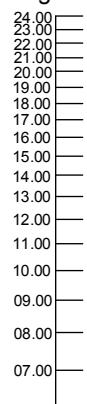
**ANSWER 1 (m.mmm)**

Y from equinox line assumes  
M = E or W radius is 1

**ANSWER 4 (Y.YYY)**



Declination in 1  
degree increments



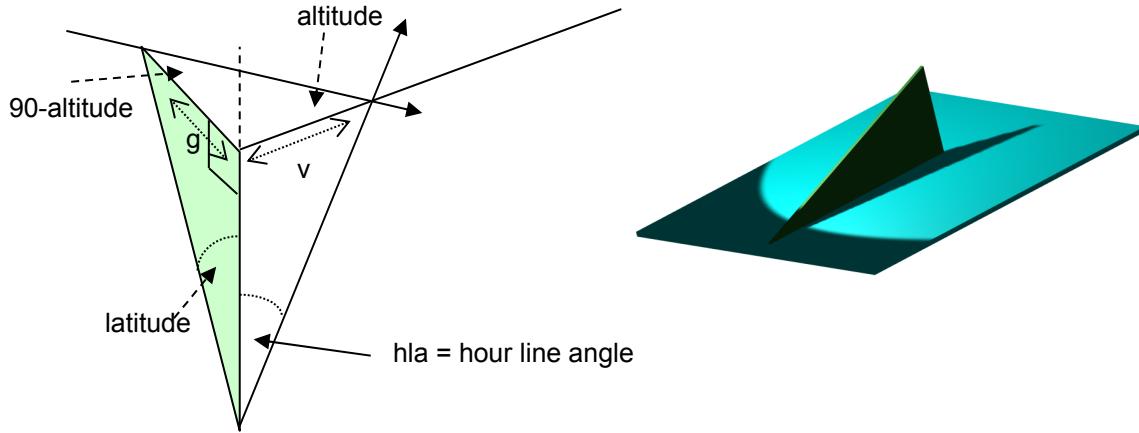
	EOT mm.m	DEC dd.d
Jan	+3.2	-22.8
	+7.4	-21.6
	+10.8	-19.7
Feb	+13.1	-17.3
	+14.1	-14.4
Mar	+13.9	-11.0
	+12.5	-7.4
	+10.2	-3.6
Apr	+7.3	0.4
	+4.1	4.3
	+1.1	8.1
May	-1.4	11.7
	-3.1	15.0
	-3.8	17.8
Jun	-3.6	20.1
	-2.5	21.9
	-0.7	23.0
Jul	+1.4	23.4
	+3.6	23.2
	+5.3	22.4
Aug	+6.4	20.9
	+6.6	18.8
	+5.8	16.2
Sep	+3.9	13.1
	+1.1	9.6
	-2.3	5.9
Oct	-6.1	2.0
	-9.7	-2.0
	-12.9	-5.9
Nov	-15.2	-9.6
	-16.3	-13.1
	-16.1	-16.2
Dec	-14.6	-18.8
	-11.8	-20.9
	-8.1	-22.4
	-3.7	-23.2

ANALEMMATIC DIAL [www.illustratingshadows.com](http://www.illustratingshadows.com)  
Relative distance on N:S line for gnomon base  
M = west to center, or center to east radius = 1  
Chapter 21 of Illustrating Times Shadow



## Declination/Calendar lines/curves for a dial plate using the sun's altitude (with certain standard declinations)

How far from the nodus base (nodus dropped perpendicular to dial plate) on a horizontal or flat dial is the declination point? Chapter 23 discussed declination or calendar curves.



$\tan(\text{alt}) = \text{gnomon linear height} / \text{distance from gnomon base} = g / v$   
 thus  $v = g / \tan(\text{alt})$  or  $v = g * \cot(\text{alt})$   
 or in spreadsheet (Excel) terms which has the formula for altitude and  
 where the declination is a fixed number such as 23.5, 0, or -23.5, or some other value

```
=gnomonlinearheight /  

(TAN(ASIN(SIN(RADIANS(deg)) * SIN(RADIANS(lat))+COS(RADIANS(deg)) *  

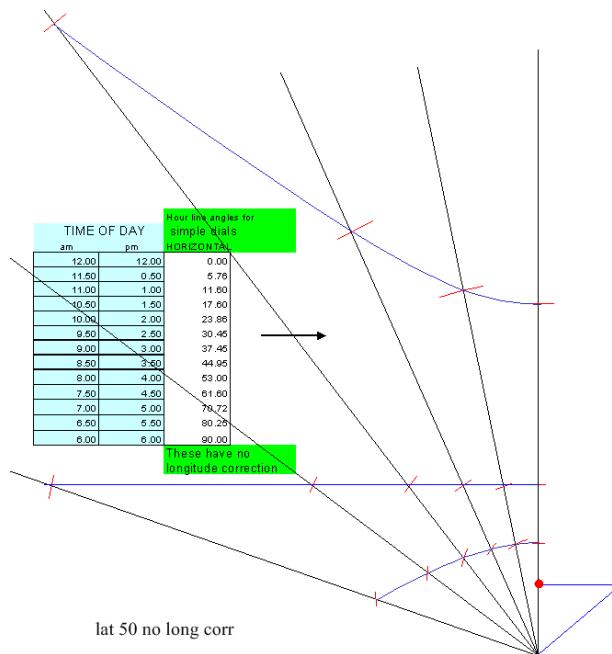
COS(RADIANS(lat)) * COS(RADIANS(15*(12-hh))))))
```

This method works for all angles, and assumes a known perpendicular distance from the nodus to the dial plate. Thus this works for all style heights (latitudes including 0).

The tables for this method are **A4.4** for latitudes 0 to 65 for declinations of 23.5, 0, and -23.5

The azimuth may be used as a vector from the nodus base, or using the law of sines, a distance from dial center along the hour line may be derived. However, using the azimuth fails when the azimuth is 0, fails at the equator, and requires the dial center to be accessible. This is often not true for great decliners. For this reason, the azimuth method is less common, but covered in a few pages.

**NOTE:** These formulae are used in the DeltaCAD and JustBASIC programs, and in the Excel spreadsheets associated with this book.



**NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES**  
The nodus base is a line from the nodus dropped perpendicular to the dial plate

Distance from gnomon base to the hour line for the relevant declination point

**Gnomon linear height:** →

1

Latitude	Decl	1200	1100	1000	900	800	700
0.0	23.50	0.435	0.524	0.765	1.174	1.938	4.093
0.0	0.00	0.000	0.268	0.577	1.000	1.732	3.732
0.0	-23.50	0.435	0.524	0.765	1.174	1.938	4.093
5.0	23.50	0.335	0.434	0.683	1.076	1.772	3.549
5.0	0.00	0.087	0.283	0.586	1.008	1.741	3.747
5.0	-23.50	0.543	0.626	0.865	1.295	2.148	4.856
10.0	23.50	0.240	0.358	0.616	0.998	1.639	3.145
10.0	0.00	0.176	0.324	0.612	1.031	1.768	3.794
10.0	-23.50	0.662	0.742	0.984	1.444	2.417	5.996
15.0	23.50	0.149	0.296	0.566	0.937	1.534	2.837
15.0	0.00	0.268	0.386	0.655	1.069	1.813	3.873
15.0	-23.50	0.795	0.875	1.126	1.629	2.769	7.869
20.0	23.50	0.061	0.256	0.532	0.893	1.452	2.596
20.0	0.00	0.364	0.462	0.714	1.125	1.879	3.988
20.0	-23.50	0.949	1.032	1.299	1.863	3.245	11.496
25.0	23.50	0.026	0.245	0.517	0.865	1.390	2.407
25.0	0.00	0.466	0.552	0.789	1.198	1.967	4.144
25.0	-23.50	1.130	1.219	1.513	2.166	3.922	21.438
30.0	23.50	0.114	0.266	0.520	0.853	1.346	2.258
30.0	0.00	0.577	0.655	0.882	1.291	2.082	4.348
30.0	-23.50	1.351	1.450	1.787	2.573	4.958	
31.0	23.50	0.132	0.273	0.523	0.852	1.339	2.232
31.0	0.00	0.601	0.677	0.903	1.312	2.108	4.395
31.0	-23.50	1.402	1.503	1.851	2.672	5.234	
32.0	23.50	0.149	0.282	0.527	0.852	1.333	2.208
32.0	0.00	0.625	0.700	0.924	1.335	2.136	4.445
32.0	-23.50	1.455	1.559	1.919	2.779	5.543	
33.0	23.50	0.167	0.291	0.531	0.852	1.327	2.184
33.0	0.00	0.649	0.724	0.946	1.358	2.165	4.497
33.0	-23.50	1.511	1.618	1.991	2.893	5.890	
34.0	23.50	0.185	0.302	0.536	0.854	1.323	2.162
34.0	0.00	0.675	0.748	0.970	1.382	2.195	4.552
34.0	-23.50	1.570	1.680	2.067	3.017	6.284	
35.0	23.50	0.203	0.313	0.542	0.855	1.318	2.141
35.0	0.00	0.700	0.773	0.994	1.407	2.227	4.609
35.0	-23.50	1.632	1.746	2.149	3.151	6.734	

Latitude	Decl	1200	100	200	300	400	500

Note: the spreadsheet: **IllustratingShadows.xls** on the web site has the nodus base distance to an hour line, as well as the dial center to hour line distance, in interactive form.

## LATITUDES 35 to 45 - Distance from gnomon base to hour line

A4.4b

## NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from gnomon base to the hour line for the relevant declination point

Gnomon linear height :



1

Latitude	Decl	1200	1100	1000	900	800	700
35.0	23.50	0.203	0.313	0.542	0.855	1.318	2.141
35.0	0.00	0.700	0.773	0.994	1.407	2.227	4.609
35.0	-23.50	1.632	1.746	2.149	3.151	6.734	
36.0	23.50	0.222	0.325	0.548	0.858	1.315	2.121
36.0	0.00	0.727	0.798	1.018	1.434	2.261	4.670
36.0	-23.50	1.698	1.817	2.237	3.297	7.253	
37.0	23.50	0.240	0.338	0.555	0.861	1.312	2.102
37.0	0.00	0.754	0.825	1.044	1.461	2.296	4.733
37.0	-23.50	1.767	1.891	2.331	3.457	7.859	
38.0	23.50	0.259	0.351	0.563	0.865	1.310	2.085
38.0	0.00	0.781	0.852	1.071	1.490	2.333	4.800
38.0	-23.50	1.842	1.971	2.432	3.631	8.575	
39.0	23.50	0.277	0.365	0.572	0.869	1.308	2.068
39.0	0.00	0.810	0.880	1.099	1.520	2.371	4.870
39.0	-23.50	1.921	2.056	2.541	3.824	9.434	
40.0	23.50	0.296	0.380	0.581	0.874	1.307	2.052
40.0	0.00	0.839	0.909	1.128	1.552	2.412	4.944
40.0	-23.50	2.006	2.147	2.658	4.037	10.485	
41.0	23.50	0.315	0.395	0.591	0.879	1.307	2.037
41.0	0.00	0.869	0.939	1.158	1.585	2.454	5.021
41.0	-23.50	2.097	2.245	2.786	4.274	11.798	
42.0	23.50	0.335	0.410	0.601	0.885	1.307	2.023
42.0	0.00	0.900	0.970	1.189	1.619	2.499	5.102
42.0	-23.50	2.194	2.351	2.926	4.541	13.488	
43.0	23.50	0.354	0.427	0.612	0.892	1.308	2.009
43.0	0.00	0.933	1.002	1.222	1.655	2.545	5.187
43.0	-23.50	2.300	2.465	3.079	4.841	15.741	
44.0	23.50	0.374	0.443	0.624	0.900	1.310	1.997
44.0	0.00	0.966	1.035	1.256	1.693	2.594	5.277
44.0	-23.50	2.414	2.590	3.247	5.182	18.897	
45.0	23.50	0.394	0.460	0.637	0.908	1.312	1.986
45.0	0.00	1.000	1.069	1.291	1.732	2.646	5.372
45.0	-23.50	2.539	2.726	3.434	5.574	23.635	
Latitude	Decl	1200	100	200	300	400	500

The azimuth method is not included in these tables because it fails for an azimuth of 0, and if using azimuth to derive a distance from dial center along an hour line, that fails when the dial center is unavailable as in great declining dials. The most common method for calendar lines is the nodus base distance to the associated hour line.

## NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from gnomon base to the hour line for the relevant declination point

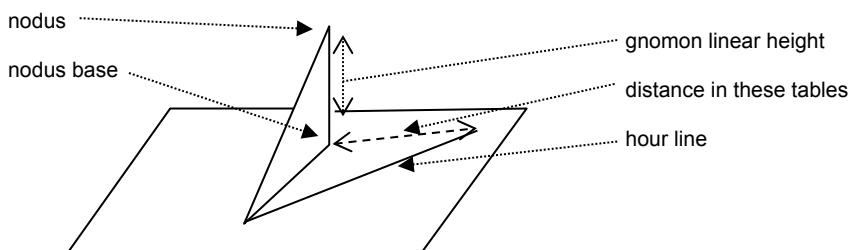
Gnomon linear height :



1

Latitude	Decl	1200	1100	1000	900	800	700
45.0	23.50	0.394	0.460	0.637	0.908	1.312	1.986
45.0	0.00	1.000	1.069	1.291	1.732	2.646	5.372
45.0	-23.50	2.539	2.726	3.434	5.574	23.635	
46.0	23.50	0.414	0.478	0.650	0.916	1.315	1.975
46.0	0.00	1.036	1.105	1.328	1.773	2.700	5.471
46.0	-23.50	2.675	2.875	3.641	6.029	31.544	
47.0	23.50	0.435	0.496	0.663	0.926	1.318	1.965
47.0	0.00	1.072	1.142	1.366	1.817	2.757	5.576
47.0	-23.50	2.824	3.040	3.873	6.563	47.405	
48.0	23.50	0.456	0.515	0.678	0.936	1.322	1.956
48.0	0.00	1.111	1.181	1.406	1.862	2.817	5.687
48.0	-23.50	2.989	3.222	4.134	7.200	95.340	
49.0	23.50	0.477	0.534	0.693	0.946	1.327	1.948
49.0	0.00	1.150	1.221	1.448	1.910	2.880	5.804
49.0	-23.50	3.172	3.426	4.432	7.970		
50.0	23.50	0.499	0.554	0.709	0.958	1.333	1.941
50.0	0.00	1.192	1.263	1.492	1.960	2.946	5.927
50.0	-23.50	3.376	3.655	4.773	8.924		
51.0	23.50	0.521	0.574	0.725	0.969	1.339	1.934
51.0	0.00	1.235	1.306	1.538	2.012	3.017	6.057
51.0	-23.50	3.606	3.913	5.170	10.134		
52.0	23.50	0.543	0.594	0.742	0.982	1.345	1.928
52.0	0.00	1.280	1.352	1.587	2.068	3.091	6.196
52.0	-23.50	3.867	4.209	5.635	11.720		
53.0	23.50	0.566	0.616	0.759	0.995	1.353	1.923
53.0	0.00	1.327	1.400	1.637	2.127	3.169	6.342
53.0	-23.50	4.165	4.550	6.189	13.892		
54.0	23.50	0.589	0.637	0.778	1.010	1.361	1.919
54.0	0.00	1.376	1.450	1.691	2.188	3.252	6.497
54.0	-23.50	4.511	4.949	6.861	17.047		
55.0	23.50	0.613	0.660	0.797	1.024	1.370	1.916
55.0	0.00	1.428	1.503	1.747	2.254	3.340	6.662
55.0	-23.50	4.915	5.420	7.693	22.050		

Latitude	Decl	1200	100	200	300	400	500
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## LATITUDES 55 to 65 - Distance from gnomon base to hour line

A4.4d

## NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from gnomon base to the hour line for the relevant declination point

Gnomon linear height :



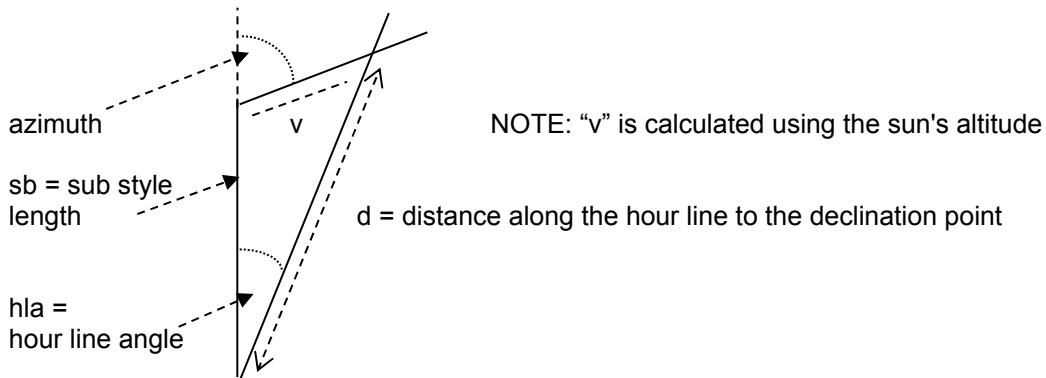
1

Latitude	Decl	1200	1100	1000	900	800	700
55.0	23.50	0.613	0.660	0.797	1.024	1.370	1.916
55.0	0.00	1.428	1.503	1.747	2.254	3.340	6.662
55.0	-23.50	4.915	5.420	7.693	22.050		
56.0	23.50	0.637	0.683	0.816	1.040	1.379	1.913
56.0	0.00	1.483	1.558	1.807	2.323	3.434	6.837
56.0	-23.50	5.396	5.986	8.751	31.199		
57.0	23.50	0.662	0.706	0.837	1.056	1.389	1.911
57.0	0.00	1.540	1.617	1.869	2.396	3.533	7.023
57.0	-23.50	5.976	6.681	10.141	53.304		
58.0	23.50	0.687	0.730	0.858	1.073	1.400	1.909
58.0	0.00	1.600	1.678	1.936	2.474	3.639	7.222
58.0	-23.50	6.691	7.553	12.050			
59.0	23.50	0.713	0.755	0.880	1.091	1.412	1.909
59.0	0.00	1.664	1.744	2.007	2.557	3.752	7.435
59.0	-23.50	7.596	8.681	14.837			
60.0	23.50	0.740	0.781	0.902	1.109	1.424	1.909
60.0	0.00	1.732	1.813	2.082	2.646	3.873	7.662
60.0	-23.50	8.777	10.199	19.290			
61.0	23.50	0.767	0.807	0.926	1.129	1.438	1.910
61.0	0.00	1.804	1.887	2.162	2.740	4.002	7.907
61.0	-23.50	10.385	12.352	27.545			
62.0	23.50	0.795	0.834	0.950	1.149	1.451	1.912
62.0	0.00	1.881	1.965	2.247	2.842	4.141	8.169
62.0	-23.50	12.706	15.645	48.116			
63.0	23.50	0.824	0.862	0.975	1.170	1.466	1.914
63.0	0.00	1.963	2.049	2.339	2.950	4.290	8.452
63.0	-23.50	16.350	21.316				
64.0	23.50	0.854	0.891	1.002	1.192	1.482	1.917
64.0	0.00	2.050	2.139	2.437	3.067	4.451	8.757
64.0	-23.50	22.904	33.406				
65.0	23.50	0.885	0.920	1.029	1.215	1.498	1.921
65.0	0.00	2.145	2.236	2.543	3.193	4.626	9.087
65.0	-23.50	38.188	77.089				
Latitude	Decl	1200	100	200	300	400	500

## Declination/Calendar lines/curves for a dial plate using the sun's altitude as well as azimuth – gnomon linear height

**How far along an hour line from the dial center is the declination point on a horizontal dial (with certain standard declinations) (using the gnomon's linear height)**

This uses the sun's azimuth as well as its altitude.

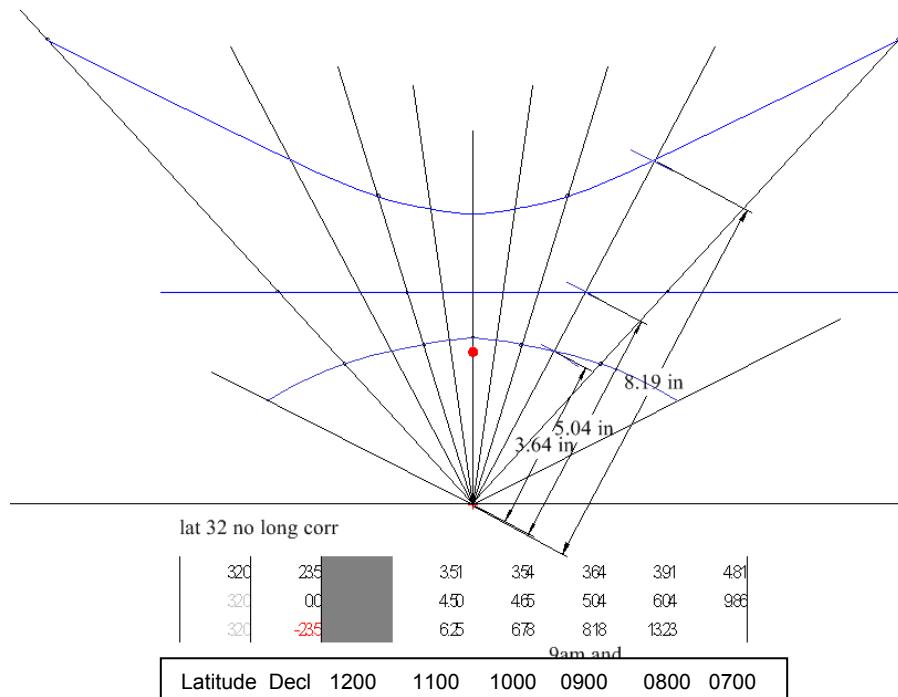


using the law of sines:  $d / (\sin(180-\text{azimuth})) = v / (\sin(\text{hla}))$   
 $d = v * \sin(180-\text{azimuth}) / (\sin(\text{hla}))$

This method fails when the azimuth is 0 or solar noon, it does not work on the equator, and is only practical if the dial center is accessible. This is often not true for great decliners. For this reason, this method is not common.

The azimuth method can be used from the gnomon base as an angle which when extended intersects the appropriate hour line, or using the law of sines can be used to develop the distance along an hour line from dial center which is the declination point, which is discussed here.

To the right is an example for latitude 32° using the distance from the dial center to the calendar point on the hour line.



LATITUDES 0 to 35 - Distance from dial center along the hour line using the gnomon linear height

**NOTE:** does not work for latitude 0, nor solar noon, this also requires the dial center be usable for measurement

A4.5a

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES							
Distance from dial center to the hour line for the relevant declination point							
Latitude	Decl	1200	1100	1000	900	800	700
0.0	23.5						
0.0	0.0						
0.0	-23.5						
5.0	23.5		11.08	11.05	10.97	10.82	10.56
5.0	0.0		11.52	11.53	11.56	11.65	12.11
5.0	-23.5		11.99	12.06	12.22	12.61	14.20
10.0	23.5		5.42	5.40	5.35	5.29	5.38
10.0	0.0		5.85	5.88	5.94	6.11	6.97
10.0	-23.5		6.36	6.45	6.66	7.21	9.90
15.0	23.5		3.58	3.56	3.55	3.56	3.83
15.0	0.0		4.01	4.04	4.13	4.38	5.56
15.0	-23.5		4.56	4.67	4.95	5.72	10.11
20.0	23.5		2.68	2.68	2.69	2.75	3.13
20.0	0.0		3.12	3.17	3.29	3.62	5.05
20.0	-23.5		3.74	3.88	4.24	5.29	12.99
25.0	23.5		2.17	2.18	2.20	2.30	2.73
25.0	0.0		2.63	2.69	2.83	3.24	4.88
25.0	-23.5		3.33	3.51	3.97	5.44	22.51
30.0	23.5		1.85	1.86	1.91	2.03	2.48
30.0	0.0		2.33	2.40	2.58	3.06	4.89
30.0	-23.5		3.15	3.38	4.00	6.14	
31.0	23.5		1.80	1.82	1.86	1.99	2.44
31.0	0.0		2.29	2.36	2.55	3.04	4.91
31.0	-23.5		3.13	3.38	4.04	6.36	
32.0	23.5		1.75	1.77	1.82	1.96	2.41
32.0	0.0		2.25	2.33	2.52	3.02	4.93
32.0	-23.5		3.13	3.39	4.09	6.62	
33.0	23.5		1.71	1.73	1.78	1.92	2.37
33.0	0.0		2.21	2.29	2.49	3.01	4.96
33.0	-23.5		3.13	3.41	4.15	6.91	
34.0	23.5		1.67	1.69	1.75	1.89	2.34
34.0	0.0		2.18	2.27	2.47	3.00	4.99
34.0	-23.5		3.13	3.43	4.22	7.26	
35.0	23.5		1.64	1.66	1.72	1.86	2.31
35.0	0.0		2.15	2.24	2.45	3.00	5.03
35.0	-23.5		3.14	3.46	4.31	7.67	
Latitude	Decl	1200	1100	1000	900	800	700

LATITUDES 35 to 45 - Distance from dial center along the hour line using the gnomon linear height

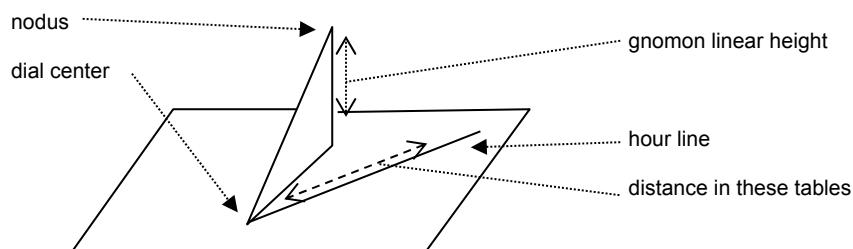
A4.5b

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from dial center to the hour line for the relevant declination point

Gnomon linear height :  1

Latitude	Decl	1200	1100	1000	900	800	700
35.0	23.5		1.64	1.66	1.72	1.86	2.31
35.0	0.0		2.15	2.24	2.45	3.00	5.03
35.0	-23.5		3.14	3.46	4.31	7.67	
36.0	23.5		1.60	1.63	1.69	1.84	2.28
36.0	0.0		2.13	2.22	2.44	3.00	5.07
36.0	-23.5		3.16	3.50	4.41	8.15	
37.0	23.5		1.57	1.60	1.66	1.82	2.26
37.0	0.0		2.11	2.20	2.43	3.01	5.12
37.0	-23.5		3.19	3.54	4.53	8.72	
38.0	23.5		1.55	1.57	1.64	1.79	2.23
38.0	0.0		2.09	2.19	2.42	3.01	5.17
38.0	-23.5		3.22	3.60	4.66	9.40	
39.0	23.5		1.52	1.55	1.61	1.77	2.21
39.0	0.0		2.07	2.18	2.42	3.02	5.22
39.0	-23.5		3.26	3.67	4.81	10.23	
40.0	23.5		1.50	1.52	1.59	1.76	2.19
40.0	0.0		2.06	2.17	2.41	3.04	5.28
40.0	-23.5		3.31	3.74	4.99	11.24	
41.0	23.5		1.47	1.50	1.57	1.74	2.17
41.0	0.0		2.05	2.16	2.42	3.06	5.34
41.0	-23.5		3.37	3.83	5.19	12.53	
42.0	23.5		1.45	1.48	1.56	1.73	2.15
42.0	0.0		2.04	2.16	2.42	3.08	5.41
42.0	-23.5		3.44	3.93	5.42	14.19	
43.0	23.5		1.44	1.47	1.54	1.71	2.14
43.0	0.0		2.04	2.15	2.43	3.10	5.48
43.0	-23.5		3.51	4.05	5.69	16.41	
44.0	23.5		1.42	1.45	1.53	1.70	2.12
44.0	0.0		2.04	2.16	2.44	3.13	5.56
44.0	-23.5		3.60	4.19	6.00	19.54	
45.0	23.5		1.40	1.44	1.52	1.69	2.11
45.0	0.0		2.04	2.16	2.45	3.16	5.64
45.0	-23.5		3.70	4.34	6.36	24.26	
Latitude	Decl	1200	1100	1000	900	800	700



LATITUDES 45 to 55 - Distance from dial center along the hour line using  
the gnomon linear height

A4.5c

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES							
		Distance from dial center to the hour line for the relevant declination point					
Latitude	Decl	1200	1100	1000	900	800	700
45.0	23.5		1.40	1.44	1.52	1.69	2.11
45.0	0.0		2.04	2.16	2.45	3.16	5.64
45.0	-23.5		3.70	4.34	6.36		
46.0	23.5		1.39	1.43	1.51	1.68	2.09
46.0	0.0		2.04	2.17	2.47	3.20	5.73
46.0	-23.5		3.82	4.51	6.79		
47.0	23.5		1.38	1.41	1.50	1.67	2.08
47.0	0.0		2.04	2.18	2.48	3.24	5.83
47.0	-23.5		3.95	4.71	7.29		
48.0	23.5		1.37	1.40	1.49	1.67	2.07
48.0	0.0		2.05	2.19	2.51	3.28	5.93
48.0	-23.5		4.10	4.95	7.90		
49.0	23.5		1.36	1.40	1.48	1.66	2.06
49.0	0.0		2.06	2.20	2.53	3.32	6.04
49.0	-23.5		4.27	5.22	8.65		
50.0	23.5		1.35	1.39	1.48	1.66	2.05
50.0	0.0		2.07	2.22	2.56	3.37	6.15
50.0	-23.5		4.47	5.53	9.58		
51.0	23.5		1.34	1.38	1.47	1.65	2.04
51.0	0.0		2.09	2.24	2.59	3.43	6.27
51.0	-23.5		4.70	5.90	10.76		
52.0	23.5		1.34	1.38	1.47	1.65	2.03
52.0	0.0		2.11	2.26	2.62	3.49	6.40
52.0	-23.5		4.97	6.34	12.32		
53.0	23.5		1.33	1.37	1.47	1.65	2.03
53.0	0.0		2.13	2.29	2.66	3.55	6.54
53.0	-23.5		5.28	6.87	14.47		
54.0	23.5		1.33	1.37	1.47	1.65	2.02
54.0	0.0		2.15	2.32	2.70	3.62	6.69
54.0	-23.5		5.66	7.51	17.61		
55.0	23.5		1.33	1.37	1.46	1.65	2.01
55.0	0.0		2.18	2.35	2.75	3.69	6.85
55.0	-23.5		6.10	8.32			
Latitude	Decl	1200	1100	1000	900	800	700

LATITUDES 55 to 65 - Distance from dial center along the hour line using  
the gnomon linear height

A4.5d

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES							
		1200	1100	1000	900	800	700
Latitude	Decl						
55.0	23.5		1.33	1.37	1.46	1.65	2.01
55.0	0.0		2.18	2.35	2.75	3.69	6.85
55.0	-23.5		6.10				
56.0	23.5		1.33	1.37	1.47	1.65	2.01
56.0	0.0		2.21	2.39	2.80	3.77	7.01
56.0	-23.5		6.64				
57.0	23.5		1.33	1.37	1.47	1.65	2.01
57.0	0.0		2.24	2.43	2.86	3.86	7.19
57.0	-23.5		7.31				
58.0	23.5		1.33	1.37	1.47	1.65	2.00
58.0	0.0		2.28	2.48	2.92	3.95	7.39
58.0	-23.5		8.16				
59.0	23.5		1.33	1.38	1.47	1.66	2.00
59.0	0.0		2.32	2.53	2.98	4.05	7.59
59.0	-23.5						
60.0	23.5		1.33	1.38	1.48	1.66	2.00
60.0	0.0		2.37	2.58	3.06	4.16	7.81
60.0	-23.5						
61.0	23.5		1.34	1.39	1.49	1.67	2.00
61.0	0.0		2.42	2.64	3.13	4.28	8.05
61.0	-23.5						
62.0	23.5		1.34	1.39	1.49	1.67	2.00
62.0	0.0		2.48	2.71	3.22	4.41	8.31
62.0	-23.5						
63.0	23.5		1.35	1.40	1.50	1.68	2.00
63.0	0.0		2.54	2.78	3.31	4.55	8.58
63.0	-23.5						
64.0	23.5		1.36	1.41	1.51	1.69	2.00
64.0	0.0		2.61	2.86	3.41	4.70	8.88
64.0	-23.5						
65.0	23.5		1.37	1.42	1.52	1.70	2.00
65.0	0.0		2.69	2.95	3.52	4.86	9.21
65.0	-23.5						
Latitude	Decl	1200	1100	1000	900	800	700

NOTE: does not work for latitude 0, does not work for solar noon, and also requires the dial center be usable for measurement

## Declination/Calendar lines/curves for a dial plate using the sun's altitude as well as azimuth – style's linear length

**How far along an hour line from the dial center is the declination point on a horizontal dial (with certain standard declinations) (using the style's linear length)**

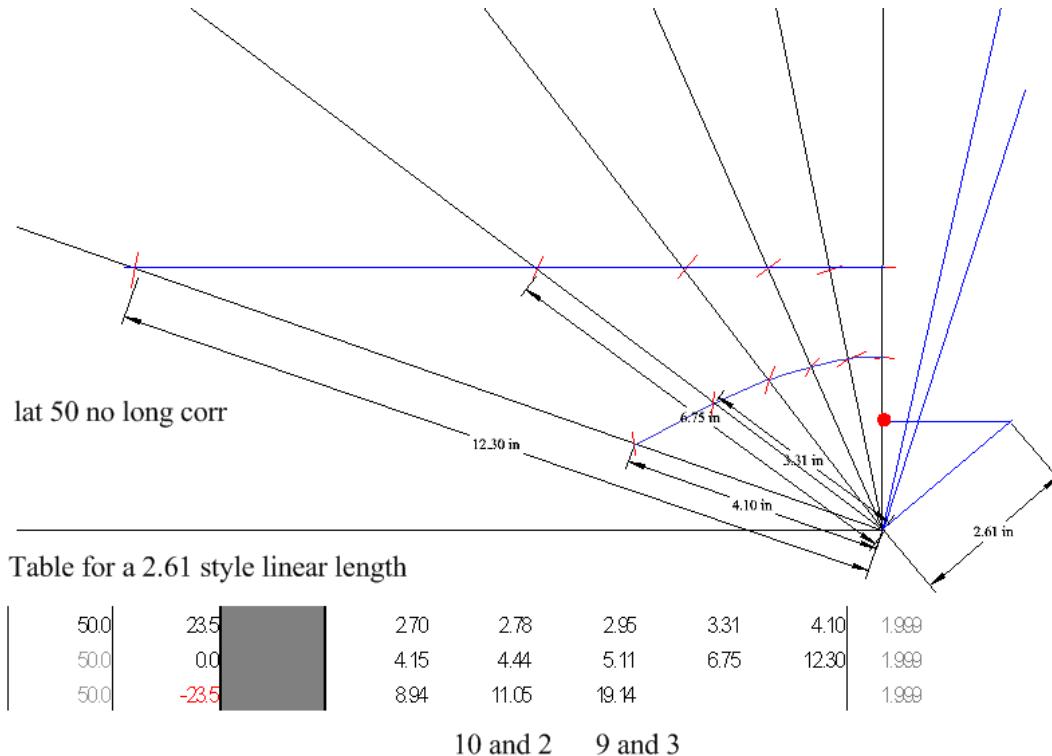
This uses the sun's azimuth as well as its altitude.

This uses the same formulae as used in the distance along an hour line using the gnomon's linear height, except that instead of a unit gnomon linear height, a unit style linear length is used.

This method fails when the azimuth is 0 or solar noon, it does not work on the equator, and is only practical if the dial center is accessible. This is often not true for great decliners. For this reason, this method is not common.

The azimuth method can be used from the gnomon base as an angle which when extended intersects the appropriate hour line, or using the law of sines can be used to develop the distance along an hour line from dial center which is the declination point. which is discussed here.

Below is an example for latitude 50 using the distance from the dial center to the calendar point on the hour line. However, for these tables (A4.6) the style's linear length is used, and not the gnomon linear height.



**Using a style linear length as opposed to gnomon linear height**

LATITUDES 0 to 35 - Distance from dial center along the hour line using the style's length

A4.6a

### DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from dial center to the hour line for the relevant declination point

Style linear length



1

Latitude	Decl	1200	1100	1000	900	800	700
0.0	23.5						
0.0	0.0						
0.0	-23.5						
5.0	23.5		0.97	0.96	0.96	0.94	0.92
5.0	0.0		1.00	1.01	1.01	1.02	1.06
5.0	-23.5		1.05	1.05	1.06	1.10	1.24
10.0	23.5		0.94	0.94	0.93	0.92	0.93
10.0	0.0		1.02	1.02	1.03	1.06	1.21
10.0	-23.5		1.10	1.12	1.16	1.25	1.72
15.0	23.5		0.93	0.92	0.92	0.92	0.99
15.0	0.0		1.04	1.05	1.07	1.13	1.44
15.0	-23.5		1.18	1.21	1.28	1.48	2.62
20.0	23.5		0.92	0.92	0.92	0.94	1.07
20.0	0.0		1.07	1.08	1.12	1.24	1.73
20.0	-23.5		1.28	1.33	1.45	1.81	4.44
25.0	23.5		0.92	0.92	0.93	0.97	1.16
25.0	0.0		1.11	1.14	1.20	1.37	2.06
25.0	-23.5		1.41	1.48	1.68	2.30	9.51
30.0	23.5		0.92	0.93	0.95	1.02	1.24
30.0	0.0		1.17	1.20	1.29	1.53	2.44
30.0	-23.5		1.57	1.69	2.00	3.07	
31.0	23.5		0.93	0.94	0.96	1.03	1.26
31.0	0.0		1.18	1.22	1.31	1.56	2.53
31.0	-23.5		1.61	1.74	2.08	3.27	
32.0	23.5		0.93	0.94	0.96	1.04	1.27
32.0	0.0		1.19	1.23	1.33	1.60	2.61
32.0	-23.5		1.66	1.80	2.17	3.51	
33.0	23.5		0.93	0.94	0.97	1.05	1.29
33.0	0.0		1.20	1.25	1.36	1.64	2.70
33.0	-23.5		1.70	1.85	2.26	3.77	
34.0	23.5		0.94	0.95	0.98	1.06	1.31
34.0	0.0		1.22	1.27	1.38	1.68	2.79
34.0	-23.5		1.75	1.92	2.36	4.06	
35.0	23.5		0.94	0.95	0.98	1.07	1.33
35.0	0.0		1.24	1.29	1.41	1.72	2.88
35.0	-23.5		1.80	1.98	2.47	4.40	
Latitude	Decl	1200	1100	1000	900	800	700

LATITUDES 35 to 45 - Distance from dial center along the hour line using  
the style's length

A4.6b

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES								
Style linear length		→	1	Distance from dial center to the hour line for the relevant declination point				
Latitude	Decl	1200	1100	1000	900	800	700	
35.0	23.5		0.94	0.95	0.98	1.07	1.33	
35.0	0.0		1.24	1.29	1.41	1.72	2.88	
35.0	-23.5		1.80	1.98	2.47	4.40		
36.0	23.5		0.94	0.96	0.99	1.08	1.34	
36.0	0.0		1.25	1.31	1.43	1.76	2.98	
36.0	-23.5		1.86	2.05	2.59	4.79		
37.0	23.5		0.95	0.96	1.00	1.09	1.36	
37.0	0.0		1.27	1.33	1.46	1.81	3.08	
37.0	-23.5		1.92	2.13	2.72	5.25		
38.0	23.5		0.95	0.97	1.01	1.10	1.38	
38.0	0.0		1.29	1.35	1.49	1.86	3.18	
38.0	-23.5		1.98	2.22	2.87	5.79		
39.0	23.5		0.96	0.97	1.01	1.12	1.39	
39.0	0.0		1.30	1.37	1.52	1.90	3.28	
39.0	-23.5		2.05	2.31	3.03	6.43		
40.0	23.5		0.96	0.98	1.02	1.13	1.41	
40.0	0.0		1.32	1.39	1.55	1.95	3.39	
40.0	-23.5		2.13	2.41	3.21	7.23		
41.0	23.5		0.97	0.99	1.03	1.14	1.42	
41.0	0.0		1.35	1.42	1.58	2.01	3.50	
41.0	-23.5		2.21	2.51	3.40	8.22		
42.0	23.5		0.97	0.99	1.04	1.16	1.44	
42.0	0.0		1.37	1.44	1.62	2.06	3.62	
42.0	-23.5		2.30	2.63	3.63	9.49		
43.0	23.5		0.98	1.00	1.05	1.17	1.46	
43.0	0.0		1.39	1.47	1.66	2.12	3.74	
43.0	-23.5		2.40	2.76	3.88	11.19		
44.0	23.5		0.99	1.01	1.06	1.18	1.47	
44.0	0.0		1.41	1.50	1.69	2.17	3.86	
44.0	-23.5		2.50	2.91	4.17	13.58		
45.0	23.5		0.99	1.02	1.07	1.20	1.49	
45.0	0.0		1.44	1.53	1.73	2.24	3.99	
45.0	-23.5		2.62	3.07	4.50	17.15		
Latitude	Decl	1200	1100	1000	900	800	700	

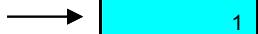
LATITUDES 45 to 55 - Distance from dial center along the hour line using  
the style's length

A4.6c

**DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES**

Distance from dial center to the hour line for the relevant declination point

Style linear length



1

Latitude	Decl	1200	1100	1000	900	800	700
45.0	23.5		0.99	1.02	1.07	1.20	1.49
45.0	0.0		1.44	1.53	1.73	2.24	3.99
45.0	-23.5		2.62	3.07	4.50		
46.0	23.5		1.00	1.03	1.08	1.21	1.51
46.0	0.0		1.47	1.56	1.77	2.30	4.12
46.0	-23.5		2.75	3.25	4.88		
47.0	23.5		1.01	1.03	1.09	1.22	1.52
47.0	0.0		1.49	1.59	1.82	2.37	4.26
47.0	-23.5		2.89	3.45	5.33		
48.0	23.5		1.02	1.04	1.11	1.24	1.54
48.0	0.0		1.52	1.63	1.86	2.44	4.41
48.0	-23.5		3.05	3.68	5.87		
49.0	23.5		1.02	1.05	1.12	1.25	1.55
49.0	0.0		1.56	1.66	1.91	2.51	4.56
49.0	-23.5		3.23	3.94	6.53		
50.0	23.5		1.03	1.06	1.13	1.27	1.57
50.0	0.0		1.59	1.70	1.96	2.58	4.71
50.0	-23.5		3.43	4.24	7.34		
51.0	23.5		1.04	1.08	1.14	1.28	1.59
51.0	0.0		1.62	1.74	2.01	2.66	4.87
51.0	-23.5		3.65	4.58	8.36		
52.0	23.5		1.05	1.09	1.16	1.30	1.60
52.0	0.0		1.66	1.78	2.07	2.75	5.05
52.0	-23.5		3.92	4.99	9.71		
53.0	23.5		1.06	1.10	1.17	1.32	1.62
53.0	0.0		1.70	1.83	2.13	2.84	5.22
53.0	-23.5		4.22	5.48	11.56		
54.0	23.5		1.07	1.11	1.19	1.33	1.63
54.0	0.0		1.74	1.88	2.19	2.93	5.41
54.0	-23.5		4.58	6.08	14.24		
55.0	23.5		1.09	1.12	1.20	1.35	1.65
55.0	0.0		1.78	1.93	2.25	3.03	5.61
55.0	-23.5		5.00	6.82	18.50		

Latitude      Decl      1200      1100      1000      900      800      700

LATITUDES 55 to 65 - Distance from dial center along the hour line using  
the style's length

A4.6d

DIAL CENTER DISTANCE ON HOUR LINE FOR A FLAT SUN DIAL'S DECLINATION LINES							
		Distance from dial center to the hour line for the relevant declination point					
Latitude	Decl	1200	1100	1000	900	800	700
55.0	23.5		1.09	1.12	1.20	1.35	1.65
55.0	0.0		1.78	1.93	2.25	3.03	5.61
55.0	-23.5		5.00				
56.0	23.5		1.10	1.14	1.22	1.37	1.67
56.0	0.0		1.83	1.98	2.32	3.13	5.81
56.0	-23.5		5.51				
57.0	23.5		1.11	1.15	1.23	1.38	1.68
57.0	0.0		1.88	2.04	2.40	3.24	6.03
57.0	-23.5		6.13				
58.0	23.5		1.12	1.17	1.25	1.40	1.70
58.0	0.0		1.94	2.10	2.47	3.35	6.26
58.0	-23.5		6.92				
59.0	23.5		1.14	1.18	1.26	1.42	1.71
59.0	0.0		1.99	2.17	2.56	3.48	6.51
59.0	-23.5		7.94				
60.0	23.5		1.15	1.20	1.28	1.44	1.73
60.0	0.0		2.05	2.24	2.65	3.61	6.77
60.0	-23.5		9.32				
61.0	23.5		1.17	1.21	1.30	1.46	1.75
61.0	0.0		2.12	2.31	2.74	3.74	7.04
61.0	-23.5						
62.0	23.5		1.19	1.23	1.32	1.48	1.76
62.0	0.0		2.19	2.39	2.84	3.89	7.34
62.0	-23.5						
63.0	23.5		1.20	1.25	1.34	1.50	1.78
63.0	0.0		2.26	2.48	2.95	4.05	7.65
63.0	-23.5						
64.0	23.5		1.22	1.27	1.36	1.52	1.80
64.0	0.0		2.35	2.57	3.07	4.22	7.98
64.0	-23.5						
65.0	23.5		1.24	1.29	1.38	1.54	1.81
65.0	0.0		2.43	2.67	3.19	4.40	8.35
65.0	-23.5						
Latitude	Decl	1200	1100	1000	900	800	700

## CONVERSION OF GNOMON TRIANGLE SIDES FOR VARIOUS LATITUDES

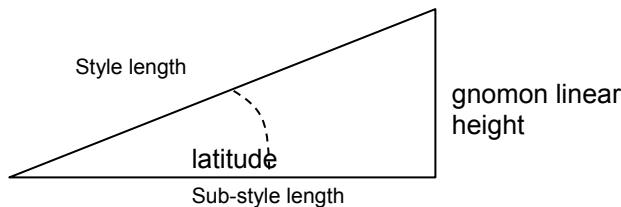
LENGTHS OF THE GNOMON FOR VARIOUS LATITUDES ASSUMING A  
GNOMON LINEAR HEIGHT OF:

1

A4.7

Latitude	Style length	1/style ln	Substyle length
0	infinite	0.000	infinite
5	11.474	0.087	11.430
10	5.759	0.174	5.671
15	3.864	0.259	3.732
20	2.924	0.342	2.747
25	2.366	0.423	2.145
30	2.000	0.500	1.732
31	1.942	0.515	1.664
32	1.887	0.530	1.600
33	1.836	0.545	1.540
34	1.788	0.559	1.483
35	1.743	0.574	1.428
36	1.701	0.588	1.376
37	1.662	0.602	1.327
38	1.624	0.616	1.280
39	1.589	0.629	1.235
40	1.556	0.643	1.192
41	1.524	0.656	1.150
42	1.494	0.669	1.111
43	1.466	0.682	1.072
44	1.440	0.695	1.036
45	1.414	0.707	1.000

Latitude	Style length	1/style ln	Substyle length
45	1.414	0.707	1.000
46	1.390	0.719	0.966
47	1.367	0.731	0.933
48	1.346	0.743	0.900
49	1.325	0.755	0.869
50	1.305	0.766	0.839
51	1.287	0.777	0.810
52	1.269	0.788	0.781
53	1.252	0.799	0.754
54	1.236	0.809	0.727
55	1.221	0.819	0.700
56	1.206	0.829	0.675
57	1.192	0.839	0.649
58	1.179	0.848	0.625
59	1.167	0.857	0.601
60	1.155	0.866	0.577
61	1.143	0.875	0.554
62	1.133	0.883	0.532
63	1.122	0.891	0.510
64	1.113	0.899	0.488
65	1.103	0.906	0.466
90	1.000	1.000	0.000



$$\sin(\text{lat}) = \text{gnomon linear height} / \text{Style length}$$

$$\text{Style length} = \text{gnomon linear height} / \sin(\text{lat})$$

$$\tan(\text{lat}) = \text{gnomon linear height} / \text{substyle length}$$

$$\text{substyle length} = \text{gnomon linear height} / \tan(\text{lat})$$

It may be easier to measure the linear length of the style than the gnomon's linear height, in which case the values above may prove beneficial.

## APPENDIX 5 – TABLES FOR THE VERTICAL DECLINERS

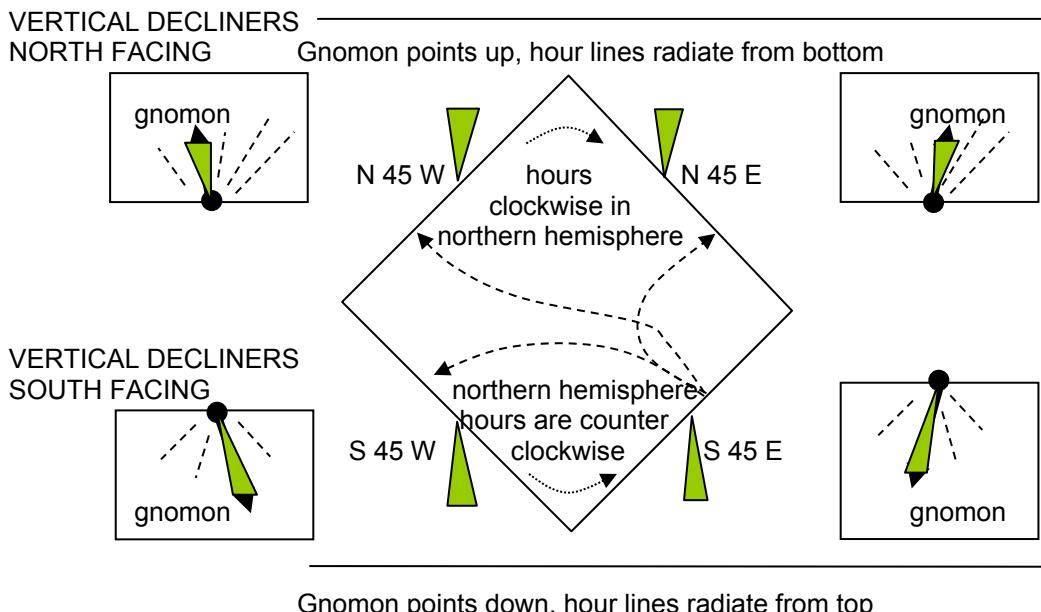
### VERTICAL DECLINERS – PREDOMINANTLY SOUTH

#### LATITUDE 30-60° hour line angle tables

Appendix 5.1 charts are useful for cases where dial center is accessible.

Hour lines for great decliners with declinations of more than 45 degrees whose dial centers are often not readily accessible are in appendix 5.2

The following tables use exactly the same formulae as in appendix A5.2, the sole difference is that these declination angles are no more than 45 degrees.



A design for South xx degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. If longitude correction is applied, care must be applied as hour lines shift. The North facing decliner gnomons are inverted, and the vertical is midnight.

Also hours are clockwise when north declining, whereas they are anti clockwise when south declining.

If an angle in the morning shows positive before going negative, or the other way around, and similarly for the afternoon, then these are angles for which the gnomon shadow may be meaningless. In which case, the cell is shaded as is the font.

Negative angles are west of the vertical

The hour line angle from vertical is:

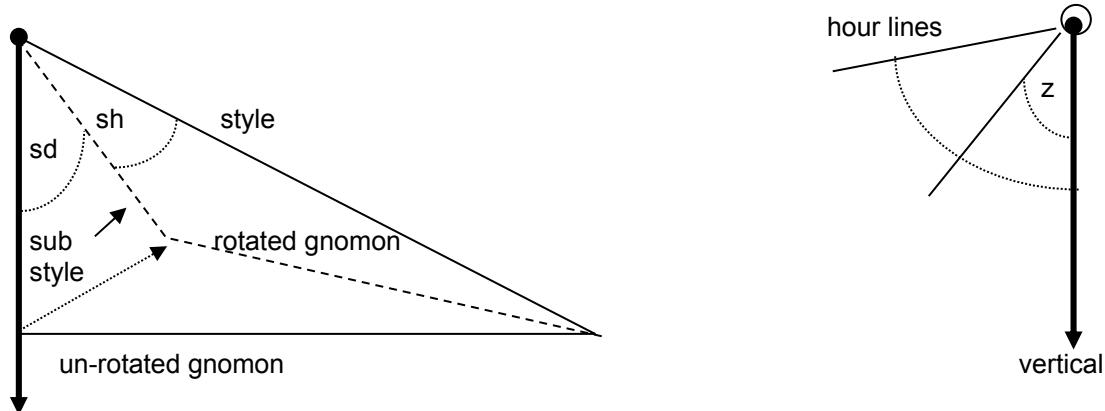
The gnomon offset from vertical is (style distance):  
Style and sub style angle is (style height)

Positive angles are east of the vertical

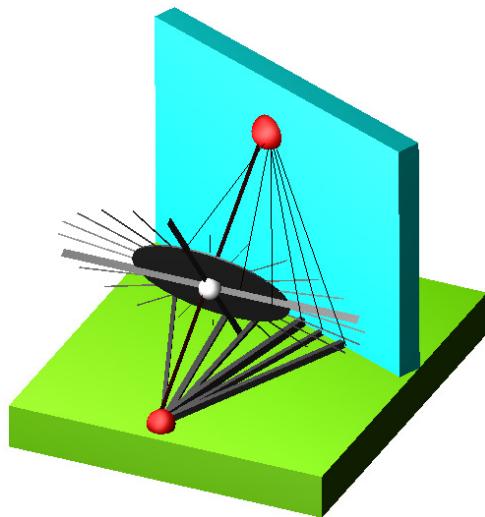
$$z = \text{atan}(\cos(\text{lat}) / (\cos(d) \cot(ha) + \sin(d) \sin(\text{lat})))$$

$$sd = \text{atan}(\sin(\text{dec}) / \tan(\text{lat}))$$

$$sh = \text{asin}(\cos(\text{lat}) * \cos(\text{dec}))$$



The gnomon does not have to be rotated, it may be chamfered and thus aligned to true north. However, if the gnomon is rotated, then it becomes easier to develop calendar or declination lines, as well as analemma figures.



The picture to the left is a reminder of what these tables are achieving. The altering of the vertical dial's hour line to match those of either a horizontal dial, or an equatorial dial.

Appendix 7 has the proof of **SD** and of **SH** and for the hour line angles

**NOTE:** An angle, **DL**, is derived when vertical decliners are designed which enables a surrogate horizontal dial designed for latitude **SH** and longitude **DL** that has analemma (and calendar, and hour line) data, to be used as a surrogate dial for the original vertical decliner. This is most meaningful when the vertical decliner's longitude is considered. This surrogate dial's local noon (not corrected for longitude noon) is aligned with the vertical decliner's SD line.

**A5.1 a**

LATITUDE:	30	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-67.8	-69.6	-71.7	-73.9	-76.3	-78.8	-81.5	-84.3	-87.1	90.0	
7.00	-57.9	-58.7	-59.7	-60.9	-62.3	-64.0	-65.9	-68.0	-70.3	-72.8	
8.00	-48.7	-48.6	-48.7	-49.1	-49.7	-50.5	-51.6	-52.9	-54.5	-56.3	
9.00	-39.2	-38.5	-38.1	-37.8	-37.8	-37.9	-38.3	-38.9	-39.8	-40.9	
10.00	-28.8	-27.7	-26.9	-26.3	-25.9	-25.7	-25.7	-25.8	-26.1	-26.6	
10.50	-22.8	-21.7	-20.9	-20.3	-19.8	-19.5	-19.4	-19.4	-19.5	-19.7	
11.00	-16.1	-15.2	-14.5	-14.0	-13.5	-13.2	-13.1	-13.0	-13.0	-13.1	
11.50	-8.6	-8.0	-7.6	-7.2	-7.0	-6.8	-6.6	-6.5	-6.5	-6.5	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	20.8	18.8	17.4	16.2	15.3	14.6	14.0	13.6	13.3	13.1	
13.50	32.6	29.5	27.1	25.2	23.7	22.4	21.5	20.7	20.1	19.7	
14.00	44.8	40.7	37.4	34.7	32.5	30.7	29.3	28.1	27.2	26.6	
14.50	56.7	52.0	48.0	44.6	41.8	39.4	37.5	35.9	34.6	33.6	
15.00	67.8	62.8	58.4	54.6	51.3	48.4	46.0	44.0	42.3	40.9	
16.00	86.4	82.1	77.9	73.9	70.2	66.8	63.7	60.9	58.5	56.3	
17.00	-79.3	-82.4	-85.6	-88.8	87.9	84.7	81.5	78.4	75.5	72.8	
18.00	-67.8	-69.6	-71.7	-73.9	-76.3	-78.8	-81.5	-84.3	-87.1	90.0	
STYLE:SD	-50.8	-48.1	-44.8	-40.9	-36.2	-30.6	-24.1	-16.7	-8.6	0.0	
STYLE:SH	37.8	41.6	45.2	48.6	51.7	54.5	56.8	58.5	59.6	60.0	
D.LONG	-63.4	-59.2	-54.5	-49.1	-43.0	-36.1	-28.2	-19.4	-9.9	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
	0	5	10	15	20	25	30	35	40	45	hh.hh
90.0	87.1	84.3	81.5	78.8	76.3	73.9	71.7	69.6	67.8	65.0	6.00
-72.8	-75.5	-78.4	-81.5	-84.7	-87.9	88.8	85.6	82.4	79.3	72.8	7.00
-56.3	-58.5	-60.9	-63.7	-66.8	-70.2	-73.9	-77.9	-82.1	-86.4	-88.4	8.00
-40.9	-42.3	-44.0	-46.0	-48.4	-51.3	-54.6	-58.4	-62.8	-67.8	-72.0	9.00
-26.6	-27.2	-28.1	-29.3	-30.7	-32.5	-34.7	-37.4	-40.7	-44.8	-48.0	10.00
-19.7	-20.1	-20.7	-21.5	-22.4	-23.7	-25.2	-27.1	-29.5	-32.6	-35.5	10.50
-13.1	-13.3	-13.6	-14.0	-14.6	-15.3	-16.2	-17.4	-18.8	-20.8	-22.8	11.00
-6.5	-6.6	-6.7	-6.9	-7.1	-7.4	-7.8	-8.3	-9.0	-9.8	-10.5	11.50
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00
13.1	13.0	13.0	13.1	13.2	13.5	14.0	14.5	15.2	16.1	17.0	13.00
19.7	19.5	19.4	19.4	19.5	19.8	20.3	20.9	21.7	22.8	23.5	13.50
26.6	26.1	25.8	25.7	25.7	25.9	26.3	26.9	27.7	28.8	29.5	14.00
33.6	32.8	32.3	32.0	31.8	31.9	32.1	32.6	33.3	34.2	35.0	14.50
40.9	39.8	38.9	38.3	37.9	37.8	37.8	38.1	38.5	39.2	40.0	15.00
56.3	54.5	52.9	51.6	50.5	49.7	49.1	48.7	48.6	48.7	49.0	16.00
72.8	70.3	68.0	65.9	64.0	62.3	60.9	59.7	58.7	57.9	56.0	17.00
90.0	87.1	84.3	81.5	78.8	76.3	73.9	71.7	69.6	67.8	65.0	18.00
0.0	8.6	16.7	24.1	30.6	36.2	40.9	44.8	48.1	50.8	SD	
60.0	59.6	58.5	56.8	54.5	51.7	48.6	45.2	41.6	37.8	SH	
0.0	9.9	19.4	28.2	36.1	43.0	49.1	54.5	59.2	63.4	DL	

**A5.1 b**

LATITUDE:	32	This table gives the hour line angles from the vertical.									
TIME	DEC		South xx degrees East ~ wall faces south east								SOUTH
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-66.2	-68.1	-70.3	-72.6	-75.2	-77.9	-80.8	-83.8	-86.9	90.0	
7.00	-56.4	-57.2	-58.3	-59.6	-61.2	-63.0	-65.0	-67.2	-69.7	-72.5	
8.00	-47.3	-47.3	-47.5	-47.9	-48.6	-49.5	-50.7	-52.1	-53.8	-55.8	
9.00	-38.1	-37.5	-37.1	-36.9	-36.9	-37.1	-37.6	-38.2	-39.1	-40.3	
10.00	-27.9	-27.0	-26.2	-25.7	-25.3	-25.1	-25.1	-25.3	-25.6	-26.1	
10.50	-22.2	-21.2	-20.4	-19.8	-19.4	-19.1	-19.0	-19.0	-19.1	-19.4	
11.00	-15.7	-14.8	-14.2	-13.6	-13.2	-12.9	-12.8	-12.7	-12.7	-12.8	
11.50	-8.4	-7.8	-7.4	-7.1	-6.8	-6.6	-6.5	-6.4	-6.4	-6.4	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	20.5	18.6	17.1	16.0	15.0	14.3	13.7	13.3	13.0	12.8	
13.50	32.5	29.3	26.9	24.9	23.4	22.1	21.1	20.4	19.8	19.4	
14.00	44.9	40.7	37.3	34.5	32.2	30.4	28.9	27.7	26.8	26.1	
14.50	57.2	52.2	48.0	44.5	41.5	39.1	37.1	35.4	34.1	33.1	
15.00	68.6	63.4	58.7	54.7	51.2	48.2	45.7	43.5	41.8	40.3	
16.00	87.7	83.2	78.7	74.5	70.6	66.9	63.6	60.7	58.0	55.8	
17.00	-77.7	-80.9	-84.3	-87.8	88.7	85.2	81.8	78.5	75.4	72.5	
18.00	-66.2	-68.1	-70.3	-72.6	-75.2	-77.9	-80.8	-83.8	-86.9	90.0	
STYLE:SD	-48.5	-45.8	-42.5	-38.7	-34.1	-28.7	-22.5	-15.5	-7.9	0.0	
STYLE:SH	36.8	40.5	44.0	47.3	50.2	52.8	55.0	56.6	57.7	58.0	
D.LONG	-62.1	-57.7	-52.9	-47.5	-41.3	-34.5	-26.8	-18.4	-9.4	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
	0	5	10	15	20	25	30	35	40	45	hh.hh
90.0	86.9	83.8	80.8	77.9	75.2	72.6	70.3	68.1	66.2	64.3	6.00
-72.5	-75.4	-78.5	-81.8	-85.2	-88.7	87.8	84.3	80.9	77.7	74.4	7.00
-55.8	-58.0	-60.7	-63.6	-66.9	-70.6	-74.5	-78.7	-83.2	-87.7	-92.3	8.00
-40.3	-41.8	-43.5	-45.7	-48.2	-51.2	-54.7	-58.7	-63.4	-68.6	-73.8	9.00
-26.1	-26.8	-27.7	-28.9	-30.4	-32.2	-34.5	-37.3	-40.7	-44.9	-49.1	10.00
-19.4	-19.8	-20.4	-21.1	-22.1	-23.4	-24.9	-26.9	-29.3	-32.5	-35.7	10.50
-12.8	-13.0	-13.3	-13.7	-14.3	-15.0	-16.0	-17.1	-18.6	-20.5	-22.4	11.00
-6.4	-6.4	-6.5	-6.7	-7.0	-7.3	-7.7	-8.2	-8.8	-9.6	-10.3	11.50
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00
12.8	12.7	12.7	12.8	12.9	13.2	13.6	14.2	14.8	15.7	16.5	13.00
19.4	19.1	19.0	19.0	19.1	19.4	19.8	20.4	21.2	22.2	23.1	13.50
26.1	25.6	25.3	25.1	25.1	25.3	25.7	26.2	27.0	27.9	28.8	14.00
33.1	32.2	31.7	31.3	31.1	31.1	31.3	31.7	32.3	33.2	34.1	14.50
40.3	39.1	38.2	37.6	37.1	36.9	36.9	37.1	37.5	38.1	38.7	15.00
55.8	53.8	52.1	50.7	49.5	48.6	47.9	47.5	47.3	47.3	47.3	16.00
72.5	69.7	67.2	65.0	63.0	61.2	59.6	58.3	57.2	56.4	55.6	17.00
90.0	86.9	83.8	80.8	77.9	75.2	72.6	70.3	68.1	66.2	64.3	18.00
0.0	7.9	15.5	22.5	28.7	34.1	38.7	42.5	45.8	48.5	SD	
58.0	57.7	56.6	55.0	52.8	50.2	47.3	44.0	40.5	36.8	SH	
0.0	9.4	18.4	26.8	34.5	41.3	47.5	52.9	57.7	62.1	DL	

**A5.1 c**

LATITUDE:	34	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-64.5	-66.6	-68.8	-71.4	-74.1	-77.0	-80.1	-83.3	-86.6	90.0	
7.00	-54.8	-55.7	-56.9	-58.3	-60.0	-61.9	-64.0	-66.5	-69.2	-72.1	
8.00	-45.9	-46.0	-46.2	-46.8	-47.5	-48.5	-49.7	-51.2	-53.0	-55.1	
9.00	-36.9	-36.4	-36.0	-35.9	-36.0	-36.2	-36.7	-37.5	-38.4	-39.7	
10.00	-27.1	-26.2	-25.5	-25.0	-24.7	-24.5	-24.5	-24.7	-25.0	-25.6	
10.50	-21.5	-20.6	-19.8	-19.3	-18.9	-18.6	-18.5	-18.5	-18.7	-19.0	
11.00	-15.3	-14.4	-13.8	-13.3	-12.9	-12.6	-12.5	-12.4	-12.4	-12.5	
11.50	-8.2	-7.6	-7.2	-6.9	-6.6	-6.5	-6.3	-6.2	-6.2	-6.2	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	20.3	18.3	16.9	15.7	14.8	14.0	13.5	13.0	12.7	12.5	
13.50	32.3	29.1	26.6	24.6	23.0	21.8	20.8	20.0	19.4	19.0	
14.00	45.0	40.6	37.1	34.2	31.9	30.0	28.5	27.3	26.3	25.6	
14.50	57.6	52.4	48.0	44.3	41.3	38.7	36.7	34.9	33.6	32.5	
15.00	69.4	63.9	59.0	54.7	51.1	47.9	45.3	43.0	41.2	39.7	
16.00	89.1	84.3	79.6	75.1	70.9	67.0	63.5	60.4	57.6	55.1	
17.00	-76.0	-79.5	-83.0	-86.7	89.5	85.8	82.2	78.6	75.3	72.1	
18.00	-64.5	-66.6	-68.8	-71.4	-74.1	-77.0	-80.1	-83.3	-86.6	90.0	
STYLE:SD	-46.4	-43.6	-40.4	-36.5	-32.1	-26.9	-21.0	-14.4	-7.4	0.0	
STYLE:SH	35.9	39.4	42.8	45.9	48.7	51.2	53.2	54.7	55.7	56.0	
D.LONG	-60.8	-56.3	-51.4	-45.9	-39.8	-33.1	-25.6	-17.5	-8.9	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
0	5	10	15	20	25	30	35	40	45	hh.hh	
90.0	86.6	83.3	80.1	77.0	74.1	71.4	68.8	66.6	64.5	6.00	
-72.1	-75.3	-78.6	-82.2	-85.8	-89.5	86.7	83.0	79.5	76.0	7.00	
-55.1	-57.6	-60.4	-63.5	-67.0	-70.9	-75.1	-79.6	-84.3	-89.1	8.00	
-39.7	-41.2	-43.0	-45.3	-47.9	-51.1	-54.7	-59.0	-63.9	-69.4	9.00	
-25.6	-26.3	-27.3	-28.5	-30.0	-31.9	-34.2	-37.1	-40.6	-45.0	10.00	
-19.0	-19.4	-20.0	-20.8	-21.8	-23.0	-24.6	-26.6	-29.1	-32.3	10.50	
-12.5	-12.7	-13.0	-13.5	-14.0	-14.8	-15.7	-16.9	-18.3	-20.3	11.00	
-6.2	-6.3	-6.4	-6.6	-6.8	-7.1	-7.5	-8.0	-8.6	-9.5	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
12.5	12.4	12.4	12.5	12.6	12.9	13.3	13.8	14.4	15.3	13.00	
19.0	18.7	18.5	18.5	18.6	18.9	19.3	19.8	20.6	21.5	13.50	
25.6	25.0	24.7	24.5	24.5	24.7	25.0	25.5	26.2	27.1	14.00	
32.5	31.6	31.0	30.6	30.4	30.3	30.5	30.8	31.4	32.2	14.50	
39.7	38.4	37.5	36.7	36.2	36.0	35.9	36.0	36.4	36.9	15.00	
55.1	53.0	51.2	49.7	48.5	47.5	46.8	46.2	46.0	45.9	16.00	
72.1	69.2	66.5	64.0	61.9	60.0	58.3	56.9	55.7	54.8	17.00	
90.0	86.6	83.3	80.1	77.0	74.1	71.4	68.8	66.6	64.5	18.00	
0.0	7.4	14.4	21.0	26.9	32.1	36.5	40.4	43.6	46.4	SD	
56.0	55.7	54.7	53.2	51.2	48.7	45.9	42.8	39.4	35.9	SH	
0.0	8.9	17.5	25.6	33.1	39.8	45.9	51.4	56.3	60.8	DL	

**A5.1 d**

LATITUDE:	36	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-62.8	-65.0	-67.4	-70.0	-72.9	-76.0	-79.4	-82.8	-86.4	90.0	
7.00	-53.2	-54.2	-55.5	-57.0	-58.7	-60.8	-63.1	-65.7	-68.5	-71.7	
8.00	-44.5	-44.6	-45.0	-45.5	-46.4	-47.4	-48.7	-50.3	-52.3	-54.5	
9.00	-35.8	-35.3	-35.0	-34.9	-35.0	-35.3	-35.9	-36.7	-37.7	-39.0	
10.00	-26.3	-25.4	-24.7	-24.3	-24.0	-23.9	-23.9	-24.1	-24.5	-25.0	
10.50	-20.9	-20.0	-19.3	-18.7	-18.4	-18.1	-18.0	-18.1	-18.2	-18.5	
11.00	-14.8	-14.0	-13.4	-12.9	-12.6	-12.3	-12.2	-12.1	-12.1	-12.2	
11.50	-8.0	-7.4	-7.0	-6.7	-6.5	-6.3	-6.2	-6.1	-6.1	-6.1	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	20.0	18.1	16.6	15.4	14.5	13.8	13.2	12.8	12.4	12.2	
13.50	32.1	28.8	26.3	24.2	22.6	21.4	20.4	19.6	19.0	18.5	
14.00	45.0	40.4	36.8	33.9	31.5	29.6	28.0	26.8	25.8	25.0	
14.50	58.0	52.5	47.9	44.1	40.9	38.3	36.2	34.4	33.0	31.8	
15.00	70.2	64.4	59.2	54.7	50.9	47.6	44.8	42.5	40.6	39.0	
16.00	-89.5	85.4	80.5	75.7	71.2	67.1	63.4	60.0	57.1	54.5	
17.00	-74.4	-78.0	-81.7	-85.6	-89.6	86.4	82.5	78.7	75.1	71.7	
18.00	-62.8	-65.0	-67.4	-70.0	-72.9	-76.0	-79.4	-82.8	-86.4	90.0	
STYLE:SD	-44.2	-41.5	-38.3	-34.5	-30.2	-25.2	-19.6	-13.4	-6.8	0.0	
STYLE:SH	34.9	38.3	41.5	44.5	47.2	49.5	51.4	52.8	53.7	54.0	
D.LONG	-59.6	-55.0	-50.0	-44.5	-38.4	-31.8	-24.5	-16.7	-8.5	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
0	5	10	15	20	25	30	35	40	45	hh.hh	
90.0	86.4	82.8	79.4	76.0	72.9	70.0	67.4	65.0	62.8	6.00	
-71.7	-75.1	-78.7	-82.5	-86.4	89.6	85.6	81.7	78.0	74.4	7.00	
-54.5	-57.1	-60.0	-63.4	-67.1	-71.2	-75.7	-80.5	-85.4	89.5	8.00	
-39.0	-40.6	-42.5	-44.8	-47.6	-50.9	-54.7	-59.2	-64.4	-70.2	9.00	
-25.0	-25.8	-26.8	-28.0	-29.6	-31.5	-33.9	-36.8	-40.4	-45.0	10.00	
-18.5	-19.0	-19.6	-20.4	-21.4	-22.6	-24.2	-26.3	-28.8	-32.1	10.50	
-12.2	-12.4	-12.8	-13.2	-13.8	-14.5	-15.4	-16.6	-18.1	-20.0	11.00	
-6.1	-6.1	-6.3	-6.4	-6.7	-7.0	-7.3	-7.8	-8.5	-9.3	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
12.2	12.1	12.1	12.2	12.3	12.6	12.9	13.4	14.0	14.8	13.00	
18.5	18.2	18.1	18.0	18.1	18.4	18.7	19.3	20.0	20.9	13.50	
25.0	24.5	24.1	23.9	23.9	24.0	24.3	24.7	25.4	26.3	14.00	
31.8	30.9	30.3	29.8	29.6	29.5	29.6	29.9	30.5	31.2	14.50	
39.0	37.7	36.7	35.9	35.3	35.0	34.9	35.0	35.3	35.8	15.00	
54.5	52.3	50.3	48.7	47.4	46.4	45.5	45.0	44.6	44.5	16.00	
71.7	68.5	65.7	63.1	60.8	58.7	57.0	55.5	54.2	53.2	17.00	
90.0	86.4	82.8	79.4	76.0	72.9	70.0	67.4	65.0	62.8	18.00	
0.0	6.8	13.4	19.6	25.2	30.2	34.5	38.3	41.5	44.2	SD	
54.0	53.7	52.8	51.4	49.5	47.2	44.5	41.5	38.3	34.9	SH	
0.0	8.5	16.7	24.5	31.8	38.4	44.5	50.0	55.0	59.6	DL	

**A5.1 e**

LATITUDE:	38	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-61.1	-63.3	-65.9	-68.7	-71.7	-75.0	-78.6	-82.3	-86.1	90.0	
7.00	-51.6	-52.7	-54.0	-55.6	-57.4	-59.6	-62.0	-64.8	-67.9	-71.2	
8.00	-43.0	-43.2	-43.6	-44.3	-45.2	-46.3	-47.7	-49.4	-51.4	-53.8	
9.00	-34.6	-34.1	-33.9	-33.9	-34.0	-34.4	-35.0	-35.8	-36.9	-38.2	
10.00	-25.4	-24.6	-24.0	-23.6	-23.3	-23.2	-23.3	-23.5	-23.9	-24.5	
10.50	-20.2	-19.3	-18.7	-18.2	-17.8	-17.6	-17.6	-17.6	-17.8	-18.1	
11.00	-14.4	-13.6	-13.0	-12.5	-12.2	-12.0	-11.8	-11.8	-11.8	-11.9	
11.50	-7.7	-7.2	-6.8	-6.5	-6.3	-6.1	-6.0	-5.9	-5.9	-5.9	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	19.7	17.7	16.2	15.1	14.2	13.4	12.9	12.5	12.1	11.9	
13.50	31.8	28.5	25.9	23.8	22.2	21.0	19.9	19.1	18.5	18.1	
14.00	44.9	40.2	36.5	33.5	31.0	29.1	27.5	26.2	25.2	24.5	
14.50	58.3	52.6	47.8	43.8	40.6	37.9	35.6	33.8	32.3	31.2	
15.00	71.0	64.8	59.4	54.7	50.7	47.2	44.3	41.9	39.9	38.2	
16.00	-88.0	86.6	81.4	76.3	71.5	67.2	63.2	59.6	56.5	53.8	
17.00	-72.7	-76.4	-80.4	-84.5	-88.7	87.0	82.8	78.7	74.9	71.2	
18.00	-61.1	-63.3	-65.9	-68.7	-71.7	-75.0	-78.6	-82.3	-86.1	90.0	
STYLE:SD	-42.1	-39.4	-36.3	-32.6	-28.4	-23.6	-18.3	-12.5	-6.4	0.0	
STYLE:SH	33.9	37.1	40.2	43.0	45.6	47.8	49.6	50.9	51.7	52.0	
D.LONG	-58.4	-53.7	-48.7	-43.2	-37.1	-30.6	-23.5	-16.0	-8.1	0.0	

	South xx degrees West ~ wall faces southwest									TIME
5	10	15	20	25	30	35	40	45	hh.hh	
86.1	82.3	78.6	75.0	71.7	68.7	65.9	63.3	61.1	6.00	
-74.9	-78.7	-82.8	-87.0	88.7	84.5	80.4	76.4	72.7	7.00	
-56.5	-59.6	-63.2	-67.2	-71.5	-76.3	-81.4	-86.6	88.0	8.00	
-39.9	-41.9	-44.3	-47.2	-50.7	-54.7	-59.4	-64.8	-71.0	9.00	
-25.2	-26.2	-27.5	-29.1	-31.0	-33.5	-36.5	-40.2	-44.9	10.00	
-18.5	-19.1	-19.9	-21.0	-22.2	-23.8	-25.9	-28.5	-31.8	10.50	
-12.1	-12.5	-12.9	-13.4	-14.2	-15.1	-16.2	-17.7	-19.7	11.00	
-6.0	-6.1	-6.3	-6.5	-6.8	-7.2	-7.6	-8.3	-9.1	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
11.8	11.8	11.8	12.0	12.2	12.5	13.0	13.6	14.4	13.00	
17.8	17.6	17.6	17.6	17.8	18.2	18.7	19.3	20.2	13.50	
23.9	23.5	23.3	23.2	23.3	23.6	24.0	24.6	25.4	14.00	
30.2	29.5	29.1	28.8	28.7	28.7	29.0	29.5	30.1	14.50	
36.9	35.8	35.0	34.4	34.0	33.9	33.9	34.1	34.6	15.00	
51.4	49.4	47.7	46.3	45.2	44.3	43.6	43.2	43.0	16.00	
67.9	64.8	62.0	59.6	57.4	55.6	54.0	52.7	51.6	17.00	
86.1	82.3	78.6	75.0	71.7	68.7	65.9	63.3	61.1	18.00	
6.4	12.5	18.3	23.6	28.4	32.6	36.3	39.4	42.1	SD	
51.7	50.9	49.6	47.8	45.6	43.0	40.2	37.1	33.9	SH	
8.1	16.0	23.5	30.6	37.1	43.2	48.7	53.7	58.4	DL	

LATITUDE:	40	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-59.3	-61.7	-64.3	-67.2	-70.5	-74.0	-77.7	-81.7	-85.8	90.0	
7.00	-49.9	-51.1	-52.5	-54.2	-56.1	-58.4	-61.0	-63.9	-67.1	-70.7	
8.00	-41.6	-41.8	-42.3	-43.0	-43.9	-45.1	-46.6	-48.4	-50.5	-53.0	
9.00	-33.4	-33.0	-32.8	-32.8	-33.0	-33.5	-34.1	-34.9	-36.1	-37.5	
10.00	-24.5	-23.8	-23.2	-22.8	-22.6	-22.5	-22.6	-22.9	-23.3	-23.9	
10.50	-19.5	-18.7	-18.1	-17.6	-17.3	-17.1	-17.0	-17.1	-17.3	-17.6	
11.00	-13.9	-13.2	-12.6	-12.2	-11.8	-11.6	-11.5	-11.4	-11.5	-11.6	
11.50	-7.5	-7.0	-6.6	-6.3	-6.1	-5.9	-5.8	-5.8	-5.7	-5.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	19.3	17.4	15.9	14.7	13.8	13.1	12.6	12.1	11.8	11.6	
13.50	31.4	28.1	25.5	23.4	21.8	20.5	19.5	18.7	18.1	17.6	
14.00	44.8	40.0	36.1	33.0	30.5	28.6	27.0	25.7	24.6	23.9	
14.50	58.6	52.6	47.6	43.5	40.1	37.3	35.0	33.2	31.7	30.4	
15.00	71.8	65.3	59.5	54.6	50.4	46.8	43.8	41.3	39.2	37.5	
16.00	-86.5	87.8	82.3	76.9	71.8	67.2	62.9	59.2	55.9	53.0	
17.00	-70.9	-74.8	-79.0	-83.3	-87.8	87.6	83.1	78.8	74.6	70.7	
18.00	-59.3	-61.7	-64.3	-67.2	-70.5	-74.0	-77.7	-81.7	-85.8	90.0	
STYLE:SD	-40.1	-37.5	-34.4	-30.8	-26.7	-22.2	-17.1	-11.7	-5.9	0.0	
STYLE:SH	32.8	35.9	38.9	41.6	44.0	46.0	47.7	49.0	49.7	50.0	
D.LONG	-57.3	-52.5	-47.4	-41.9	-36.0	-29.5	-22.6	-15.3	-7.8	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
0	5	10	15	20	25	30	35	40	45	hh.hh	
90.0	85.8	81.7	77.7	74.0	70.5	67.2	64.3	61.7	59.3	6.00	
-70.7	-74.6	-78.8	-83.1	-87.6	87.8	83.3	79.0	74.8	70.9	7.00	
-53.0	-55.9	-59.2	-62.9	-67.2	-71.8	-76.9	-82.3	-87.8	-86.5	8.00	
-37.5	-39.2	-41.3	-43.8	-46.8	-50.4	-54.6	-59.5	-65.3	-71.8	9.00	
-23.9	-24.6	-25.7	-27.0	-28.6	-30.5	-33.0	-36.1	-40.0	-44.8	10.00	
-17.6	-18.1	-18.7	-19.5	-20.5	-21.8	-23.4	-25.5	-28.1	-31.4	10.50	
-11.6	-11.8	-12.1	-12.6	-13.1	-13.8	-14.7	-15.9	-17.4	-19.3	11.00	
-5.8	-5.8	-5.9	-6.1	-6.3	-6.6	-7.0	-7.5	-8.1	-8.9	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
11.6	11.5	11.4	11.5	11.6	11.8	12.2	12.6	13.2	13.9	13.00	
17.6	17.3	17.1	17.0	17.1	17.3	17.6	18.1	18.7	19.5	13.50	
23.9	23.3	22.9	22.6	22.5	22.6	22.8	23.2	23.8	24.5	14.00	
30.4	29.5	28.8	28.3	27.9	27.8	27.8	28.1	28.5	29.1	14.50	
37.5	36.1	34.9	34.1	33.5	33.0	32.8	32.8	33.0	33.4	15.00	
53.0	50.5	48.4	46.6	45.1	43.9	43.0	42.3	41.8	41.6	16.00	
70.7	67.1	63.9	61.0	58.4	56.1	54.2	52.5	51.1	49.9	17.00	
90.0	85.8	81.7	77.7	74.0	70.5	67.2	64.3	61.7	59.3	18.00	
0.0	5.9	11.7	17.1	22.2	26.7	30.8	34.4	37.5	40.1	SD	
50.0	49.7	49.0	47.7	46.0	44.0	41.6	38.9	35.9	32.8	SH	
0.0	7.8	15.3	22.6	29.5	36.0	41.9	47.4	52.5	57.3	DL	

**A5.1 g**

LATITUDE:	45	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-54.7	-57.3	-60.2	-63.4	-67.1	-71.1	-75.5	-80.1	-85.0	90.0	
7.00	-45.7	-47.0	-48.5	-50.4	-52.5	-55.1	-58.0	-61.3	-65.1	-69.2	
8.00	-37.9	-38.3	-38.8	-39.6	-40.7	-42.0	-43.7	-45.6	-48.0	-50.8	
9.00	-30.4	-30.1	-30.0	-30.1	-30.4	-30.9	-31.6	-32.6	-33.8	-35.3	
10.00	-22.3	-21.7	-21.2	-20.9	-20.7	-20.7	-20.9	-21.1	-21.6	-22.2	
10.50	-17.8	-17.1	-16.5	-16.1	-15.9	-15.7	-15.7	-15.8	-16.0	-16.3	
11.00	-12.7	-12.0	-11.5	-11.2	-10.9	-10.7	-10.6	-10.5	-10.6	-10.7	
11.50	-6.9	-6.4	-6.1	-5.8	-5.6	-5.5	-5.4	-5.3	-5.3	-5.3	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	18.3	16.4	14.9	13.8	12.9	12.2	11.7	11.3	10.9	10.7	
13.50	30.4	26.9	24.2	22.1	20.5	19.2	18.2	17.4	16.8	16.3	
14.00	44.3	39.0	34.9	31.7	29.1	27.0	25.4	24.1	23.0	22.2	
14.50	59.2	52.4	46.9	42.4	38.7	35.7	33.3	31.4	29.8	28.5	
15.00	73.7	66.2	59.7	54.1	49.3	45.4	42.1	39.4	37.1	35.3	
16.00	-82.6	-89.0	84.6	78.3	72.4	67.0	62.1	57.8	54.0	50.8	
17.00	-66.3	-70.6	-75.3	-80.3	-85.5	89.2	83.9	78.7	73.8	69.2	
18.00	-54.7	-57.3	-60.2	-63.4	-67.1	-71.1	-75.5	-80.1	-85.0	90.0	
STYLE:SD	-35.3	-32.7	-29.8	-26.6	-22.9	-18.9	-14.5	-9.9	-5.0	0.0	
STYLE:SH	30.0	32.8	35.4	37.8	39.9	41.6	43.1	44.1	44.8	45.0	
D.LONG	-54.7	-49.9	-44.7	-39.2	-33.4	-27.2	-20.8	-14.0	-7.1	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
0	5	10	15	20	25	30	35	40	45	hh.hh	
90.0	85.0	80.1	75.5	71.1	67.1	63.4	60.2	57.3	54.7	6.00	
-69.2	-73.8	-78.7	-83.9	-89.2	85.5	80.3	75.3	70.6	66.3	7.00	
-50.8	-54.0	-57.8	-62.1	-67.0	-72.4	-78.3	-84.6	89.0	82.6	8.00	
-35.3	-37.1	-39.4	-42.1	-45.4	-49.3	-54.1	-59.7	-66.2	-73.7	9.00	
-22.2	-23.0	-24.1	-25.4	-27.0	-29.1	-31.7	-34.9	-39.0	-44.3	10.00	
-16.3	-16.8	-17.4	-18.2	-19.2	-20.5	-22.1	-24.2	-26.9	-30.4	10.50	
-10.7	-10.9	-11.3	-11.7	-12.2	-12.9	-13.8	-14.9	-16.4	-18.3	11.00	
-5.3	-5.4	-5.5	-5.6	-5.9	-6.1	-6.5	-6.9	-7.5	-8.3	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
10.7	10.6	10.5	10.6	10.7	10.9	11.2	11.5	12.0	12.7	13.00	
16.3	16.0	15.8	15.7	15.7	15.9	16.1	16.5	17.1	17.8	13.50	
22.2	21.6	21.1	20.9	20.7	20.7	20.9	21.2	21.7	22.3	14.00	
28.5	27.5	26.7	26.1	25.7	25.5	25.5	25.6	26.0	26.4	14.50	
35.3	33.8	32.6	31.6	30.9	30.4	30.1	30.0	30.1	30.4	15.00	
50.8	48.0	45.6	43.7	42.0	40.7	39.6	38.8	38.3	37.9	16.00	
69.2	65.1	61.3	58.0	55.1	52.5	50.4	48.5	47.0	45.7	17.00	
90.0	85.0	80.1	75.5	71.1	67.1	63.4	60.2	57.3	54.7	18.00	
0.0	5.0	9.9	14.5	18.9	22.9	26.6	29.8	32.7	35.3	SD	
45.0	44.8	44.1	43.1	41.6	39.9	37.8	35.4	32.8	30.0	SH	
0.0	7.1	14.0	20.8	27.2	33.4	39.2	44.7	49.9	54.7	DL	

**A5.1 h**

LATITUDE:	50	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-49.9	-52.5	-55.6	-59.2	-63.3	-67.8	-72.9	-78.3	-84.1	90.0	
7.00	-41.3	-42.7	-44.3	-46.3	-48.6	-51.4	-54.6	-58.3	-62.6	-67.4	
8.00	-34.1	-34.5	-35.2	-36.1	-37.2	-38.6	-40.4	-42.5	-45.0	-48.1	
9.00	-27.2	-27.1	-27.1	-27.2	-27.6	-28.1	-28.9	-29.9	-31.2	-32.7	
10.00	-20.0	-19.5	-19.1	-18.8	-18.8	-18.8	-19.0	-19.3	-19.7	-20.4	
10.50	-16.0	-15.3	-14.9	-14.6	-14.4	-14.3	-14.3	-14.4	-14.6	-14.9	
11.00	-11.4	-10.9	-10.4	-10.1	-9.8	-9.7	-9.6	-9.6	-9.6	-9.8	
11.50	-6.2	-5.8	-5.5	-5.3	-5.1	-5.0	-4.9	-4.8	-4.8	-4.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	17.0	15.2	13.8	12.7	11.9	11.2	10.7	10.3	10.0	9.8	
13.50	28.9	25.3	22.7	20.6	19.0	17.8	16.8	16.0	15.4	14.9	
14.00	43.3	37.6	33.3	29.9	27.3	25.2	23.6	22.2	21.2	20.4	
14.50	59.4	51.8	45.7	40.8	36.9	33.7	31.2	29.2	27.6	26.3	
15.00	75.6	66.9	59.4	53.1	47.8	43.5	39.9	37.0	34.7	32.7	
16.00	-78.3	-85.5	87.0	79.7	72.8	66.4	60.8	55.9	51.7	48.1	
17.00	-61.3	-65.9	-71.1	-76.8	-82.8	-89.1	84.6	78.5	72.7	67.4	
18.00	-49.9	-52.5	-55.6	-59.2	-63.3	-67.8	-72.9	-78.3	-84.1	90.0	
STYLE:SD	-30.7	-28.3	-25.7	-22.8	-19.5	-16.0	-12.3	-8.3	-4.2	0.0	
STYLE:SH	27.0	29.5	31.8	33.8	35.6	37.2	38.4	39.3	39.8	40.0	
D.LONG	-52.5	-47.6	-42.4	-37.0	-31.3	-25.4	-19.3	-13.0	-6.5	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
0	5	10	15	20	25	30	35	40	45	hh.hh	
90.0	84.1	78.3	72.9	67.8	63.3	59.2	55.6	52.5	49.9	6.00	
-67.4	-72.7	-78.5	-84.6	89.1	82.8	76.8	71.1	65.9	61.3	7.00	
-48.1	-51.7	-55.9	-60.8	-66.4	-72.8	-79.7	-87.0	85.5	78.3	8.00	
-32.7	-34.7	-37.0	-39.9	-43.5	-47.8	-53.1	-59.4	-66.9	-75.6	9.00	
-20.4	-21.2	-22.2	-23.6	-25.2	-27.3	-29.9	-33.3	-37.6	-43.3	10.00	
-14.9	-15.4	-16.0	-16.8	-17.8	-19.0	-20.6	-22.7	-25.3	-28.9	10.50	
-9.8	-10.0	-10.3	-10.7	-11.2	-11.9	-12.7	-13.8	-15.2	-17.0	11.00	
-4.8	-4.9	-5.0	-5.1	-5.3	-5.6	-5.9	-6.3	-6.9	-7.6	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
9.8	9.6	9.6	9.6	9.7	9.8	10.1	10.4	10.9	11.4	13.00	
14.9	14.6	14.4	14.3	14.3	14.4	14.6	14.9	15.3	16.0	13.50	
20.4	19.7	19.3	19.0	18.8	18.8	18.8	19.1	19.5	20.0	14.00	
26.3	25.2	24.4	23.8	23.4	23.1	23.0	23.1	23.3	23.7	14.50	
32.7	31.2	29.9	28.9	28.1	27.6	27.2	27.1	27.1	27.2	15.00	
48.1	45.0	42.5	40.4	38.6	37.2	36.1	35.2	34.5	34.1	16.00	
67.4	62.6	58.3	54.6	51.4	48.6	46.3	44.3	42.7	41.3	17.00	
90.0	84.1	78.3	72.9	67.8	63.3	59.2	55.6	52.5	49.9	18.00	
0.0	4.2	8.3	12.3	16.0	19.5	22.8	25.7	28.3	30.7	SD	
40.0	39.8	39.3	38.4	37.2	35.6	33.8	31.8	29.5	27.0	SH	
0.0	6.5	13.0	19.3	25.4	31.3	37.0	42.4	47.6	52.5	DL	

LATITUDE:	60	This table gives the hour line angles from the vertical.									
TIME	DEC	South xx degrees East ~ wall faces south east								SOUTH	
hh.hh	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	
6.00	-39.2	-41.9	-45.2	-49.1	-53.8	-59.4	-65.9	-73.3	-81.4	90.0	
7.00	-31.9	-33.3	-34.9	-36.9	-39.4	-42.4	-46.0	-50.4	-55.6	-61.8	
8.00	-26.1	-26.6	-27.3	-28.2	-29.3	-30.8	-32.6	-34.8	-37.5	-40.9	
9.00	-20.8	-20.7	-20.8	-21.1	-21.5	-22.0	-22.8	-23.8	-25.0	-26.6	
10.00	-15.2	-14.9	-14.6	-14.5	-14.5	-14.6	-14.8	-15.1	-15.5	-16.1	
10.50	-12.2	-11.7	-11.4	-11.2	-11.1	-11.0	-11.1	-11.2	-11.4	-11.7	
11.00	-8.7	-8.3	-8.0	-7.8	-7.6	-7.5	-7.4	-7.4	-7.5	-7.6	
11.50	-4.8	-4.5	-4.3	-4.1	-3.9	-3.8	-3.8	-3.7	-3.7	-3.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	13.9	12.3	11.0	10.1	9.4	8.9	8.4	8.1	7.8	7.6	
13.50	24.5	21.1	18.7	16.8	15.3	14.2	13.3	12.7	12.1	11.7	
14.00	39.2	33.0	28.5	25.1	22.6	20.6	19.0	17.8	16.9	16.1	
14.50	58.3	48.5	41.2	35.7	31.5	28.3	25.8	23.8	22.2	21.0	
15.00	79.3	67.3	57.2	49.1	42.8	37.8	34.0	30.9	28.5	26.6	
16.00	-67.8	-77.1	-87.3	82.4	72.5	63.8	56.3	50.1	45.0	40.9	
17.00	-49.8	-54.9	-61.0	-68.1	-76.2	-84.9	86.0	77.2	69.0	61.8	
18.00	-39.2	-41.9	-45.2	-49.1	-53.8	-59.4	-65.9	-73.3	-81.4	90.0	
STYLE:SD	-22.2	-20.4	-18.3	-16.1	-13.7	-11.2	-8.5	-5.7	-2.9	0.0	
STYLE:SH	20.7	22.5	24.2	25.7	26.9	28.0	28.9	29.5	29.9	30.0	
D.LONG	-49.1	-44.1	-39.0	-33.7	-28.3	-22.8	-17.2	-11.5	-5.8	0.0	

SOUTH	South xx degrees West ~ wall faces southwest										TIME
	0	5	10	15	20	25	30	35	40	45	hh.hh
90.0	81.4	73.3	65.9	59.4	53.8	49.1	45.2	41.9	39.2	36.0	6.00
-61.8	-69.0	-77.2	-86.0	84.9	76.2	68.1	61.0	54.9	49.8	42.0	7.00
-40.9	-45.0	-50.1	-56.3	-63.8	-72.5	-82.4	87.3	77.1	67.8	58.0	8.00
-26.6	-28.5	-30.9	-34.0	-37.8	-42.8	-49.1	-57.2	-67.3	-79.3	9.00	
-16.1	-16.9	-17.8	-19.0	-20.6	-22.6	-25.1	-28.5	-33.0	-39.2	10.00	
-11.7	-12.1	-12.7	-13.3	-14.2	-15.3	-16.8	-18.7	-21.1	-24.5	10.50	
-7.6	-7.8	-8.1	-8.4	-8.9	-9.4	-10.1	-11.0	-12.3	-13.9	11.00	
-3.8	-3.8	-3.9	-4.0	-4.2	-4.4	-4.7	-5.0	-5.4	-6.0	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
7.6	7.5	7.4	7.4	7.5	7.6	7.8	8.0	8.3	8.7	13.00	
11.7	11.4	11.2	11.1	11.0	11.1	11.2	11.4	11.7	12.2	13.50	
16.1	15.5	15.1	14.8	14.6	14.5	14.5	14.6	14.9	15.2	14.00	
21.0	20.0	19.2	18.6	18.2	17.9	17.8	17.7	17.8	18.1	14.50	
26.6	25.0	23.8	22.8	22.0	21.5	21.1	20.8	20.7	20.8	15.00	
40.9	37.5	34.8	32.6	30.8	29.3	28.2	27.3	26.6	26.1	16.00	
61.8	55.6	50.4	46.0	42.4	39.4	36.9	34.9	33.3	31.9	17.00	
90.0	81.4	73.3	65.9	59.4	53.8	49.1	45.2	41.9	39.2	36.0	18.00
0.0	2.9	5.7	8.5	11.2	13.7	16.1	18.3	20.4	22.2	SD	
30.0	29.9	29.5	28.9	28.0	26.9	25.7	24.2	22.5	20.7	SH	
0.0	5.8	11.5	17.2	22.8	28.3	33.7	39.0	44.1	49.1	DL	

## GREAT DECLINERS – PREDOMINANTLY EAST/WEST

### LATITUDE 30-60° hour line angle tables

(for hour lines for south decliners with declinations of -45 to +45 – see appendix 5.1)

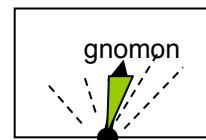
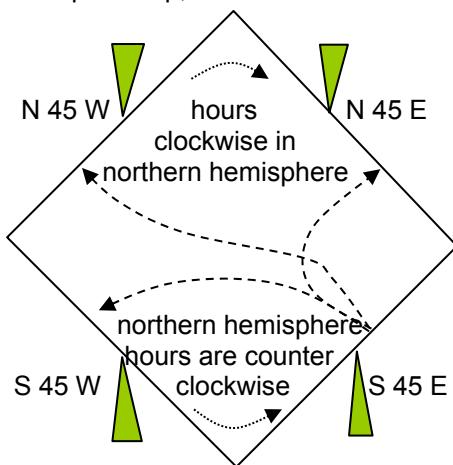
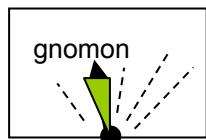
This chart is useful for cases where dial center is not readily accessible.

The following tables use exactly the same formulae as in appendix A5.1, the sole difference is that these declination angles are greater than 45 degrees. Both tables include 45 degrees so they can be cross checked as being the same.

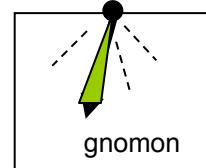
#### VERTICAL DECLINERS

NORTH FACING

Gnomon points up, hour lines radiate from bottom



#### VERTICAL DECLINERS SOUTH FACING



Gnomon points down, hour lines radiate from top

A design for South xx degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. If longitude correction is applied, care must be applied as hour lines shift. The North facing decliner gnomons are inverted, and the vertical is midnight.

Also hours are clockwise when north declining, whereas they are anti clockwise when south declining.

The dial centers of great decliners are not easily accessible, however the techniques to manage this are addressed in the big book.

**A5.2 a**

LATITUDE:	30	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-60.1	-60.4	-60.9	-61.5	-62.4	-63.4	-64.7	-66.1	-67.8	90.0	
7.00	-58.9	-58.1	-57.5	-57.0	-56.8	-56.8	-57.0	-57.3	-57.9	-72.8	
8.00	-57.7	-55.6	-53.9	-52.4	-51.2	-50.2	-49.5	-49.0	-48.7	-56.3	
9.00	-55.9	-52.4	-49.4	-46.8	-44.7	-42.9	-41.4	-40.2	-39.2	-40.9	
10.00	-53.1	-47.5	-42.9	-39.2	-36.2	-33.7	-31.7	-30.1	-28.8	-26.6	
10.50	-50.7	-43.5	-38.0	-33.8	-30.4	-27.8	-25.8	-24.1	-22.8	-19.7	
11.00	-46.4	-37.2	-30.9	-26.4	-23.1	-20.6	-18.8	-17.3	-16.1	-13.1	
11.50	-36.7	-25.6	-19.5	-15.8	-13.3	-11.6	-10.3	-9.3	-8.6	-6.5	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-78.7	79.8	60.9	47.0	37.6	31.1	26.6	23.2	20.8	13.1	
13.50	-71.6	-85.2	80.7	67.7	56.8	48.2	41.6	36.5	32.6	19.7	
14.00	-68.2	-77.5	-87.7	81.9	72.2	63.4	56.0	49.9	44.8	26.6	
14.50	-66.1	-72.9	-80.5	-88.4	83.6	75.8	68.7	62.3	56.7	33.6	
15.00	-64.6	-69.8	-75.5	-81.6	-88.0	85.6	79.3	73.3	67.8	40.9	
16.00	-62.7	-65.6	-68.9	-72.5	-76.4	-80.5	-84.8	-89.2	86.4	56.3	
17.00	-61.3	-62.8	-64.5	-66.4	-68.6	-71.0	-73.5	-76.3	-79.3	72.8	
18.00	-60.1	-60.4	-60.9	-61.5	-62.4	-63.4	-64.7	-66.1	-67.8	90.0	
STYLE:SD	-59.9	-59.6	-59.1	-58.4	-57.5	-56.3	-54.8	-53.0	-50.8	0.0	
STYLE:SH	4.3	8.6	13.0	17.2	21.5	25.7	29.8	33.8	37.8	60.0	
D.LONG	-87.5	-85.0	-82.4	-79.7	-76.9	-73.9	-70.7	-67.2	-63.4	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	67.8	66.1	64.7	63.4	62.4	61.5	60.9	60.4	60.1	6.00	
-72.8	79.3	76.3	73.5	71.0	68.6	66.4	64.5	62.8	61.3	7.00	
-56.3	-86.4	89.2	84.8	80.5	76.4	72.5	68.9	65.6	62.7	8.00	
-40.9	-67.8	-73.3	-79.3	-85.6	88.0	81.6	75.5	69.8	64.6	9.00	
-26.6	-44.8	-49.9	-56.0	-63.4	-72.2	-81.9	87.7	77.5	68.2	10.00	
-19.7	-32.6	-36.5	-41.6	-48.2	-56.8	-67.7	-80.7	85.2	71.6	10.50	
-13.1	-20.8	-23.2	-26.6	-31.1	-37.6	-47.0	-60.9	-79.8	78.7	11.00	
-6.5	-9.8	-10.9	-12.4	-14.4	-17.4	-22.1	-30.3	-46.3	-79.3	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
13.1	16.1	17.3	18.8	20.6	23.1	26.4	30.9	37.2	46.4	13.00	
19.7	22.8	24.1	25.8	27.8	30.4	33.8	38.0	43.5	50.7	13.50	
26.6	28.8	30.1	31.7	33.7	36.2	39.2	42.9	47.5	53.1	14.00	
33.6	34.2	35.4	36.8	38.6	40.8	43.4	46.6	50.3	54.8	14.50	
40.9	39.2	40.2	41.4	42.9	44.7	46.8	49.4	52.4	55.9	15.00	
56.3	48.7	49.0	49.5	50.2	51.2	52.4	53.9	55.6	57.7	16.00	
72.8	57.9	57.3	57.0	56.8	56.8	57.0	57.5	58.1	58.9	17.00	
90.0	67.8	66.1	64.7	63.4	62.4	61.5	60.9	60.4	60.1	18.00	
0.0	50.8	53.0	54.8	56.3	57.5	58.4	59.1	59.6	59.9	SD	
60.0	37.8	33.8	29.8	25.7	21.5	17.2	13.0	8.6	4.3	SH	
0.0	63.4	67.2	70.7	73.9	76.9	79.7	82.4	85.0	87.5	DL	

**A5.2 b**

LATITUDE:	32	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-58.1	-58.4	-58.9	-59.6	-60.5	-61.6	-62.9	-64.4	-66.2	90.0	
7.00	-57.0	-56.2	-55.6	-55.2	-55.0	-55.0	-55.3	-55.7	-56.4	-72.5	
8.00	-55.7	-53.7	-52.1	-50.6	-49.5	-48.6	-47.9	-47.5	-47.3	-55.8	
9.00	-54.0	-50.6	-47.7	-45.3	-43.2	-41.5	-40.1	-39.0	-38.1	-40.3	
10.00	-51.3	-45.9	-41.5	-37.9	-35.0	-32.6	-30.7	-29.2	-27.9	-26.1	
10.50	-49.0	-42.0	-36.7	-32.6	-29.5	-27.0	-25.0	-23.4	-22.2	-19.4	
11.00	-44.8	-35.9	-29.8	-25.5	-22.4	-20.0	-18.2	-16.8	-15.7	-12.8	
11.50	-35.5	-24.7	-18.9	-15.3	-12.9	-11.3	-10.0	-9.1	-8.4	-6.4	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-76.6	81.5	61.8	47.4	37.7	31.1	26.4	23.1	20.5	12.8	
13.50	-69.5	-83.1	82.4	68.9	57.5	48.6	41.7	36.5	32.5	19.4	
14.00	-66.0	-75.4	-85.7	83.6	73.5	64.4	56.6	50.2	44.9	26.1	
14.50	-64.0	-70.8	-78.4	-86.5	85.2	77.2	69.7	63.0	57.2	33.1	
15.00	-62.5	-67.7	-73.4	-79.6	-86.1	87.2	80.7	74.4	68.6	40.3	
16.00	-60.6	-63.6	-66.9	-70.5	-74.4	-78.6	-83.1	-87.6	87.7	55.8	
17.00	-59.2	-60.7	-62.4	-64.4	-66.6	-69.0	-71.7	-74.6	-77.7	72.5	
18.00	-58.1	-58.4	-58.9	-59.6	-60.5	-61.6	-62.9	-64.4	-66.2	90.0	
STYLE:SD	-57.9	-57.6	-57.1	-56.4	-55.4	-54.2	-52.7	-50.8	-48.5	0.0	
STYLE:SH	4.2	8.5	12.7	16.9	21.0	25.1	29.1	33.0	36.8	58.0	
D.LONG	-87.3	-84.7	-81.9	-79.1	-76.1	-73.0	-69.6	-66.0	-62.1	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	66.2	64.4	62.9	61.6	60.5	59.6	58.9	58.4	58.1	6.00	
-72.5	77.7	74.6	71.7	69.0	66.6	64.4	62.4	60.7	59.2	7.00	
-55.8	-87.7	87.6	83.1	78.6	74.4	70.5	66.9	63.6	60.6	8.00	
-40.3	-68.6	-74.4	-80.7	-87.2	86.1	79.6	73.4	67.7	62.5	9.00	
-26.1	-44.9	-50.2	-56.6	-64.4	-73.5	-83.6	85.7	75.4	66.0	10.00	
-19.4	-32.5	-36.5	-41.7	-48.6	-57.5	-68.9	-82.4	83.1	69.5	10.50	
-12.8	-20.5	-23.1	-26.4	-31.1	-37.7	-47.4	-61.8	-81.5	76.6	11.00	
-6.4	-9.6	-10.7	-12.2	-14.3	-17.3	-22.0	-30.3	-46.8	-81.0	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
12.8	15.7	16.8	18.2	20.0	22.4	25.5	29.8	35.9	44.8	13.00	
19.4	22.2	23.4	25.0	27.0	29.5	32.6	36.7	42.0	49.0	13.50	
26.1	27.9	29.2	30.7	32.6	35.0	37.9	41.5	45.9	51.3	14.00	
33.1	33.2	34.3	35.7	37.4	39.4	41.9	45.0	48.6	52.9	14.50	
40.3	38.1	39.0	40.1	41.5	43.2	45.3	47.7	50.6	54.0	15.00	
55.8	47.3	47.5	47.9	48.6	49.5	50.6	52.1	53.7	55.7	16.00	
72.5	56.4	55.7	55.3	55.0	55.0	55.2	55.6	56.2	57.0	17.00	
90.0	66.2	64.4	62.9	61.6	60.5	59.6	58.9	58.4	58.1	18.00	
0.0	48.5	50.8	52.7	54.2	55.4	56.4	57.1	57.6	57.9	SD	
58.0	36.8	33.0	29.1	25.1	21.0	16.9	12.7	8.5	4.2	SH	
0.0	62.1	66.0	69.6	73.0	76.1	79.1	81.9	84.7	87.3	DL	

**A5.2 c**

LATITUDE:	34	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-56.1	-56.4	-56.9	-57.6	-58.6	-59.7	-61.1	-62.7	-64.5	90.0	
7.00	-55.0	-54.2	-53.7	-53.3	-53.2	-53.3	-53.6	-54.1	-54.8	-52.1	
8.00	-53.8	-51.9	-50.2	-48.9	-47.8	-47.0	-46.4	-46.0	-45.9	-55.1	
9.00	-52.2	-48.9	-46.1	-43.7	-41.7	-40.1	-38.8	-37.7	-36.9	-39.7	
10.00	-49.5	-44.2	-40.0	-36.6	-33.8	-31.5	-29.7	-28.3	-27.1	-25.6	
10.50	-47.2	-40.5	-35.4	-31.5	-28.5	-26.1	-24.2	-22.7	-21.5	-19.0	
11.00	-43.2	-34.7	-28.8	-24.7	-21.7	-19.4	-17.7	-16.3	-15.3	-12.5	
11.50	-34.2	-23.9	-18.3	-14.9	-12.6	-11.0	-9.8	-8.9	-8.2	-6.2	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-74.4	83.3	62.8	47.8	37.8	31.0	26.2	22.8	20.3	12.5	
13.50	-67.3	-81.0	84.2	70.1	58.2	48.9	41.8	36.4	32.3	19.0	
14.00	-63.9	-73.2	-83.7	85.4	74.8	65.3	57.1	50.4	45.0	25.6	
14.50	-61.9	-68.6	-76.3	-84.5	87.0	78.6	70.8	63.7	57.6	32.5	
15.00	-60.5	-65.5	-71.3	-77.5	-84.2	88.9	82.1	75.5	69.4	39.7	
16.00	-58.6	-61.5	-64.8	-68.4	-72.4	-76.7	-81.3	-86.0	89.1	55.1	
17.00	-57.2	-58.7	-60.4	-62.4	-64.6	-67.1	-69.8	-72.8	-76.0	72.1	
18.00	-56.1	-56.4	-56.9	-57.6	-58.6	-59.7	-61.1	-62.7	-64.5	90.0	
STYLE:SD	-55.9	-55.6	-55.1	-54.3	-53.3	-52.1	-50.5	-48.6	-46.4	0.0	
STYLE:SH	4.1	8.3	12.4	16.5	20.5	24.5	28.4	32.2	35.9	56.0	
D.LONG	-87.2	-84.4	-81.5	-78.5	-75.4	-72.1	-68.6	-64.9	-60.8	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	64.5	62.7	61.1	59.7	58.6	57.6	56.9	56.4	56.1	6.00	
-72.1	76.0	72.8	69.8	67.1	64.6	62.4	60.4	58.7	57.2	7.00	
-55.1	-89.1	86.0	81.3	76.7	72.4	68.4	64.8	61.5	58.6	8.00	
-39.7	-69.4	-75.5	-82.1	-88.9	84.2	77.5	71.3	65.5	60.5	9.00	
-25.6	-45.0	-50.4	-57.1	-65.3	-74.8	-85.4	83.7	73.2	63.9	10.00	
-19.0	-32.3	-36.4	-41.8	-48.9	-58.2	-70.1	-84.2	81.0	67.3	10.50	
-12.5	-20.3	-22.8	-26.2	-31.0	-37.8	-47.8	-62.8	-83.3	74.4	11.00	
-6.2	-9.5	-10.5	-12.0	-14.0	-17.0	-21.8	-30.2	-47.2	-82.8	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
12.5	15.3	16.3	17.7	19.4	21.7	24.7	28.8	34.7	43.2	13.00	
19.0	21.5	22.7	24.2	26.1	28.5	31.5	35.4	40.5	47.2	13.50	
25.6	27.1	28.3	29.7	31.5	33.8	36.6	40.0	44.2	49.5	14.00	
32.5	32.2	33.2	34.5	36.1	38.1	40.5	43.4	46.9	51.0	14.50	
39.7	36.9	37.7	38.8	40.1	41.7	43.7	46.1	48.9	52.2	15.00	
55.1	45.9	46.0	46.4	47.0	47.8	48.9	50.2	51.9	53.8	16.00	
72.1	54.8	54.1	53.6	53.3	53.2	53.3	53.7	54.2	55.0	17.00	
90.0	64.5	62.7	61.1	59.7	58.6	57.6	56.9	56.4	56.1	18.00	
0.0	46.4	48.6	50.5	52.1	53.3	54.3	55.1	55.6	55.9	SD	
56.0	35.9	32.2	28.4	24.5	20.5	16.5	12.4	8.3	4.1	SH	
0.0	60.8	64.9	68.6	72.1	75.4	78.5	81.5	84.4	87.2	DL	

**A5.2 d**

LATITUDE:	36	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-54.1	-54.4	-54.9	-55.7	-56.6	-57.8	-59.2	-60.9	-62.8	90.0	
7.00	-53.0	-52.3	-51.8	-51.5	-51.4	-51.5	-51.9	-52.4	-53.2	-71.7	
8.00	-51.8	-50.0	-48.4	-47.2	-46.2	-45.4	-44.9	-44.6	-44.5	-54.5	
9.00	-50.3	-47.1	-44.4	-42.1	-40.3	-38.7	-37.5	-36.5	-35.8	-39.0	
10.00	-47.7	-42.6	-38.5	-35.2	-32.6	-30.5	-28.7	-27.4	-26.3	-25.0	
10.50	-45.5	-39.0	-34.2	-30.4	-27.5	-25.2	-23.4	-22.0	-20.9	-18.5	
11.00	-41.6	-33.4	-27.8	-23.9	-21.0	-18.8	-17.1	-15.9	-14.8	-12.2	
11.50	-33.0	-23.1	-17.7	-14.4	-12.2	-10.6	-9.5	-8.6	-8.0	-6.1	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-72.2	85.1	63.8	48.2	37.8	30.8	26.0	22.5	20.0	12.2	
13.50	-65.1	-78.8	86.0	71.3	58.9	49.2	41.8	36.3	32.1	18.5	
14.00	-61.8	-71.0	-81.6	87.2	76.2	66.2	57.7	50.7	45.0	25.0	
14.50	-59.7	-66.5	-74.1	-82.5	88.7	80.0	71.8	64.4	58.0	31.8	
15.00	-58.4	-63.4	-69.1	-75.4	-82.3	-89.4	83.5	76.6	70.2	39.0	
16.00	-56.5	-59.4	-62.7	-66.3	-70.4	-74.8	-79.5	-84.4	-89.5	54.5	
17.00	-55.2	-56.7	-58.4	-60.3	-62.6	-65.1	-67.9	-71.0	-74.4	71.7	
18.00	-54.1	-54.4	-54.9	-55.7	-56.6	-57.8	-59.2	-60.9	-62.8	90.0	
STYLE:SD	-53.9	-53.6	-53.1	-52.3	-51.3	-50.0	-48.4	-46.5	-44.2	0.0	
STYLE:SH	4.0	8.1	12.1	16.1	20.0	23.9	27.6	31.3	34.9	54.0	
D.LONG	-87.1	-84.1	-81.0	-77.9	-74.7	-71.3	-67.6	-63.7	-59.6	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	62.8	60.9	59.2	57.8	56.6	55.7	54.9	54.4	54.1	6.00	
-71.7	74.4	71.0	67.9	65.1	62.6	60.3	58.4	56.7	55.2	7.00	
-54.5	89.5	84.4	79.5	74.8	70.4	66.3	62.7	59.4	56.5	8.00	
-39.0	-70.2	-76.6	-83.5	89.4	82.3	75.4	69.1	63.4	58.4	9.00	
-25.0	-45.0	-50.7	-57.7	-66.2	-76.2	-87.2	81.6	71.0	61.8	10.00	
-18.5	-32.1	-36.3	-41.8	-49.2	-58.9	-71.3	-86.0	78.8	65.1	10.50	
-12.2	-20.0	-22.5	-26.0	-30.8	-37.8	-48.2	-63.8	-85.1	-72.2	11.00	
-6.1	-9.3	-10.3	-11.8	-13.8	-16.8	-21.6	-30.1	-47.5	-84.6	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
12.2	14.8	15.9	17.1	18.8	21.0	23.9	27.8	33.4	41.6	13.00	
18.5	20.9	22.0	23.4	25.2	27.5	30.4	34.2	39.0	45.5	13.50	
25.0	26.3	27.4	28.7	30.5	32.6	35.2	38.5	42.6	47.7	14.00	
31.8	31.2	32.1	33.4	34.9	36.7	39.0	41.8	45.1	49.2	14.50	
39.0	35.8	36.5	37.5	38.7	40.3	42.1	44.4	47.1	50.3	15.00	
54.5	44.5	44.6	44.9	45.4	46.2	47.2	48.4	50.0	51.8	16.00	
71.7	53.2	52.4	51.9	51.5	51.4	51.5	51.8	52.3	53.0	17.00	
90.0	62.8	60.9	59.2	57.8	56.6	55.7	54.9	54.4	54.1	18.00	
0.0	44.2	46.5	48.4	50.0	51.3	52.3	53.1	53.6	53.9	SD	
54.0	34.9	31.3	27.6	23.9	20.0	16.1	12.1	8.1	4.0	SH	
0.0	59.6	63.7	67.6	71.3	74.7	77.9	81.0	84.1	87.1	DL	

LATITUDE:	38	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-52.1	-52.4	-53.0	-53.7	-54.7	-55.9	-57.4	-59.1	-61.1	90.0	
7.00	-51.1	-50.4	-49.9	-49.6	-49.6	-49.7	-50.1	-50.7	-51.6	-71.2	
8.00	-49.9	-48.1	-46.6	-45.4	-44.5	-43.8	-43.3	-43.1	-43.0	-53.8	
9.00	-48.4	-45.3	-42.7	-40.6	-38.8	-37.3	-36.2	-35.3	-34.6	-38.2	
10.00	-45.9	-41.0	-37.1	-33.9	-31.4	-29.4	-27.7	-26.4	-25.4	-24.5	
10.50	-43.7	-37.5	-32.9	-29.3	-26.5	-24.4	-22.6	-21.3	-20.2	-18.1	
11.00	-40.0	-32.1	-26.8	-23.0	-20.3	-18.2	-16.6	-15.4	-14.4	-11.9	
11.50	-31.7	-22.3	-17.1	-13.9	-11.8	-10.3	-9.2	-8.4	-7.7	-5.9	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-69.9	87.0	64.8	48.5	37.7	30.6	25.7	22.2	19.7	11.9	
13.50	-62.9	-76.6	87.8	72.6	59.6	49.5	41.8	36.1	31.8	18.1	
14.00	-59.6	-68.8	-79.5	89.0	77.5	67.1	58.2	50.8	44.9	24.5	
14.50	-57.6	-64.3	-71.9	-80.4	-89.5	81.5	72.8	65.1	58.3	31.2	
15.00	-56.3	-61.2	-66.9	-73.3	-80.3	-87.6	85.0	77.7	71.0	38.2	
16.00	-54.5	-57.3	-60.5	-64.2	-68.3	-72.8	-77.6	-82.7	-88.0	53.8	
17.00	-53.2	-54.6	-56.3	-58.3	-60.6	-63.1	-66.0	-69.2	-72.7	71.2	
18.00	-52.1	-52.4	-53.0	-53.7	-54.7	-55.9	-57.4	-59.1	-61.1	90.0	
STYLE:SD	-51.9	-51.6	-51.0	-50.3	-49.2	-47.9	-46.4	-44.4	-42.1	0.0	
STYLE:SH	3.9	7.9	11.8	15.6	19.5	23.2	26.9	30.4	33.9	52.0	
D.LONG	-86.9	-83.8	-80.6	-77.4	-74.0	-70.4	-66.7	-62.7	-58.4	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	61.1	59.1	57.4	55.9	54.7	53.7	53.0	52.4	52.1	6.00	
-71.2	72.7	69.2	66.0	63.1	60.6	58.3	56.3	54.6	53.2	7.00	
-53.8	88.0	82.7	77.6	72.8	68.3	64.2	60.5	57.3	54.5	8.00	
-38.2	-71.0	-77.7	-85.0	87.6	80.3	73.3	66.9	61.2	56.3	9.00	
-24.5	-44.9	-50.8	-58.2	-67.1	-77.5	-89.0	79.5	68.8	59.6	10.00	
-18.1	-31.8	-36.1	-41.8	-49.5	-59.6	-72.6	-87.8	76.6	62.9	10.50	
-11.9	-19.7	-22.2	-25.7	-30.6	-37.7	-48.5	-64.8	-87.0	69.9	11.00	
-5.9	-9.1	-10.1	-11.6	-13.6	-16.5	-21.3	-29.9	-47.9	-86.5	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
11.9	14.4	15.4	16.6	18.2	20.3	23.0	26.8	32.1	40.0	13.00	
18.1	20.2	21.3	22.6	24.4	26.5	29.3	32.9	37.5	43.7	13.50	
24.5	25.4	26.4	27.7	29.4	31.4	33.9	37.1	41.0	45.9	14.00	
31.2	30.1	31.0	32.2	33.6	35.4	37.6	40.2	43.4	47.3	14.50	
38.2	34.6	35.3	36.2	37.3	38.8	40.6	42.7	45.3	48.4	15.00	
53.8	43.0	43.1	43.3	43.8	44.5	45.4	46.6	48.1	49.9	16.00	
71.2	51.6	50.7	50.1	49.7	49.6	49.6	49.9	50.4	51.1	17.00	
90.0	61.1	59.1	57.4	55.9	54.7	53.7	53.0	52.4	52.1	18.00	
0.0	42.1	44.4	46.4	47.9	49.2	50.3	51.0	51.6	51.9	SD	
52.0	33.9	30.4	26.9	23.2	19.5	15.6	11.8	7.9	3.9	SH	
0.0	58.4	62.7	66.7	70.4	74.0	77.4	80.6	83.8	86.9	DL	

LATITUDE:	40	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-50.1	-50.4	-51.0	-51.7	-52.7	-54.0	-55.5	-57.3	-59.3	90.0	
7.00	-49.1	-48.4	-48.0	-47.8	-47.8	-48.0	-48.4	-49.1	-49.9	-70.7	
8.00	-48.0	-46.3	-44.8	-43.7	-42.8	-42.2	-41.8	-41.6	-41.6	-53.0	
9.00	-46.5	-43.5	-41.0	-39.0	-37.3	-35.9	-34.9	-34.0	-33.4	-37.5	
10.00	-44.1	-39.4	-35.6	-32.6	-30.2	-28.3	-26.7	-25.5	-24.5	-23.9	
10.50	-42.0	-36.1	-31.6	-28.2	-25.5	-23.5	-21.8	-20.5	-19.5	-17.6	
11.00	-38.4	-30.9	-25.8	-22.2	-19.5	-17.5	-16.0	-14.8	-13.9	-11.6	
11.50	-30.5	-21.4	-16.5	-13.5	-11.4	-10.0	-8.9	-8.1	-7.5	-5.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-67.6	88.9	65.8	48.7	37.6	30.3	25.4	21.9	19.3	11.6	
13.50	-60.7	-74.4	89.7	73.9	60.3	49.7	41.8	35.9	31.4	17.6	
14.00	-57.4	-66.6	-77.3	-89.1	79.0	68.0	58.6	51.0	44.8	23.9	
14.50	-55.5	-62.0	-69.7	-78.3	-87.6	82.9	73.9	65.7	58.6	30.4	
15.00	-54.2	-59.1	-64.7	-71.1	-78.2	-85.8	86.5	78.9	71.8	37.5	
16.00	-52.4	-55.2	-58.4	-62.0	-66.2	-70.7	-75.7	-81.0	-86.5	53.0	
17.00	-51.2	-52.6	-54.2	-56.2	-58.5	-61.1	-64.0	-67.3	-70.9	70.7	
18.00	-50.1	-50.4	-51.0	-51.7	-52.7	-54.0	-55.5	-57.3	-59.3	90.0	
STYLE:SD	-49.9	-49.6	-49.0	-48.2	-47.2	-45.9	-44.3	-42.4	-40.1	0.0	
STYLE:SH	3.8	7.6	11.4	15.2	18.9	22.5	26.1	29.5	32.8	50.0	
D.LONG	-86.8	-83.5	-80.2	-76.8	-73.3	-69.6	-65.8	-61.7	-57.3	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	59.3	57.3	55.5	54.0	52.7	51.7	51.0	50.4	50.1	6.00	
-70.7	70.9	67.3	64.0	61.1	58.5	56.2	54.2	52.6	51.2	7.00	
-53.0	86.5	81.0	75.7	70.7	66.2	62.0	58.4	55.2	52.4	8.00	
-37.5	-71.8	-78.9	-86.5	-85.8	-78.2	-71.1	-64.7	-59.1	-54.2	9.00	
-23.9	-44.8	-51.0	-58.6	-68.0	-79.0	89.1	77.3	66.6	57.4	10.00	
-17.6	-31.4	-35.9	-41.8	-49.7	-60.3	-73.9	-89.7	74.4	60.7	10.50	
-11.6	-19.3	-21.9	-25.4	-30.3	-37.6	-48.7	-65.8	-88.9	67.6	11.00	
-5.8	-8.9	-9.9	-11.3	-13.3	-16.3	-21.0	-29.7	-48.2	-88.4	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
11.6	13.9	14.8	16.0	17.5	19.5	22.2	25.8	30.9	38.4	13.00	
17.6	19.5	20.5	21.8	23.5	25.5	28.2	31.6	36.1	42.0	13.50	
23.9	24.5	25.5	26.7	28.3	30.2	32.6	35.6	39.4	44.1	14.00	
30.4	29.1	29.9	31.0	32.4	34.1	36.1	38.6	41.7	45.5	14.50	
37.5	33.4	34.0	34.9	35.9	37.3	39.0	41.0	43.5	46.5	15.00	
53.0	41.6	41.6	41.8	42.2	42.8	43.7	44.8	46.3	48.0	16.00	
70.7	49.9	49.1	48.4	48.0	47.8	47.8	48.0	48.4	49.1	17.00	
90.0	59.3	57.3	55.5	54.0	52.7	51.7	51.0	50.4	50.1	18.00	
0.0	40.1	42.4	44.3	45.9	47.2	48.2	49.0	49.6	49.9	SD	
50.0	32.8	29.5	26.1	22.5	18.9	15.2	11.4	7.6	3.8	SH	
0.0	57.3	61.7	65.8	69.6	73.3	76.8	80.2	83.5	86.8	DL	

**A5.2 g**

LATITUDE:	45	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-45.1	-45.4	-46.0	-46.8	-47.8	-49.1	-50.7	-52.5	-54.7	90.0	
7.00	-44.2	-43.6	-43.2	-43.1	-43.2	-43.5	-44.0	-44.7	-45.7	-69.2	
8.00	-43.1	-41.6	-40.3	-39.4	-38.6	-38.1	-37.8	-37.8	-37.9	-50.8	
9.00	-41.8	-39.1	-36.9	-35.1	-33.6	-32.4	-31.5	-30.8	-30.4	-35.3	
10.00	-39.6	-35.3	-32.0	-29.4	-27.3	-25.6	-24.2	-23.1	-22.3	-22.2	
10.50	-37.7	-32.4	-28.4	-25.4	-23.1	-21.2	-19.8	-18.7	-17.8	-16.3	
11.00	-34.5	-27.7	-23.2	-20.0	-17.7	-15.9	-14.6	-13.5	-12.7	-10.7	
11.50	-27.4	-19.3	-14.9	-12.2	-10.4	-9.1	-8.2	-7.4	-6.9	-5.3	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-61.8	-86.1	68.2	49.1	37.1	29.4	24.4	20.8	18.3	10.7	
13.50	-55.1	-68.6	-85.3	77.2	61.8	49.9	41.3	35.0	30.4	16.3	
14.00	-51.9	-60.8	-71.6	-84.2	82.7	70.3	59.6	51.0	44.3	22.2	
14.50	-50.1	-56.4	-63.9	-72.8	-82.7	86.8	76.6	67.3	59.2	28.5	
15.00	-48.9	-53.5	-59.0	-65.5	-72.8	-81.0	-89.5	81.9	73.7	35.3	
16.00	-47.2	-49.9	-53.0	-56.6	-60.7	-65.4	-70.7	-76.4	-82.6	50.8	
17.00	-46.1	-47.4	-49.0	-51.0	-53.3	-55.9	-59.0	-62.4	-66.3	69.2	
18.00	-45.1	-45.4	-46.0	-46.8	-47.8	-49.1	-50.7	-52.5	-54.7	90.0	
STYLE:SD	-44.9	-44.6	-44.0	-43.2	-42.2	-40.9	-39.3	-37.5	-35.3	0.0	
STYLE:SH	3.5	7.1	10.5	14.0	17.4	20.7	23.9	27.0	30.0	45.0	
D.LONG	-86.5	-82.9	-79.3	-75.6	-71.8	-67.8	-63.7	-59.3	-54.7	0.0	

PURE S	0	45	50	55	60	65	70	75	80	85	TIME hh.hh
90.0	54.7	52.5	50.7	49.1	47.8	46.8	46.0	45.4	45.1	6.00	
-69.2	66.3	62.4	59.0	55.9	53.3	51.0	49.0	47.4	46.1	7.00	
-50.8	82.6	76.4	70.7	65.4	60.7	56.6	53.0	49.9	47.2	8.00	
-35.3	-73.7	-81.9	89.5	81.0	72.8	65.5	59.0	53.5	48.9	9.00	
-22.2	-44.3	-51.0	-59.6	-70.3	-82.7	84.2	71.6	60.8	51.9	10.00	
-16.3	-30.4	-35.0	-41.3	-49.9	-61.8	-77.2	85.3	68.6	55.1	10.50	
-10.7	-18.3	-20.8	-24.4	-29.4	-37.1	-49.1	-68.2	86.1	61.8	11.00	
-5.3	-8.3	-9.3	-10.6	-12.5	-15.4	-20.1	-28.9	-48.6	86.6	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
10.7	12.7	13.5	14.6	15.9	17.7	20.0	23.2	27.7	34.5	13.00	
16.3	17.8	18.7	19.8	21.2	23.1	25.4	28.4	32.4	37.7	13.50	
22.2	22.3	23.1	24.2	25.6	27.3	29.4	32.0	35.3	39.6	14.00	
28.5	26.4	27.1	28.1	29.2	30.7	32.5	34.7	37.5	40.8	14.50	
35.3	30.4	30.8	31.5	32.4	33.6	35.1	36.9	39.1	41.8	15.00	
50.8	37.9	37.8	37.8	38.1	38.6	39.4	40.3	41.6	43.1	16.00	
69.2	45.7	44.7	44.0	43.5	43.2	43.1	43.2	43.6	44.2	17.00	
90.0	54.7	52.5	50.7	49.1	47.8	46.8	46.0	45.4	45.1	18.00	
0.0	35.3	37.5	39.3	40.9	42.2	43.2	44.0	44.6	44.9	SD	
45.0	30.0	27.0	23.9	20.7	17.4	14.0	10.5	7.1	3.5	SH	
0.0	54.7	59.3	63.7	67.8	71.8	75.6	79.3	82.9	86.5	DL	

**A5.2 h**

LATITUDE:	50	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-40.1	-40.4	-41.0	-41.8	-42.8	-44.1	-45.7	-47.6	-49.9	90.0	
7.00	-39.3	-38.7	-38.5	-38.4	-38.5	-38.9	-39.4	-40.3	-41.3	-67.4	
8.00	-38.3	-36.9	-35.9	-35.0	-34.4	-34.0	-33.8	-33.9	-34.1	-48.1	
9.00	-37.1	-34.7	-32.8	-31.2	-29.9	-28.9	-28.2	-27.6	-27.2	-32.7	
10.00	-35.1	-31.3	-28.4	-26.1	-24.3	-22.8	-21.6	-20.7	-20.0	-20.4	
10.50	-33.4	-28.7	-25.2	-22.6	-20.6	-19.0	-17.7	-16.7	-16.0	-14.9	
11.00	-30.6	-24.6	-20.6	-17.8	-15.8	-14.3	-13.1	-12.1	-11.4	-9.8	
11.50	-24.3	-17.2	-13.4	-11.0	-9.3	-8.2	-7.3	-6.7	-6.2	-4.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	-55.7	-80.6	70.6	49.1	36.1	28.1	23.0	19.5	17.0	9.8	
13.50	-49.3	-62.5	-79.8	80.6	63.1	49.8	40.3	33.7	28.9	14.9	
14.00	-46.4	-54.8	-65.6	-78.8	86.6	72.5	60.3	50.7	43.3	20.4	
14.50	-44.7	-50.6	-57.9	-66.9	-77.4	-88.9	79.4	68.7	59.4	26.3	
15.00	-43.6	-47.9	-53.2	-59.6	-67.1	-75.7	-85.2	85.0	75.6	32.7	
16.00	-42.0	-44.5	-47.4	-50.9	-55.0	-59.8	-65.2	-71.4	-78.3	48.1	
17.00	-41.0	-42.2	-43.8	-45.7	-47.9	-50.5	-53.6	-57.2	-61.3	67.4	
18.00	-40.1	-40.4	-41.0	-41.8	-42.8	-44.1	-45.7	-47.6	-49.9	90.0	
STYLE:SD	-39.9	-39.6	-39.0	-38.3	-37.3	-36.0	-34.5	-32.7	-30.7	0.0	
STYLE:SH	3.2	6.4	9.6	12.7	15.8	18.7	21.6	24.4	27.0	40.0	
D.LONG	-86.2	-82.3	-78.4	-74.4	-70.3	-66.1	-61.8	-57.3	-52.5	0.0	

PURE S	0	45	50	55	60	65	70	75	80	85	TIME hh.hh
90.0	49.9	47.6	45.7	44.1	42.8	41.8	41.0	40.4	40.1	6.00	
-67.4	61.3	57.2	53.6	50.5	47.9	45.7	43.8	42.2	41.0	7.00	
-48.1	78.3	71.4	65.2	59.8	55.0	50.9	47.4	44.5	42.0	8.00	
-32.7	-75.6	-85.0	85.2	75.7	67.1	59.6	53.2	47.9	43.6	9.00	
-20.4	-43.3	-50.7	-60.3	-72.5	-86.6	78.8	65.6	54.8	46.4	10.00	
-14.9	-28.9	-33.7	-40.3	-49.8	-63.1	-80.6	79.8	62.5	49.3	10.50	
-9.8	-17.0	-19.5	-23.0	-28.1	-36.1	-49.1	-70.6	80.6	55.7	11.00	
-4.8	-7.6	-8.5	-9.8	-11.6	-14.3	-18.9	-27.7	-48.7	81.1	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
9.8	11.4	12.1	13.1	14.3	15.8	17.8	20.6	24.6	30.6	13.00	
14.9	16.0	16.7	17.7	19.0	20.6	22.6	25.2	28.7	33.4	13.50	
20.4	20.0	20.7	21.6	22.8	24.3	26.1	28.4	31.3	35.1	14.00	
26.3	23.7	24.3	25.1	26.0	27.3	28.9	30.8	33.2	36.2	14.50	
32.7	27.2	27.6	28.2	28.9	29.9	31.2	32.8	34.7	37.1	15.00	
48.1	34.1	33.9	33.8	34.0	34.4	35.0	35.9	36.9	38.3	16.00	
67.4	41.3	40.3	39.4	38.9	38.5	38.4	38.5	38.7	39.3	17.00	
90.0	49.9	47.6	45.7	44.1	42.8	41.8	41.0	40.4	40.1	18.00	
0.0	30.7	32.7	34.5	36.0	37.3	38.3	39.0	39.6	39.9	SD	
40.0	27.0	24.4	21.6	18.7	15.8	12.7	9.6	6.4	3.2	SH	
0.0	52.5	57.3	61.8	66.1	70.3	74.4	78.4	82.3	86.2	DL	

**A5.2 i**

LATITUDE:	60	This table gives the hour line angles from the vertical.									
TIME hh.mm	DEC		South xx degrees East ~ wall faces south east								PURE S
	-85	-80	-75	-70	-65	-60	-55	-50	-45	0	
6.00	-30.1	-30.4	-30.9	-31.6	-32.5	-33.7	-35.2	-37.0	-39.2	90.0	
7.00	-29.4	-29.1	-28.9	-28.9	-29.1	-29.5	-30.1	-30.9	-31.9	-61.8	
8.00	-28.7	-27.7	-26.9	-26.3	-25.9	-25.7	-25.7	-25.8	-26.1	-40.9	
9.00	-27.8	-26.0	-24.5	-23.4	-22.5	-21.8	-21.3	-20.9	-20.8	-26.6	
10.00	-26.3	-23.4	-21.3	-19.6	-18.2	-17.2	-16.4	-15.7	-15.2	-16.1	
10.50	-25.0	-21.5	-18.9	-17.0	-15.5	-14.3	-13.4	-12.7	-12.2	-11.7	
11.00	-22.8	-18.4	-15.5	-13.5	-12.0	-10.8	-10.0	-9.3	-8.7	-7.6	
11.50	-18.2	-13.0	-10.1	-8.3	-7.1	-6.3	-5.6	-5.2	-4.8	-3.8	
12.00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
13.00	42.9	67.7	75.5	47.2	32.3	24.1	19.3	16.1	13.9	7.6	
13.50	37.5	49.1	67.1	88.6	64.8	47.6	36.5	29.4	24.5	11.7	
14.00	35.1	42.2	52.2	66.1	84.0	76.9	60.4	48.0	39.2	16.1	
14.50	33.7	38.6	45.0	53.6	64.9	78.9	85.6	70.8	58.3	21.0	
15.00	32.8	36.4	40.9	46.7	54.1	63.4	74.8	87.6	79.3	26.6	
16.00	31.6	33.6	36.0	39.1	42.8	47.3	52.9	59.7	67.8	40.9	
17.00	30.8	31.8	33.1	34.7	36.7	39.1	42.0	45.5	49.8	61.8	
18.00	30.1	30.4	30.9	31.6	32.5	33.7	35.2	37.0	39.2	90.0	
STYLE:SD	-29.9	-29.6	-29.1	-28.5	-27.6	-26.6	-25.3	-23.9	-22.2	0.0	
STYLE:SH	2.5	5.0	7.4	9.8	12.2	14.5	16.7	18.7	20.7	30.0	
D.LONG	-85.7	-81.3	-76.9	-72.5	-68.0	-63.4	-58.8	-54.0	-49.1	0.0	

PURE S	South xx degrees West ~ wall faces southwest										TIME hh.hh
0	45	50	55	60	65	70	75	80	85		
90.0	39.2	37.0	35.2	33.7	32.5	31.6	30.9	30.4	30.1	6.00	
-61.8	49.8	45.5	42.0	39.1	36.7	34.7	33.1	31.8	30.8	7.00	
-40.9	67.8	59.7	52.9	47.3	42.8	39.1	36.0	33.6	31.6	8.00	
-26.6	79.3	87.6	74.8	63.4	54.1	46.7	40.9	36.4	32.8	9.00	
-16.1	-39.2	-48.0	-60.4	-76.9	84.0	66.1	52.2	42.2	35.1	10.00	
-11.7	-24.5	-29.4	-36.5	-47.6	-64.8	-88.6	67.1	49.1	37.5	10.50	
-7.6	-13.9	-16.1	-19.3	-24.1	-32.3	-47.2	-75.5	67.7	42.9	11.00	
-3.8	-6.0	-6.8	-7.8	-9.3	-11.6	-15.7	-23.9	-47.0	68.1	11.50	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	12.00	
7.6	8.7	9.3	10.0	10.8	12.0	13.5	15.5	18.4	22.8	13.00	
11.7	12.2	12.7	13.4	14.3	15.5	17.0	18.9	21.5	25.0	13.50	
16.1	15.2	15.7	16.4	17.2	18.2	19.6	21.3	23.4	26.3	14.00	
21.0	18.1	18.4	18.9	19.6	20.5	21.7	23.1	24.9	27.1	14.50	
26.6	20.8	20.9	21.3	21.8	22.5	23.4	24.5	26.0	27.8	15.00	
40.9	26.1	25.8	25.7	25.7	25.9	26.3	26.9	27.7	28.7	16.00	
61.8	31.9	30.9	30.1	29.5	29.1	28.9	28.9	29.1	29.4	17.00	
90.0	39.2	37.0	35.2	33.7	32.5	31.6	30.9	30.4	30.1	18.00	
0.0	22.2	23.9	25.3	26.6	27.6	28.5	29.1	29.6	29.9	SD	
30.0	20.7	18.7	16.7	14.5	12.2	9.8	7.4	5.0	2.5	SH	
0.0	49.1	54.0	58.8	63.4	68.0	72.5	76.9	81.3	85.7	DL	

SD      Style Distance – This angle is the gnomon offset from the vertical ~ negative angle is towards the west ~ positive angle is towards the east

SH      Style Height - This is the angle between the style and the sub-style ~ remember the gnomon is offset from the vertical by the angle above.

Positive angles are east of the vertical  
Negative angles are west of the vertical

If an angle in the morning shows positive before going negative, or the other way around, and similarly for the afternoon, then these are angles for which the gnomon shadow may be meaningless. In which case, the cell is shaded as is the font.

Hour angle:

$$z = \text{atan}(\cos(\text{lat}) / (\cos(d) \cot(\text{ha}) + \sin(d) \sin(\text{lat})) \quad \text{angle is from vertical noon line}$$

another variant is derived in appendix 7 which produces the same results and uses an extension of a horizontal dial, where the hour lines are then extended to meet the declining vertical dial.

Style Distance:

$$sd = \text{atan}(\sin(\text{dec}) / \tan(\text{lat})) \quad \text{angle is from vertical noon}$$

Style Height:

$$sh = 90 - \text{asin}(\cos(\text{lat}) * \cos(\text{dec})) \quad \text{angle between style and sub-style (wall)}$$

**NOTE:** An angle, **DL**, is derived when vertical decliners are designed. This enables a surrogate horizontal dial designed for latitude **SH** and longitude **DL**, with complete calendar and analemma data, to be employed as a surrogate dial for the original vertical decliner. This is most meaningful when the vertical decliner's longitude is considered, thus it is less meaningful for generic tables, which is why it is not presented in the tables here. See the big book for details.

$$DL = -(\text{Atn}(\text{Tan}(\text{dec})/\text{Sin}(\text{lat}))) - (\text{Ing} - \text{ref})$$

DL in essence is a longitude displaced from longitude 0, and when used with SH as a latitude, for a horizontal dial, then the hour lines and thus analemmas, match the vertical decliner's hour lines. Illustrating Times Shadow discusses this process, and discusses the need for DL to include both a raw generic DL for a vertical decliner as well as the design longitude for the final dial. The big book gives an example of a vertical decliner with analemmas on each hour line.

NOTE: Other software may have different signs for east and west, and so on.

## FOR CROSS CHECKING V-DEC DIAL "DL" VALUES

A5.3

VERTICAL DECLINER "DL" difference in longitude for latitudes 30 to 60, declinations 1 to 80

A5.3	LATITUDE							
DECL	30	32	34	36	38	40	42	44
1	2.00	1.89	1.79	1.70	1.62	1.56	1.49	1.44
2	4.00	3.77	3.57	3.40	3.25	3.11	2.99	2.88
4	7.96	7.52	7.13	6.78	6.48	6.21	5.97	5.75
6	11.87	11.22	10.64	10.14	9.69	9.29	8.93	8.60
8	15.70	14.85	14.11	13.45	12.86	12.33	11.86	11.44
10	19.43	18.40	17.50	16.70	15.98	15.34	14.76	14.24
15	28.19	26.82	25.60	24.51	23.52	22.63	21.82	21.09
20	36.05	34.48	33.06	31.77	30.59	29.52	28.54	27.65
25	43.00	41.35	39.82	38.43	37.14	35.96	34.87	33.87
30	49.11	47.45	45.92	44.49	43.16	41.93	40.79	39.73
35	54.47	52.88	51.39	49.99	48.68	47.45	46.30	45.23
40	59.21	57.73	56.32	54.99	53.73	52.55	51.43	50.38
45	63.43	62.08	60.79	59.55	58.38	57.27	56.21	55.21
50	67.24	66.03	64.86	63.75	62.68	61.66	60.69	59.76
55	70.70	69.64	68.62	67.63	66.68	65.77	64.90	64.06
60	73.90	72.99	72.11	71.25	70.43	69.64	68.88	68.15
65	76.88	76.12	75.39	74.67	73.98	73.31	72.67	72.05
70	79.69	79.08	78.50	77.92	77.37	76.83	76.31	75.81
75	82.37	81.92	81.48	81.05	80.63	80.23	79.84	79.46
80	84.96	84.66	84.37	84.08	83.80	83.53	83.27	83.02

	46	48	50	52	54	56	58	60
1	1.39	1.35	1.31	1.27	1.24	1.21	1.18	1.15
2	2.78	2.69	2.61	2.54	2.47	2.41	2.36	2.31
4	5.55	5.38	5.22	5.07	4.94	4.82	4.71	4.62
6	8.31	8.05	7.81	7.60	7.40	7.23	7.07	6.92
8	11.05	10.71	10.40	10.11	9.85	9.62	9.41	9.22
10	13.77	13.35	12.96	12.61	12.30	12.01	11.75	11.51
15	20.43	19.83	19.28	18.78	18.33	17.91	17.53	17.19
20	26.84	26.09	25.41	24.79	24.22	23.70	23.23	22.80
25	32.95	32.11	31.33	30.62	29.96	29.36	28.80	28.30
30	38.75	37.84	37.00	36.23	35.51	34.85	34.25	33.69
35	44.23	43.30	42.43	41.62	40.88	40.18	39.55	38.96
40	49.39	48.47	47.61	46.80	46.05	45.35	44.70	44.10
45	54.27	53.38	52.55	51.76	51.03	50.34	49.70	49.11
50	58.88	58.05	57.27	56.53	55.83	55.18	54.56	53.99
55	63.27	62.51	61.79	61.11	60.47	59.86	59.30	58.77
60	67.45	66.78	66.14	65.54	64.96	64.42	63.91	63.43
65	71.46	70.89	70.34	69.82	69.33	68.86	68.42	68.01
70	75.33	74.86	74.42	74.00	73.59	73.21	72.85	72.50
75	79.09	78.74	78.40	78.08	77.77	77.48	77.20	76.94
80	82.77	82.53	82.31	82.09	81.88	81.68	81.50	81.32

This table is an aid to cross check more detailed work. Modify this with the vertical decliner's longitude difference from the legal meridian. Chapter 24 discusses this.

## FOR CROSS CHECKING V-DEC DIAL "SD" VALUES

A5.4

VERTICAL DECLINER "SD" style distance offset for latitudes 30 to 60, declinations 1 to 80

A5.4	LATITUDE							
DECL	30	32	34	36	38	40	42	44
1	1.73	1.60	1.48	1.38	1.28	1.19	1.11	1.04
2	3.46	3.20	2.96	2.75	2.56	2.38	2.22	2.07
4	6.89	6.37	5.90	5.48	5.10	4.75	4.43	4.13
6	10.26	9.50	8.81	8.19	7.62	7.10	6.62	6.18
8	13.55	12.56	11.66	10.84	10.10	9.42	8.79	8.20
10	16.74	15.53	14.44	13.44	12.53	11.69	10.92	10.19
15	24.15	22.50	20.99	19.61	18.33	17.14	16.04	15.00
20	30.64	28.69	26.89	25.21	23.64	22.18	20.80	19.50
25	36.20	34.07	32.07	30.19	28.41	26.73	25.14	23.64
30	40.89	38.67	36.55	34.54	32.62	30.79	29.04	27.37
35	44.81	42.55	40.38	38.29	36.28	34.36	32.50	30.71
40	48.07	45.81	43.62	41.50	39.45	37.45	35.52	33.65
45	50.77	48.53	46.35	44.22	42.15	40.12	38.14	36.21
50	53.00	50.80	48.64	46.52	44.44	42.39	40.39	38.42
55	54.82	52.66	50.53	48.43	46.36	44.31	42.29	40.31
60	56.31	54.19	52.09	50.01	47.94	45.90	43.89	41.89
65	57.50	55.42	53.34	51.28	49.24	47.21	45.19	43.18
70	58.43	56.38	54.33	52.29	50.26	48.24	46.22	44.22
75	59.13	57.10	55.07	53.05	51.03	49.02	47.01	45.01
80	59.62	57.60	55.59	53.58	51.57	49.57	47.56	45.56

	46	48	50	52	54	56	58	60
1	0.97	0.90	0.84	0.78	0.73	0.67	0.62	0.58
2	1.93	1.80	1.68	1.56	1.45	1.35	1.25	1.15
4	3.85	3.59	3.35	3.12	2.90	2.69	2.50	2.31
6	5.76	5.38	5.01	4.67	4.34	4.03	3.74	3.45
8	7.65	7.14	6.66	6.21	5.77	5.36	4.97	4.59
10	9.52	8.89	8.29	7.73	7.19	6.68	6.19	5.73
15	14.03	13.12	12.25	11.43	10.65	9.90	9.19	8.50
20	18.28	17.12	16.01	14.96	13.95	12.99	12.06	11.17
25	22.20	20.83	19.53	18.27	17.07	15.91	14.79	13.71
30	25.77	24.24	22.76	21.34	19.96	18.64	17.35	16.10
35	28.98	27.31	25.70	24.14	22.62	21.15	19.72	18.32
40	31.83	30.06	28.34	26.67	25.03	23.44	21.88	20.36
45	34.33	32.48	30.68	28.92	27.19	25.50	23.84	22.21
50	36.49	34.60	32.73	30.90	29.10	27.33	25.58	23.86
55	38.35	36.41	34.50	32.62	30.76	28.92	27.11	25.31
60	39.91	37.95	36.01	34.08	32.18	30.29	28.42	26.57
65	41.19	39.22	37.25	35.30	33.36	31.44	29.52	27.62
70	42.22	40.23	38.26	36.28	34.32	32.37	30.42	28.48
75	43.01	41.01	39.03	37.04	35.06	33.09	31.11	29.15
80	43.56	41.56	39.57	37.58	35.58	33.59	31.61	29.62

This table is an aid to cross check more detailed work.

## FOR CROSS CHECKING V-DEC DIAL "SH" VALUES

A5.5

VERTICAL DECLINER "SH" style height for latitudes 30 to 60, declinations 1 to 80

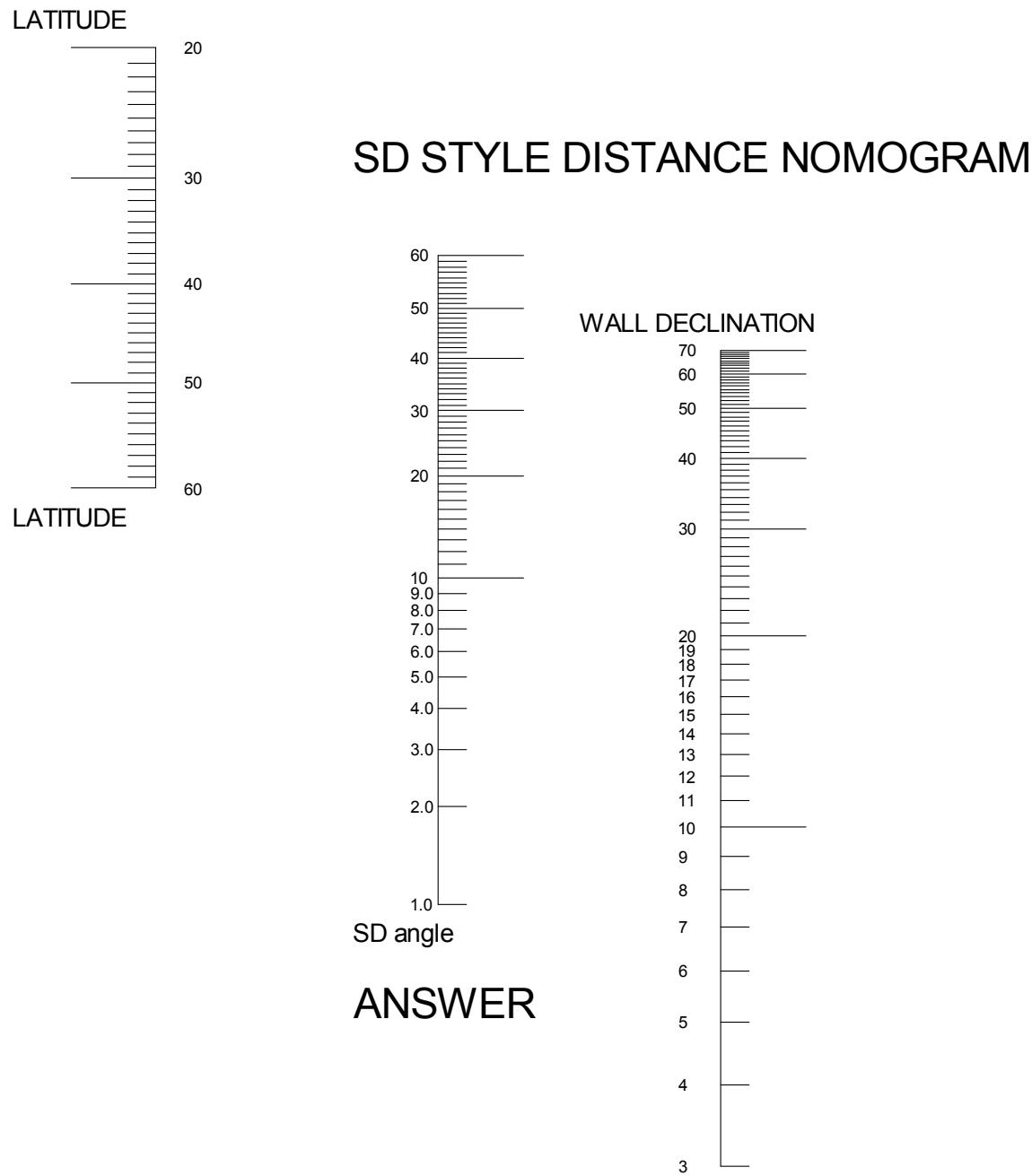
A5.5	LATITUDE							
DECL	30	32	34	36	38	40	42	44
1	59.98	57.99	55.99	53.99	51.99	49.99	47.99	45.99
2	59.94	57.94	55.95	53.95	51.96	49.96	47.96	45.96
4	59.76	57.78	55.79	53.81	51.82	49.83	47.85	45.86
6	59.46	57.50	55.54	53.57	51.60	49.63	47.65	45.68
8	59.05	57.12	55.18	53.24	51.29	49.34	47.38	45.43
10	58.53	56.63	54.73	52.82	50.90	48.97	47.04	45.11
15	56.77	55.00	53.21	51.39	49.57	47.73	45.88	44.01
20	54.47	52.84	51.17	49.48	47.77	46.04	44.29	42.53
25	51.71	50.23	48.71	47.16	45.58	43.97	42.34	40.69
30	48.59	47.26	45.89	44.48	43.03	41.56	40.06	38.53
35	45.19	44.00	42.77	41.51	40.20	38.87	37.50	36.10
40	41.56	40.51	39.43	38.30	37.13	35.93	34.70	33.44
45	37.76	36.85	35.89	34.89	33.86	32.80	31.70	30.57
50	33.83	33.03	32.20	31.33	30.43	29.50	28.53	27.54
55	29.78	29.11	28.39	27.65	26.87	26.06	25.23	24.37
60	25.66	25.09	24.49	23.86	23.20	22.52	21.81	21.08
65	21.47	21.00	20.51	19.99	19.45	18.89	18.30	17.70
70	17.23	16.86	16.47	16.06	15.64	15.19	14.72	14.24
75	12.95	12.68	12.39	12.09	11.77	11.44	11.09	10.73
80	8.65	8.47	8.28	8.08	7.86	7.64	7.41	7.18

	46	48	50	52	54	56	58	60
1	43.99	41.99	39.99	37.99	35.99	33.99	31.99	29.99
2	43.97	41.97	39.97	37.97	35.97	33.98	31.98	29.98
4	43.87	41.87	39.88	37.89	35.90	33.91	31.91	29.92
6	43.70	41.72	39.74	37.76	35.77	33.79	31.80	29.82
8	43.46	41.50	39.53	37.57	35.60	33.62	31.65	29.68
10	43.17	41.22	39.27	37.32	35.37	33.41	31.46	29.50
15	42.14	40.27	38.38	36.49	34.59	32.69	30.79	28.88
20	40.75	38.96	37.16	35.35	33.53	31.70	29.87	28.02
25	39.02	37.33	35.63	33.92	32.19	30.45	28.70	26.95
30	36.98	35.41	33.83	32.22	30.60	28.97	27.32	25.66
35	34.68	33.24	31.77	30.29	28.78	27.26	25.73	24.18
40	32.15	30.84	29.50	28.14	26.76	25.36	23.95	22.52
45	29.42	28.24	27.03	25.81	24.56	23.29	22.01	20.70
50	26.52	25.47	24.40	23.31	22.20	21.07	19.91	18.75
55	23.48	22.57	21.63	20.68	19.70	18.71	17.69	16.67
60	20.32	19.55	18.75	17.93	17.09	16.24	15.36	14.48
65	17.07	16.43	15.76	15.08	14.38	13.67	12.94	12.20
70	13.74	13.23	12.70	12.16	11.60	11.03	10.44	9.85
75	10.36	9.97	9.58	9.17	8.75	8.32	7.88	7.44
80	6.93	6.67	6.41	6.14	5.86	5.57	5.28	4.98

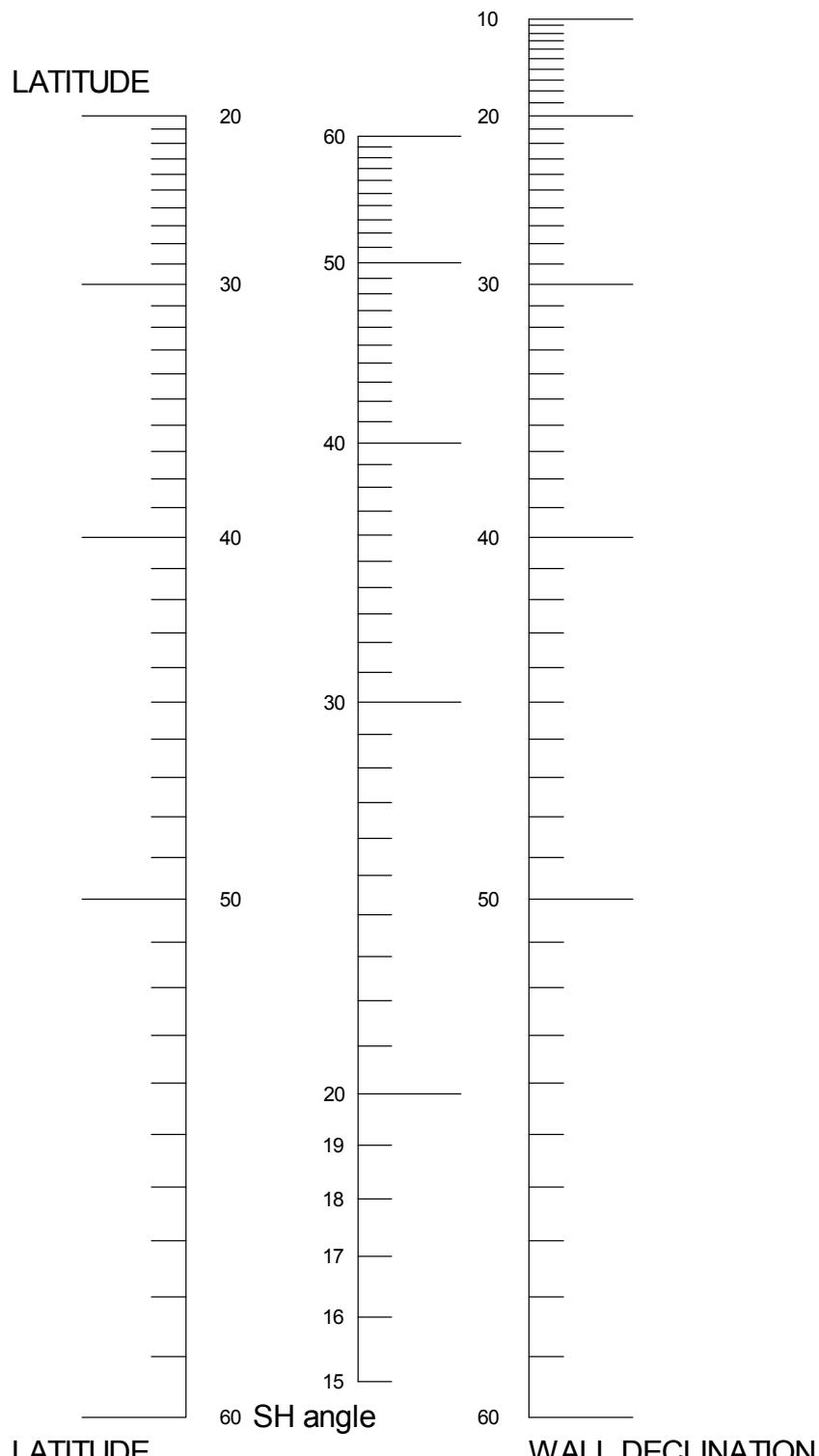
This table is an aid to cross check more detailed work.

## SD, SH, and DL

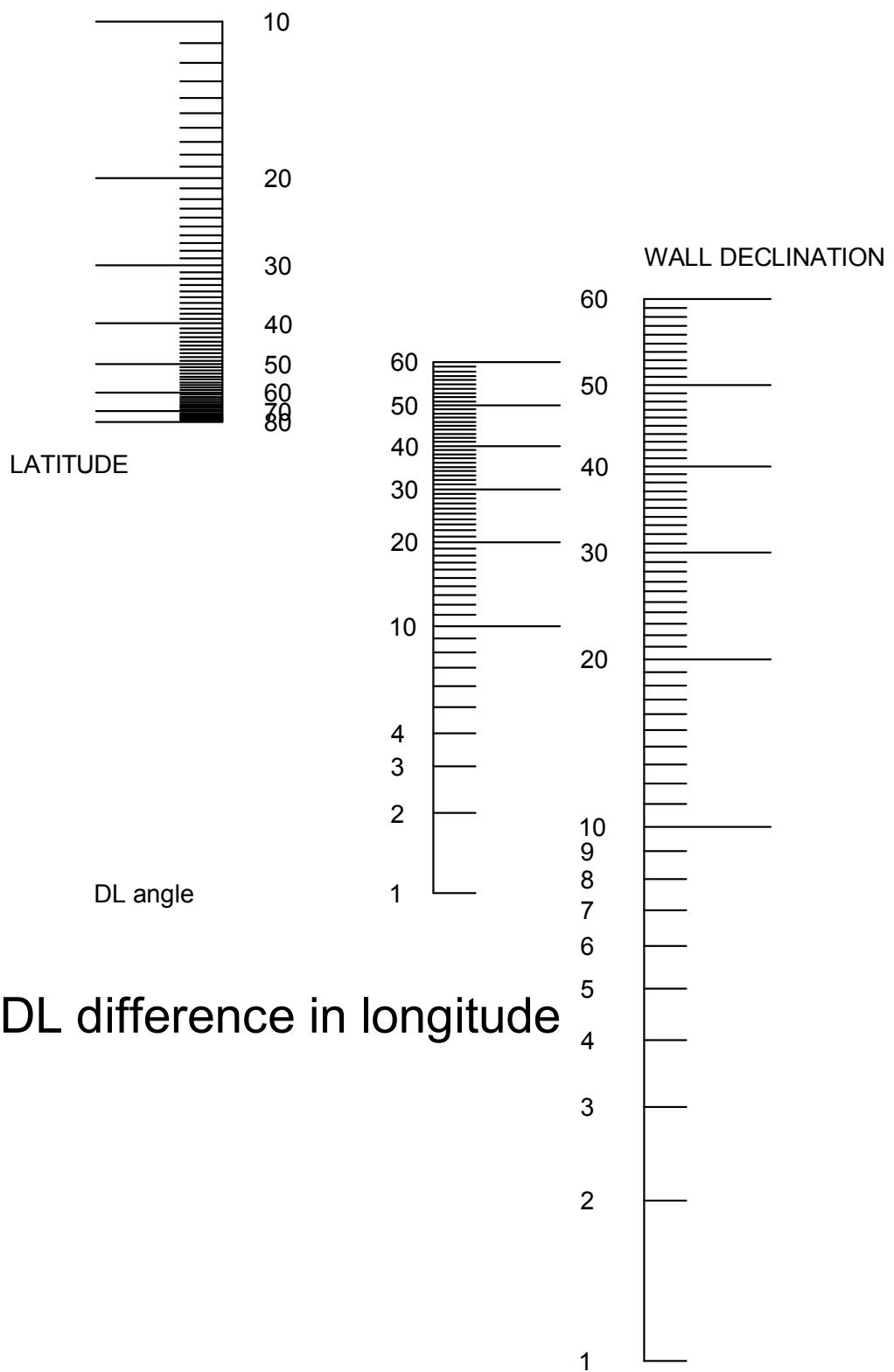
The following three nomograms can be used to construct a vertical decliner, see chapter 32 and the use of "DL". In essence, to construct a vertical decliner, calculate the SD, SH, and DL for the vertical decliner which these three nomograms do. Then design a horizontal dial for a latitude of SH, slide its local apparent noon line onto the SD line of the vertical decliner's dial plate. And that horizontal dial's hour lines are altered by an assumed longitude of "DL" with a legal meridian of 0. Remember that horizontal and vertical dial shadows rotate in opposite directions, so the sense of the surrogate dial's hours needs reversing. **NOTE:** a vertical dial of latitude " $90 - SH$ " can be used in place of a horizontal dial, when the sense of the hours would not need reversing.



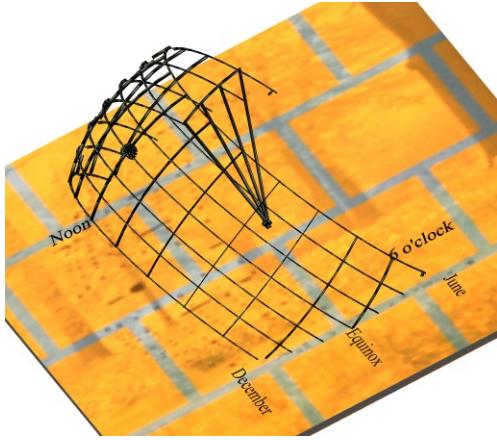
## SH STYLE HEIGHT NOMOGRAM



**ANSWER**



## APPENDIX 6 – SUNRISES AND SUNSETS and ITALIAN, BABYLONIAN, AND DAY-LENGTH LINES



Almanacs with sunrise and sunset times can be found online or can be purchased. These are accurate ways of determining when the sun wakes up or goes to sleep. However, other methods exist, maybe quaint, maybe using different definitions of sunrise, yet each is of interest. The picture to the left shows the solar-travel mesh tilted at latitude North 33°, with 6 am and 6 pm of the equinox on the horizon. It shows how sunrise and sunset is earlier at the winter solstice and later at the summer solstice. Their times can even be estimated!

A formula can be used, and the formula sequence below is very close to almanac accurate.

First calculate the "day angle" (in radians, an intermediate figure),

$$da = 2 * \pi * (j-1) / 365$$

[ j = Julian day ]

Next calculate the "Sun Declination"

$$\begin{aligned} dec &= \text{degrees} (0.006918 - 0.399912 * \cos(da) + 0.070257 * \sin(da) \\ &\quad - 0.006758 * \cos(2 * da) + 0.000907 * \sin(2 * da) \\ &\quad - 0.002697 * \cos(3 * da) + 0.001480 * \sin(3 * da)) \end{aligned}$$

Then calculate the "Azimuth of rising/setting sun"

$$\text{aziRiseSet} = \arccos (\sin(dec) / \cos(lat))$$

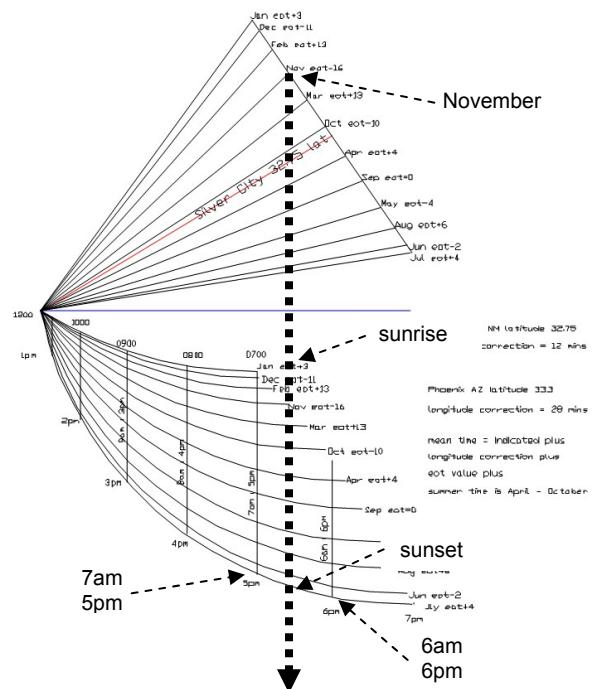
And finally the "Hour angle of rising/setting sun"

$$\text{hraRiseSet} = \arccos(\tan(lat) * \tan(dec))$$

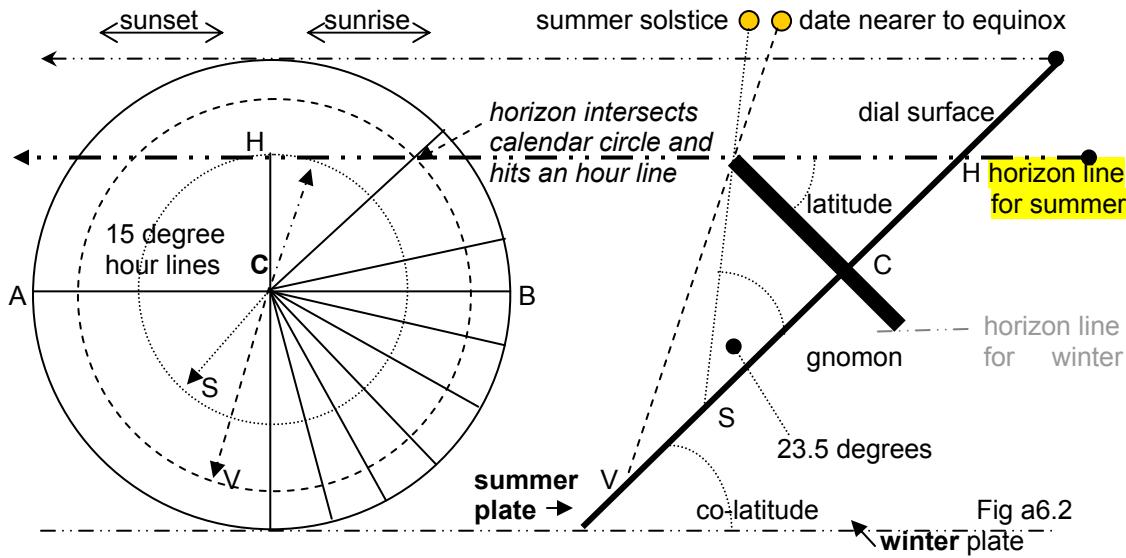
This needs longitude and EOT correction.

An astrolabe can be used to determine sunrise and set times by placing the date on the rete to the 0 degree altitude line, and the extension of that intersection to the mater will indicate L.A.T. for sunrise or set.

A Capuchin dial can be used. The template is made, and a line is drawn from the date desired vertically down to the hour line verticals. In the example shown, which is for latitude 32 degrees, sunset is about 5:30pm, so sunrise is thus about 6:30am. To which must be added November's EOT of about -16, and then the longitude correction.



An equatorial dial may also be used to display sunrise and sunset times. Again these are approximations.



An equatorial dial is drawn on paper, and from its center radiate the 15-degree hour lines. And to the side is drawn a side view of the dial, its diameter equals that of the circular dial plate. It has a gnomon drawn of a convenient size. On the side view is drawn the horizon line, this is a line touching the tip of the gnomon extending parallel to the Earth's surface. From this comes a distance, CH, which is transcribed to the circular dial. This indicates on the dial the sun's lowest visible altitude. Distances are then measured based on the sun's declination, the solstice line is easy, it is a 23.5 degree line, shown on the side view as CS, this is transcribed to the plate. Radius CS is equal to the gnomon linear height divided by the tan of the declination, and the vertical distance CH is equal to the gnomon linear height times the tan of the latitude.

The equinox will parallel the dial surface so a date between the equinox and the solstice is chosen, "V", whose angle is taken from the solar declination tables, or calculated from the solar declination formula in the appendices. And any date can be selected, its declination found, that angle drawn, and its radius transcribed.

The above diagram shows rather early hours for sunrise and late hours for sunset because the diagram is showing the upper summer side of the equinoctial dial. Sunrise is shown to the west of the nodus, sunset to the east. For a given date or declination circle, there will be an intersection with the horizon line, and that intersection is the sunrise or set time for that date. Winter would use the lower nodus, thus have later sunrises and earlier sunsets. Longitude corrections are needed for sunrise/sunset calculations. The EOT must be considered as well, since the sun runs fast and slow as the year moves on.

There are several different definitions of sunrise and sunset, so that must be considered.

There are some very complex formulae that will work over the millennia, however it maybe that these approximations are acceptable.

Illustrating Time's Shadow discusses these lines.

Phoenix AZ ~ SUNRISE AND SUNSET TIMES daylight saving not considered

A6.3a

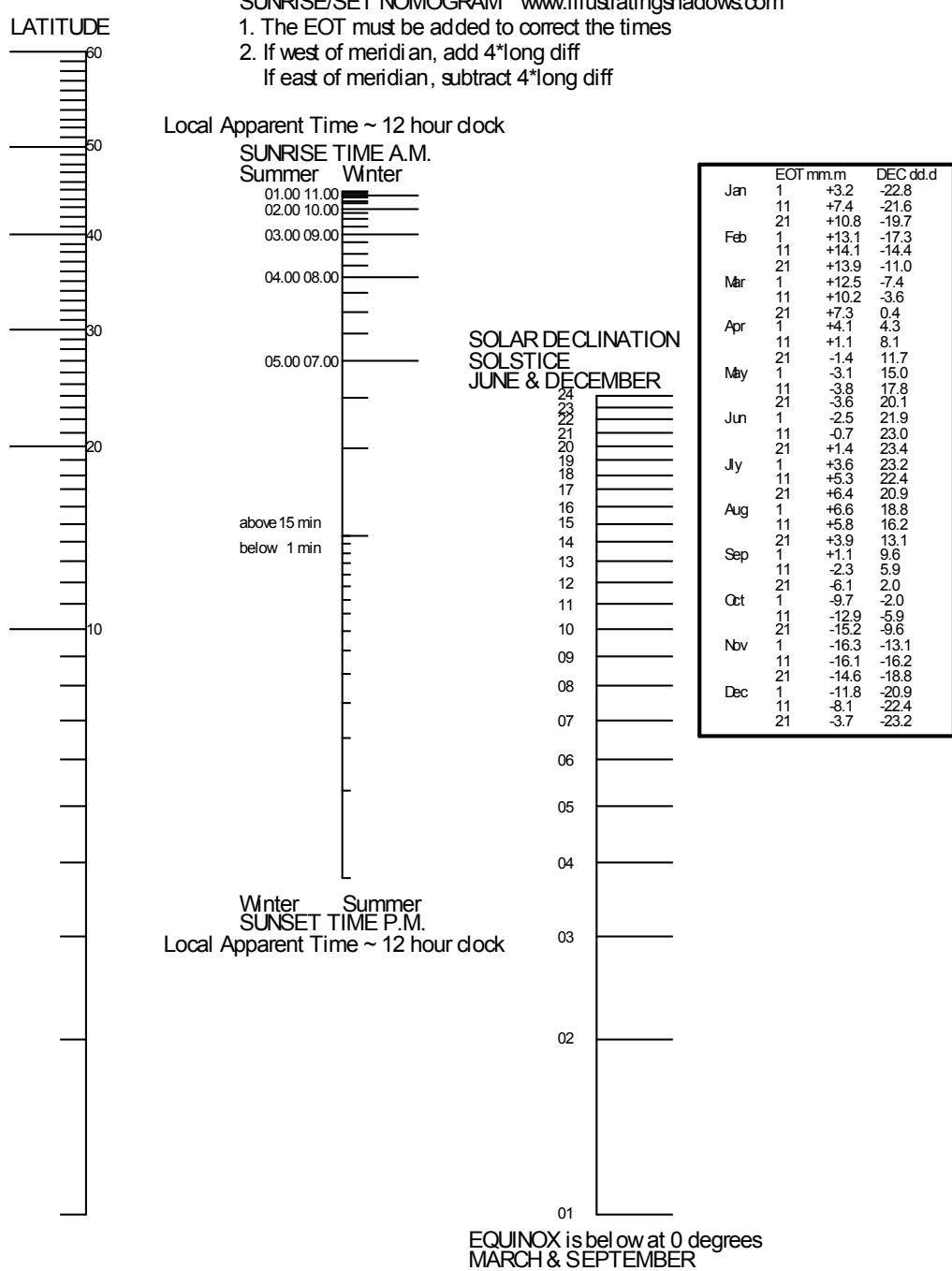
LAT	LONG	ref:long
33.50	112.10	105
Summer time is not considered.		

SUNRISE/SUNSET time LEGAL hh.hh (decimal)  
EOT mm.mm [4 yr astro avg] is used.

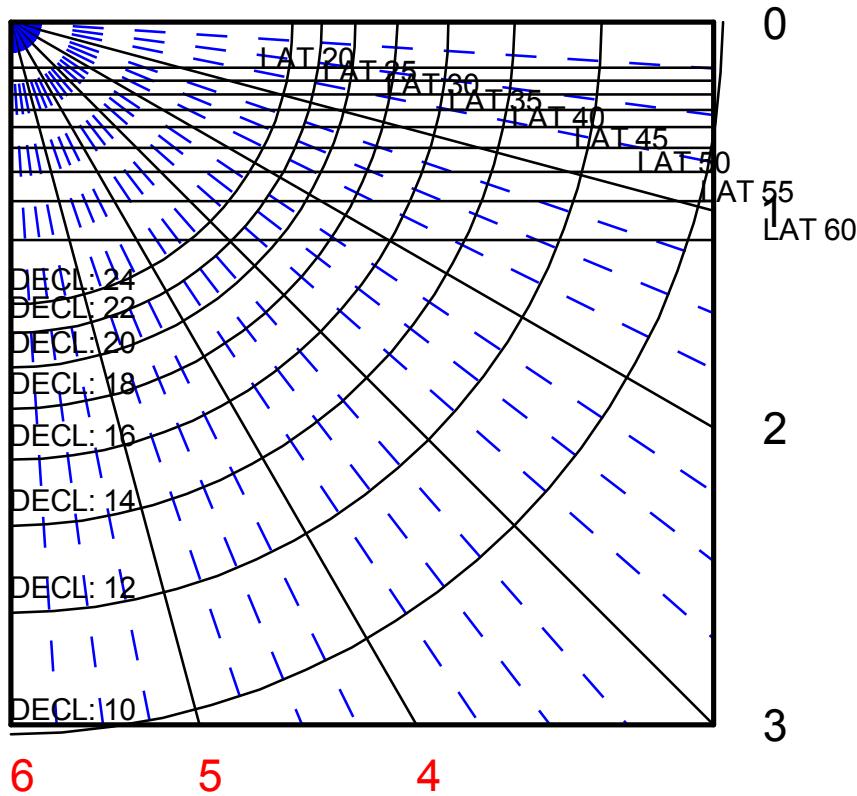
Date	Julian	decl	eot m.m	LEGAL TIME rise h.hh	set h.hh	Daylight duration	Sunrise azimuth	Noon altitude
3/20	79	-0.5	7.6	6.6	18.6	11.96	89.4	56.0
6/21	172	23.5	1.7	5.4	19.6	14.22	118.5	80.0
9/22	265	0.6	-7.1	6.3	18.4	12.06	90.8	57.1
12/21	355	-23.4	-2.1	7.5	17.3	9.78	61.5	33.1
1/5	5	-22.7	5.1	7.63	17.49	9.86	62.4	33.8
1/15	15	-21.3	9.2	7.62	17.63	10.01	64.2	35.2
1/25	25	-19.2	12.2	7.56	17.79	10.23	66.8	37.3
2/5	36	-16.2	14.0	7.44	17.97	10.52	70.5	40.3
2/15	46	-13.0	14.2	7.29	18.13	10.83	74.4	43.5
2/25	56	-9.4	13.2	7.11	18.27	11.16	78.7	47.1
3/5	64	-6.3	11.6	6.95	18.38	11.44	82.4	50.2
3/15	74	-2.4	9.0	6.73	18.52	11.78	87.1	54.1
3/25	84	1.5	6.1	6.51	18.64	12.13	91.8	58.0
4/5	95	5.8	2.8	6.26	18.78	12.51	96.9	62.3
4/15	105	9.5	0.1	6.05	18.90	12.85	101.4	66.0
4/25	115	12.9	-2.0	5.86	19.02	13.16	105.6	69.4
5/5	125	16.0	-3.3	5.69	19.15	13.46	109.3	72.5
5/15	135	18.7	-3.7	5.55	19.27	13.72	112.6	75.2
5/25	145	20.8	-3.1	5.45	19.39	13.94	115.2	77.3
6/5	156	22.5	-1.6	5.39	19.51	14.12	117.3	79.0
6/15	166	23.3	0.4	5.38	19.58	14.21	118.3	79.8
6/25	176	23.4	2.5	5.40	19.63	14.22	118.5	79.9
7/5	186	22.9	4.5	5.47	19.63	14.16	117.8	79.4
7/15	196	21.7	5.9	5.56	19.59	14.03	116.3	78.2
7/25	206	19.8	6.5	5.66	19.50	13.84	114.0	76.3
8/5	217	17.2	6.0	5.79	19.36	13.58	110.8	73.7
8/15	227	14.3	4.6	5.90	19.20	13.30	107.2	70.8
8/25	237	11.0	2.2	6.02	19.00	12.99	103.3	67.5
9/5	248	7.1	-1.1	6.14	18.77	12.63	98.5	63.6
9/15	258	3.3	-4.6	6.25	18.54	12.30	94.0	59.8
9/25	268	-0.5	-8.1	6.36	18.31	11.95	89.4	56.0
10/5	278	-4.4	-11.4	6.48	18.09	11.61	84.7	52.1
10/15	288	-8.2	-14.1	6.60	17.87	11.27	80.1	48.3
10/25	298	-11.8	-15.9	6.74	17.68	10.94	75.8	44.7
11/5	309	-15.5	-16.5	6.90	17.50	10.59	71.4	41.0
11/15	319	-18.3	-15.5	7.06	17.37	10.31	67.9	38.2
11/25	329	-20.6	-13.2	7.21	17.29	10.08	65.0	35.9
12/5	339	-22.3	-9.6	7.36	17.27	9.90	63.0	34.2
12/15	349	-23.2	-5.1	7.49	17.29	9.80	61.8	33.3
12/25	359	-23.4	-0.1	7.58	17.36	9.78	61.6	33.1

SIX HOUR SPREAD FOR BABYLONIAN & ITALIAN HOUR LINES for							33.50	112.10
Standard sunrise	BABYLONIAN		DECIMAL, NOT HH.MM			NO EOT USED		
	hh.h	+1 hour	+2 hrs	+3 hrs	+4 hrs	+5hrs	+6 HRS	
	7.6	8.6	9.6	10.6	11.6	12.6	13.6	
	6.5	7.5	8.5	9.5	10.5	11.5	12.5	
Standard sunset	5.4	6.4	7.4	8.4	9.4	10.4	11.4	
	ITALIAN		DECIMAL, NOT HH.MM			NO EOT USED		
	hh.h	-1 hour	-2 hrs	-3 hrs	-4 hrs	-5hrs	-6 HRS	
	17.4	16.4	15.4	14.4	13.4	12.4	11.4	
Winter solstice	18.5	17.5	16.5	15.5	14.5	13.5	12.5	
	19.6	18.6	17.6	16.6	15.6	14.6	13.6	

EOT (mm.mm) is the average of four years of astronomically accurate EOTs, and sunrise uses formula A8.6. The spreadsheet [illustratingShadows.xls](#) is available on: [www.illustratingshadows.com](http://www.illustratingshadows.com) and you may configure it for your location.



Alternative nomogram for sunrise or set as hours from 0600 or 1800 (6 pm)



For the theory of this, refer to Illustrating Time's Shadow. Also refer to appendix 7 for the proof of the sunrise/set formula which uses the above as the geometric model.

This nomogram is one quadrant of an equatorial dial.

**SUNRISE:** this shows the hour from 6 am for sunrise. You add the hour depicted to 0600 if this is winter as sunrise is later, and subtract it from 0600 for summer as sunrise is earlier.

**SUNSET:** this also shows the hour from 1800 (6 pm) for sunset. You subtract the hour depicted from 1800 if this is winter as sunset is earlier, and add it to 1800 for summer as sunset is later.

The longitude and EOT corrections must still be applied. Similarly, the summer time correction would need to be applied.

## ITALIAN AND BABYLONIAN HOUR LINES

ON A SUN DIAL PLATE USE SUNSET AND SUNRISE DATA RESPECTIVELY.

A6.4

Babylonian and Italian values by latitude. Times are hh.mm Local Apparent Time

Solstice Sunrise and Sunset (equinox is 6am/pm). No longitude correction. No EOT correction.

Winter solstice:

Declination:

-23.5

latitude	Rise	Set	Day length hrs
30	6.58	17.02	10.04
31	7.00	17.00	10.00
32	7.03	16.57	9.54
33	7.05	16.55	9.50
34	7.08	16.52	9.44
35	7.10	16.50	9.40
36	7.13	16.47	9.34
37	7.16	16.44	9.28
38	7.19	16.41	9.22
39	7.22	16.38	9.16
40	7.25	16.35	9.10
41	7.28	16.32	9.04
42	7.32	16.28	8.56
43	7.35	16.25	8.50
44	7.39	16.21	8.42
45	7.43	16.17	8.34
46	7.47	16.13	8.26
47	7.51	16.09	8.18
48	7.55	16.05	8.10
49	8.00	16.00	8.00
50	8.04	15.56	7.52
51	8.09	15.51	7.42
52	8.15	15.45	7.30
53	8.20	15.40	7.20
54	8.27	15.33	7.06
55	8.33	15.27	6.54
56	8.40	15.20	6.40
57	8.48	15.12	6.24
58	8.56	15.04	6.08
59	9.05	14.55	5.50
60	9.15	14.45	5.30

Summer solstice:

Declination:

+23.5

latitude	Rise	Set	Day length hrs
30	5.01	18.59	13.58
31	4.59	19.01	14.02
32	4.56	19.04	14.08
33	4.54	19.06	14.12
34	4.51	19.09	14.18
35	4.49	19.11	14.22
36	4.46	19.14	14.28
37	4.43	19.17	14.34
38	4.40	19.20	14.40
39	4.37	19.23	14.46
40	4.34	19.26	14.52
41	4.31	19.29	14.58
42	4.27	19.33	15.06
43	4.24	19.36	15.12
44	4.20	19.40	15.20
45	4.16	19.44	15.28
46	4.12	19.48	15.36
47	4.08	19.52	15.44
48	4.04	19.56	15.52
49	3.59	20.01	16.02
50	3.55	20.05	16.10
51	3.50	20.10	16.20
52	3.44	20.16	16.32
53	3.39	20.21	16.42
54	3.32	20.28	16.56
55	3.26	20.34	17.08
56	3.19	20.41	17.22
57	3.11	20.49	17.38
58	3.03	20.57	17.54
59	2.54	21.06	18.12
60	2.44	21.16	18.32

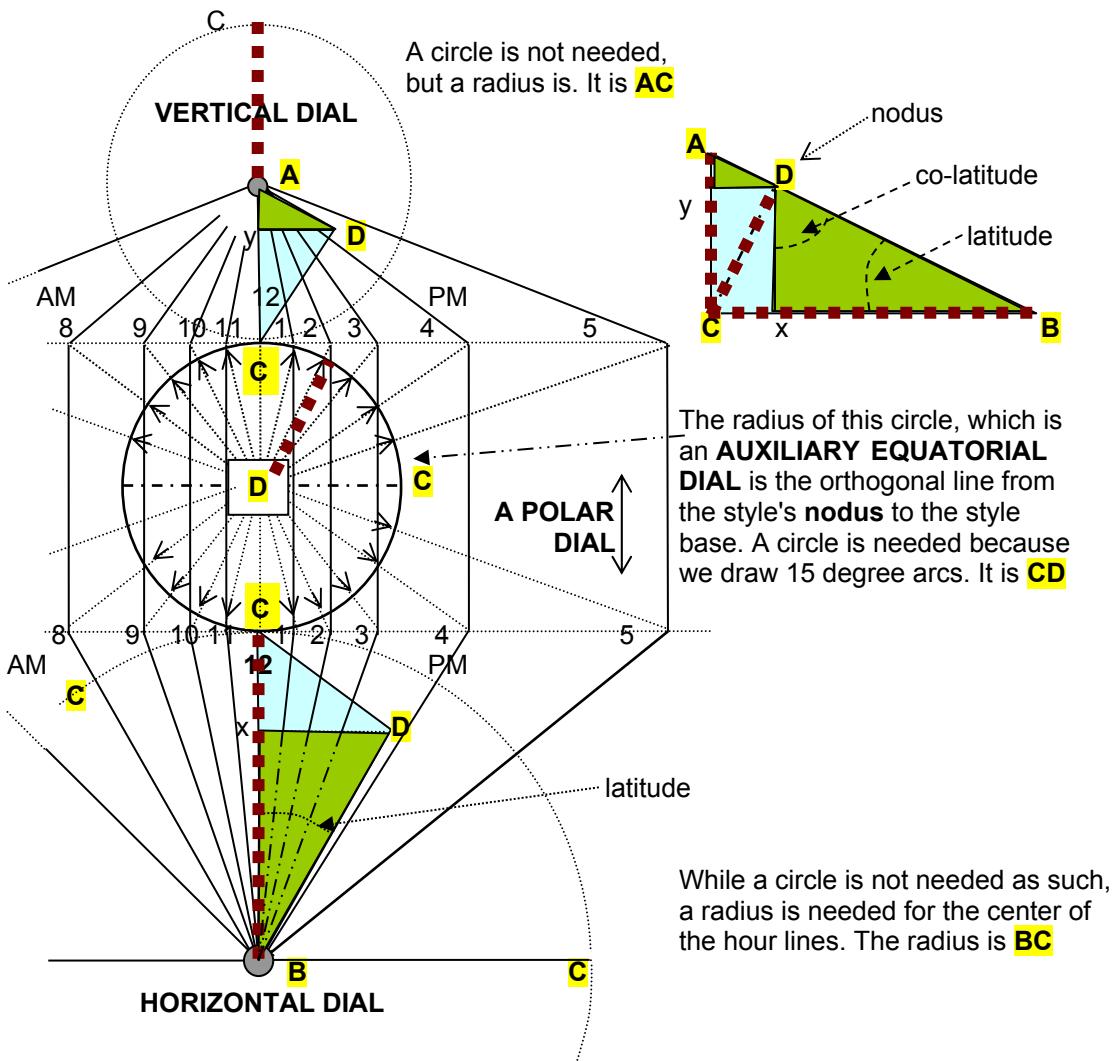
Sunset (true time or local apparent time) occurs the same number of hours after noon that sunrise happens before. An 0605 sunrise is 5 hours 55 minutes before noon, thus sunset is 5 hours 55 minutes after it, or 1755. Except for on the hour or half hour, the minutes do not match. For standard time, then the time is shifted by the longitude correction and then by the equation of time, thus the March and September equinoxes do not match due to differing EOT values.

Italian hour lines are commonly used to indicate the number of hours until sunset. For Italian hour lines, exclude the EOT. And exclude the longitude correction unless the dial already has considered longitude in its design. The spreadsheets on the web site allow you to do this, and of course, the above table A6.4 provides all the data needed for Italian and Babylonian lines. The equinox is not shown since true sunrise and sunset occurs at 6am and 6pm L.A.T.

## APPENDIX 7 – MISCELLANEOUS PROOFS

### The general geometric model of the previous dials

This is intended to show a natural symmetry in geometric dial design. This model correlates the equatorial, polar, horizontal, vertical and meridian dials. This model when extended also provides the basis for the vertical declining dial. See the first few pages of Appendix 1 for pictorials of each dial type.



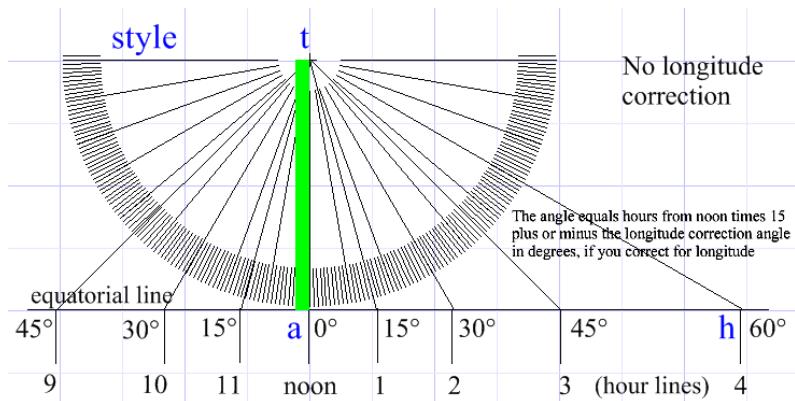
The above general geometrical model shows a symmetry in the design of the polar, vertical, and horizontal dials in the context of their gnomon's style. For a polar or a meridian dial, CD is the linear height of the style's nodus above the dial plate. In this model, CD does different things depending on dial type.

For a horizontal dial, this model is based on the ratio of the DC to CB. For a vertical dial the ratio is based on DC to CA. In all cases, DC is the equinoctial ray going to the dial plate at C from nodus D.

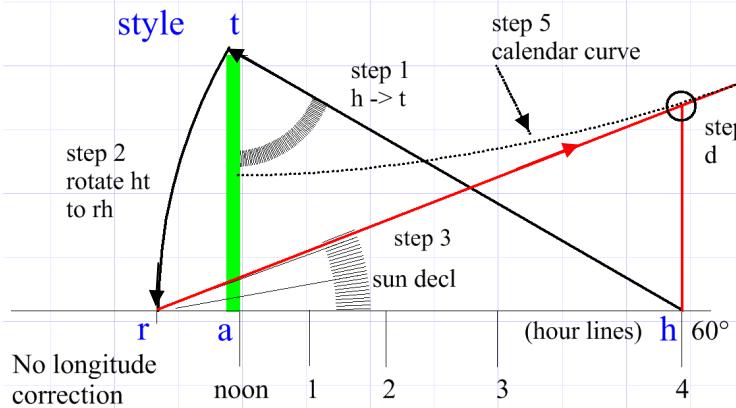
**GEOMETRIC MODELS CONTINUED** - The equatorial dial construction is obvious as its hour line angles are  $15^\circ$  apart, and the calendar information consists of circles, see the appendix discussing sunset and sunrise times for a depiction. The polar dial is shown on the preceding page and summarized below.

#### POLAR AND MERIDIAN DIAL HOURS:

The equatorial line is along "ah" and the style is "t", where "at" equals the linear height of the style above the dial plate. Angle "ath" is the time in hours from noon times  $15^\circ$  per hour giving the linear distance to the hour line as "ah".



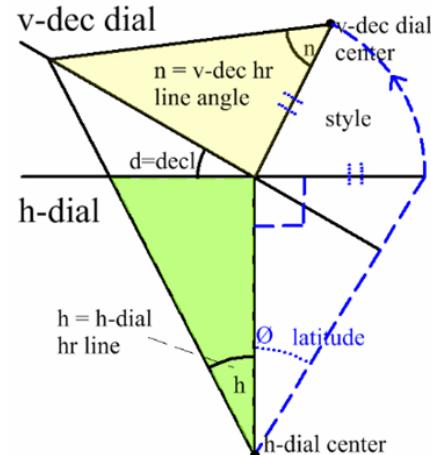
#### POLAR AND MERIDIAN DIAL CALENDAR DECLINATION LINES



From the hour line "h" go back up to the style "t" and then, rotate line "ht" until it falls back to the equatorial line "ah" at a point "r", then at point "r" draw a line at an angle equal to the desired suns declination until it intersects the original hour line at "d". Now, length "hd" is the calendar points distance from "h"

**THE VERTICAL DECLINER** dial is shown to the right and discussed in detail in the proof of its hour line angles. A surrogate horizontal dial can be used, which is the simplest form. A surrogate equatorial dial can be used, which is an extra step since that equatorial dial ends up being converted to a horizontal dial, and from thence to the vertical decliner.

Calendar curve information for these hour angle dials consists of a straight line for the equinox and hyperbolic curves for other dates (except the armillary and equatorial dials), and the construction is discussed in the appendix relating to calendar information. Except for the armillary and equatorial dials, they center on the L.A.T. noon line, or the SD (style distance) if declining.



## Proof that the sun is at 90 degrees to the style on the equinox.

Many of the geometric projections project a line from the style which is at 90 degrees from it. Why? Because that is where the sun comes from at the equinox. In other words when the day and night time are equal, March 21 and September 23 or thereabouts.

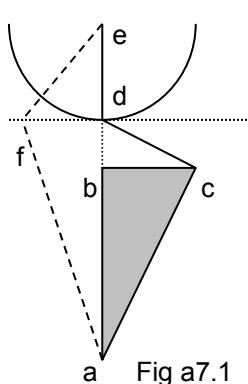


Fig a7.1

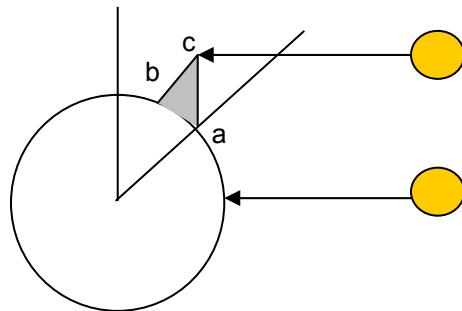


Fig a7.2

The top left figure is a very common projection. "ab" is the sub style, the style or gnomon is rotated to the left around "ab" and winds up at "c". A 90 degree line is drawn to intercept the extension of "ab", and the line length, "cd" is used for a half circle, whose radius "de" is the same as "cd". An angle is drawn from "e" being an hour angle, it intercepts the horizontal line touching the circle and the orthogonal line (namely at "d"). And from thence, an hour line is drawn "fa". This is a very common projection.

But why is angle acd 90 degrees? Look at the diagram on the top right, it is for the equinox. To make life simple, the Earth and sun are rotated as if the Earth axis was not at an angle of 23.5 degrees. When the sun is at the equinox position, it is on the equator. So it is 90 degrees above the equator. And its angle is 90 degrees to the Earth's polar axis. Since the gnomon parallels the Earth axis, that is the whole point of the style being set at an angle equal to the latitude, then the sun's rays are also at 90 degrees to the style of the gnomon.

## Proof that cotan (angle) = defined as tan (90-angle) is = 1/tan(angle)

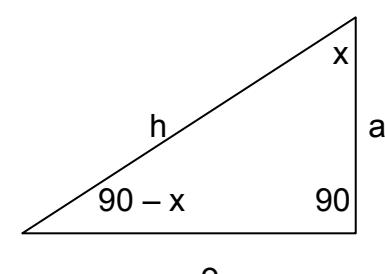


Fig a7.3

$$\tan(x) = o/a \quad \text{thus} \quad a/o = 1/\tan(x)$$

$$\text{by definition } \cotan(x) = \tan(90-x)$$

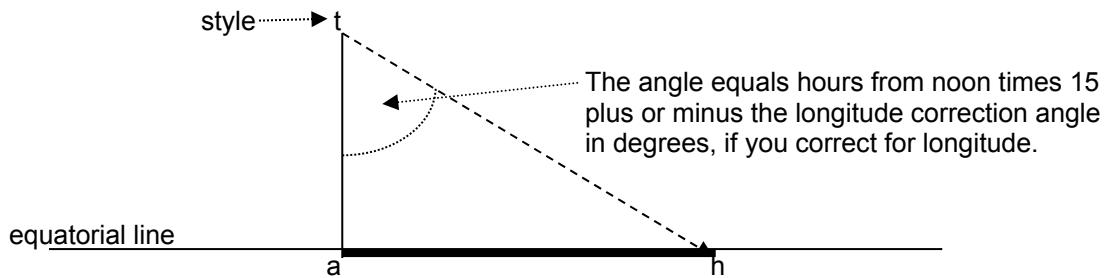
$$\tan(90-x) = a/o \text{ which} = 1/\tan(x)$$

thus:

$$\cotan(x) = \tan(90-x) = 1/\tan(x)$$

## POLAR DIAL – hour line and calendar curve derivations

HOUR LINES:



The equatorial line is along "ah", and the style is "t", where "at" equals the linear height of the style above the dial plate.

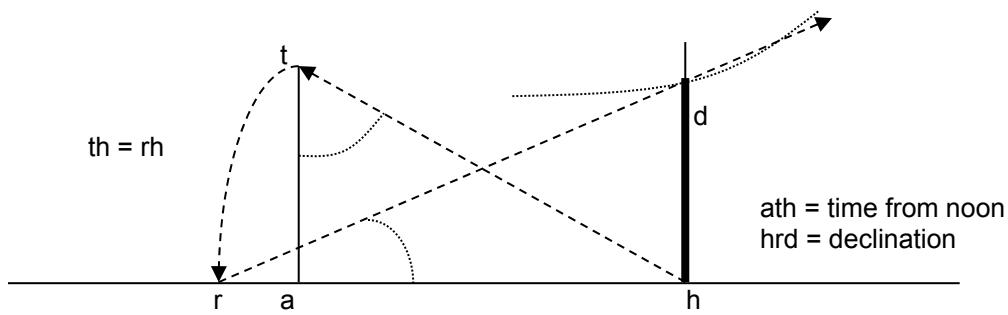
The angle "ath" is the time in hours from noon times 15 (there are 15 degrees in an hour).

$$\text{we see that: } \tan(\text{ath}) = \text{ah} / \text{at}, \quad \text{thus} \quad \text{ah} = \text{at} * \tan(\text{ath}) \quad (1)$$

in other words the distance along the equatorial or equinox line to the hour line is equal to the style height times the tangent of the time.

And the angle can be adjusted for longitude by adding or subtracting the difference between the dial's location and the standard time meridian in degrees before calculating the tangent. Or it can be omitted and then deferred to an EOT table specific to the dial's longitude.

EQUINOX, OR ANY OTHER CALENDAR DECLINATION LINES (such as length of day):



$$\text{we see that: } \tan(\text{hrd}) = \text{dh} / \text{rh} = \text{dh} / \text{th}, \quad \text{thus} \quad \text{dh} = \text{th} * \tan(\text{declination})$$

$$\text{and: } \cos(\text{ath}) = \text{ta} / \text{th} \quad \text{thus} \quad \text{th} = \text{ta} / \cos(\text{ath})$$

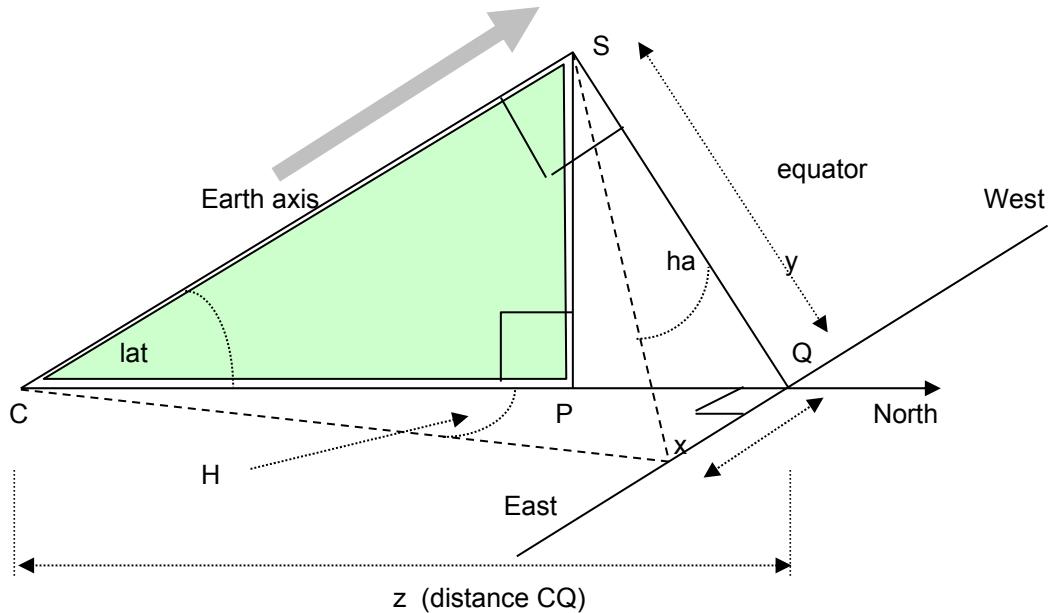
$$\text{so given: } \text{dh} = \text{th} * \tan(\text{declination})$$

$$\text{then: } \text{dh} = \text{ta} * \tan(\text{declination}) / \cos(\text{time}) \quad (2)$$

in other words the distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines.

## HORIZONTAL DIAL – hour line derivation

Consider CSP to be the gnomon, CS being the style, CP the sub-style. From S extend a perpendicular to a new point Q on the East-West line. As the sun moves by "ha" degrees from noon, a new hour line is drawn whose angle is H.



- |     |   |  |
|-----|---|--|
| (1) | $\tan(ha) = x/y$  | $\tan$ of hour angle ( 15 degrees * hour ) |
| (2) | $x = y * \tan(ha)$  | thusly                                     |
| (3) | $\tan(H) = x/z$   | $\tan$ of the hour line angle              |
| (4) | $\sin(lat) = y/z$   | $\sin$ of latitude or $\emptyset$          |
| (5) | $y = z * \sin(lat)$   | thusly                                     |
| (6) | $\begin{aligned} H &= \text{atan}(x/z) = \text{atan}((y * \tan(ha)) / z) \\ &= \text{atan}((z * \sin(lat) * \tan(ha)) / z) \\ &= \text{atan}(\sin(lat) * \tan(ha)) \end{aligned}$ |  |

So, to calculate any hour line from the sub style:  $H = \arctan(\sin(lat) * \tan(ha))$

The hours used in the hour line formula may be adjusted for longitude. If you use a spreadsheet then trigonometric functions use radians, and must be converted back using degrees. The formula is then something like:-

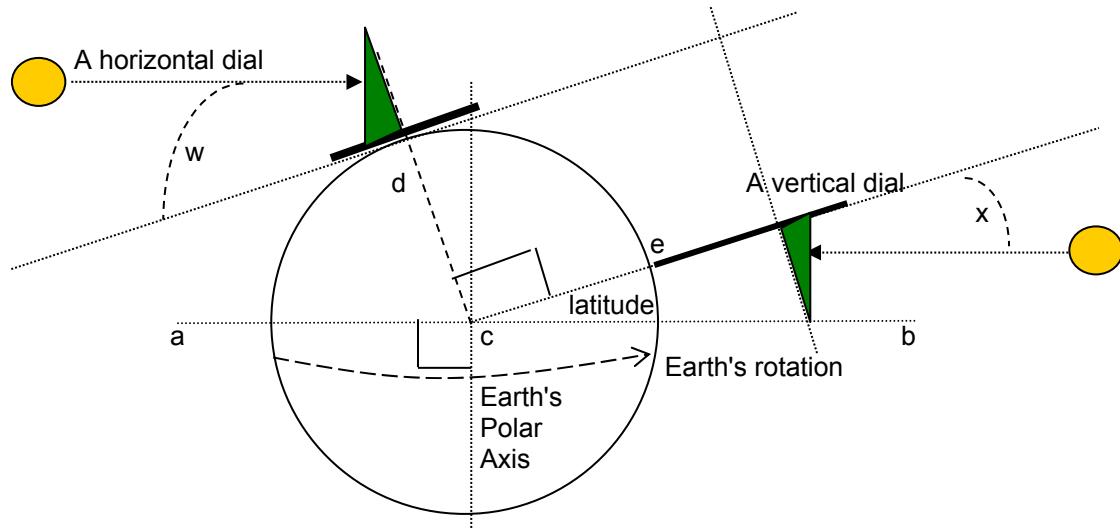
```
DEGREES(ATAN(TAN(RADIANS(15*time))*SIN(RADIANS(latitude))))
```

**Tables can be used which are trigonometric in nature**

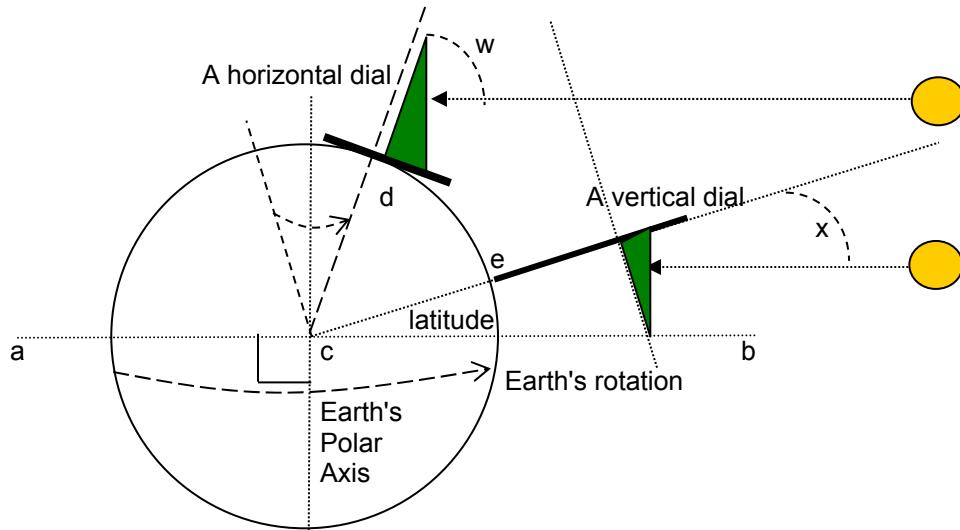
Appendix 3 has tables with hour line angles calculated for various latitudes.

## Proof a vertical dial design matches a horizontal design for the co-latitude

Consider the following diagram of the planet Earth. At latitude 20 degrees for example there is a vertical dial plate. For simplicity, the polar axis is shown as vertical.



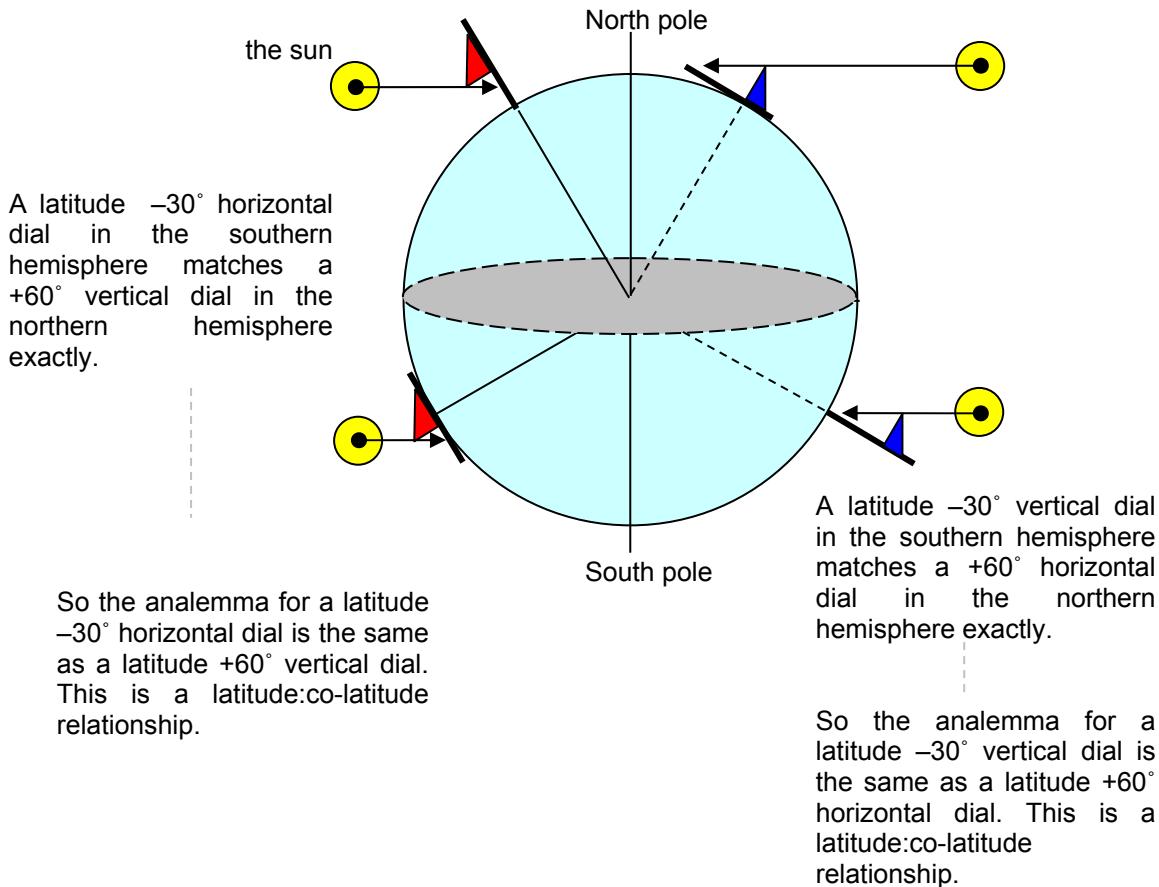
Angle  $ecb$  is the latitude, and we set angle  $dce$  to 90 degrees, so angle  $acd$  is the co-latitude. The angle  $x$  is the vertical dial's dial-plate angle with the sun. In the horizontal dial, angle  $w$  is also by definition equal to  $x$ . Thus the hour lines for the vertical dial at latitude  $x$  match those of a horizontal dial of latitude  $90-x$ . By definition the reverse is also true. The lower figure is the same as the upper figure except that the horizontal dial is now rotated around the polar axis to be at the same time or longitude as the vertical dial, angle  $bcd$  being the co-latitude.



**NOTE:** While a horizontal dial may be used for a vertical dial at the co-latitude, other things are not the same. For example the calendar lines, while they remain in the same positions, their meaning reverses, summer and winter change places. Hour line labeling reverses. Shadow movement around dial center moves from clockwise to counter clockwise. Analemmas must be rotated left to right and top to bottom. And, hour line shifts for longitude correction are also reversed.

## Empirical proof of north and south hemisphere relationships

This pictorial shows the simplest way to consider analemma drafting for the two hemispheres. It can be used to see other relationships as well.



The spreadsheets for the analemma:  
and:  
and:  
the various DeltaCAD macros

[illustratingShadows.xls](#)

[analemma.xls](#)

the various DeltaCAD macros

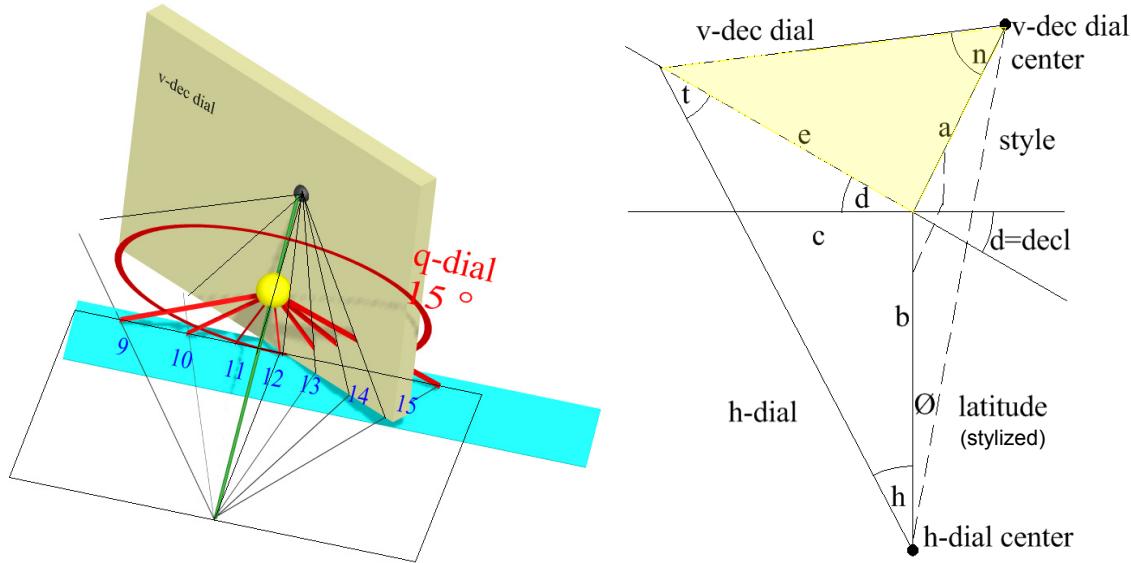
were designed for the northern hemisphere. Using the about notes and concepts, they can be used for the southern hemisphere as well.

As a rule, using the pictorial above, if you can move a dial upwards from the southern to the northern hemisphere, as in either the bottom left to top left, or bottom right to top right, then the northern hemisphere dial is usable exactly as is. But, if you have to cross the polar axis, as in bottom left to top right, or bottom right to top left, then things have to be reversed, such as which way the shadow moves, longitude correction, am and pm, and so on. This can be seen on the previous page

These concepts can be used when using the DeltaCAD **analemma.bas** and other macros designed for the northern hemisphere when designing dials for the southern hemisphere.

## Proof of Decliner/Great Decliner Hour Line angles

In the stylized figure to the right below, the triangle "h-dial" has sides "b", and "c" and angle "h". Side "b" is the horizontal dial's sub-style and a selected horizontal dial's hour line angle "h". Triangle "v-dec dial" has two named sides, with angle "n" being the vertical decliner's equivalent hour line angle that is associated with the horizontal dial's angle "h". Both dials share a style that connects the h-dial and v-dec dial centers, shown by a depicted dashed line. The vertical decliner's sub-style is not depicted in the figures below, and their "SD" and "SH" (style angular distance and angular height) are derived elsewhere. Declination is "d" and " $\emptyset$ " is latitude.



$$a = b * \tan(\emptyset) \quad [1]$$

$$\tan(n) = e / a \quad \text{thus ...} \quad n = \arctan(e / a) \quad [2] \text{ desired}$$

$$\text{so } n = \arctan(e / b * \tan(\emptyset)) \quad [3]$$

$$e / \sin(h) = b / \sin(180 - (h + (90 + d))) \quad [4] \text{ law of sines}$$

$$\text{so } e / \sin(h) = b / \sin(90 - h - d)$$

$$\text{thus } e = b * \sin(h) / \sin(90 - h - d) \quad [5]$$

$$n = \arctan(e / a) = \arctan \left[ \frac{b * \sin(h) / \sin(90 - h - d)}{b * \tan(\emptyset)} \right] \quad [\text{using 2, 5, 1}]$$

$$\text{so } n = \arctan \left[ \frac{b * \sin(h)}{\sin(90 - h - d) * b * \tan(\emptyset)} \right]$$

$$\text{thus } n = \arctan \left[ \frac{\sin(h)}{\sin(90 - h - d) * \tan(\emptyset)} \right] \quad [6]$$

and using

$$h = \text{atan}(\sin(\theta) * \tan(\text{sun hour angle})) \quad [\text{for h-dial}]$$

and since from the prior page

$$n = \text{atan}(\sin(h) / \tan(\theta) * \sin(90 - h - d)) \quad [\text{from 6}]$$

then using the sun's hour angle as opposed to a surrogate horizontal dial's hour line angles

then

$$n = \text{atan}\left[\frac{\sin(\text{atan}(\sin(\theta) * \tan(\text{sun hour angle})))}{\tan(\theta) * \sin(90 - d - \text{atan}(\sin(\theta) * \tan(\text{sun hour angle})))}\right]$$

[ 7 ]

Hence, considering a spreadsheet or a procedural program implementation of a vertical dial that declines, it has hour line angles "n" equal to:-

$$\begin{aligned} \text{i.e. } n = & \text{DEGREES(ATAN(SIN(RADIANS((DEGREES(ATAN(TAN \\ (RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat)))) / \\ (TAN(RADIANS(lat))*SIN(RADIANS(90-dec- \\ (DEGREES(ATAN(TAN(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat)) \\ )))))))))}) \end{aligned}$$

where the hour itself, and longitude corrections are all considered.

**NOTE:** The formula derived above can be converted to the simpler form using methods beyond the scope of the trigonometry discussed in this book. However, the formula is presented as is in order to show the derivation in a logical and simple form.

The results of the formula above match the formula usually published which is:-

$$n = \text{atan}(\cos(\theta) / (\cos(\text{dec}) \cot(\text{ha}) + \sin(\text{dec}) \sin(\theta)))$$

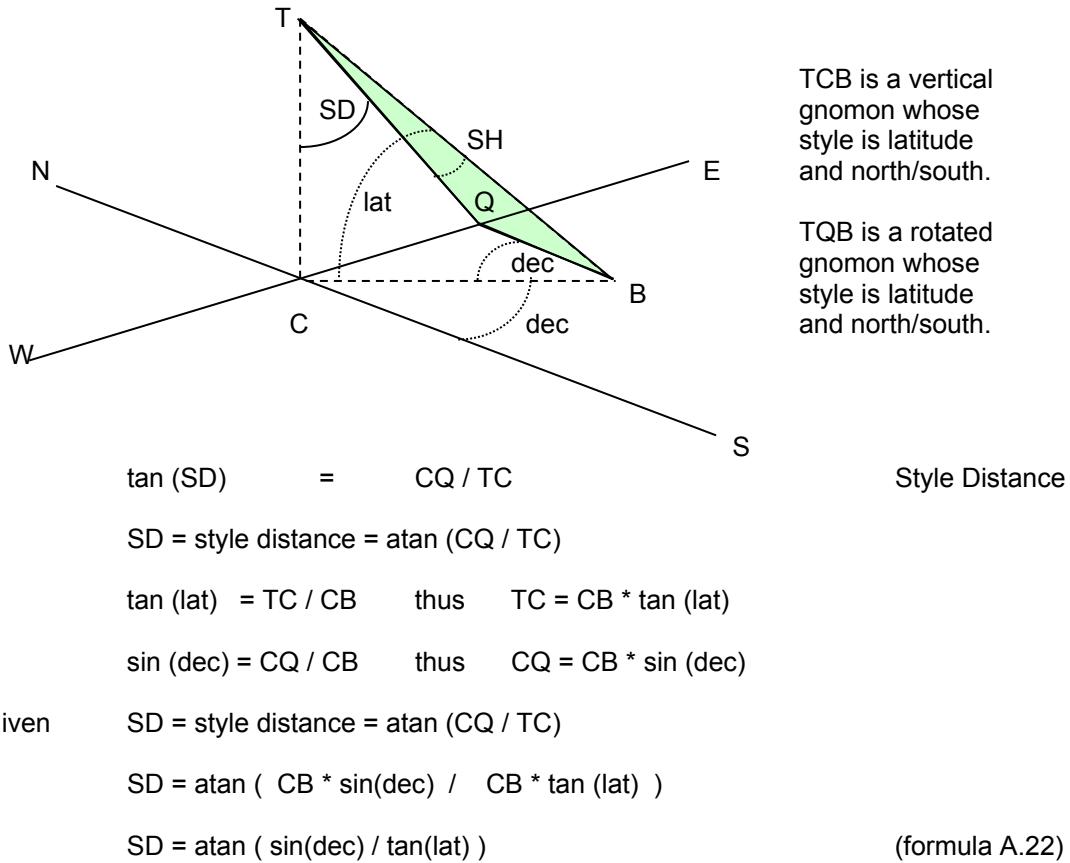
The Illustrating Shadows formula derived above is used in one of the vertical decliner sheets in:-

IllustratingShadows.xls

and the standard common formula is also used in another vertical decliner sheet. The index of sheets as well as the individual sheets themselves clearly state which formula is used.

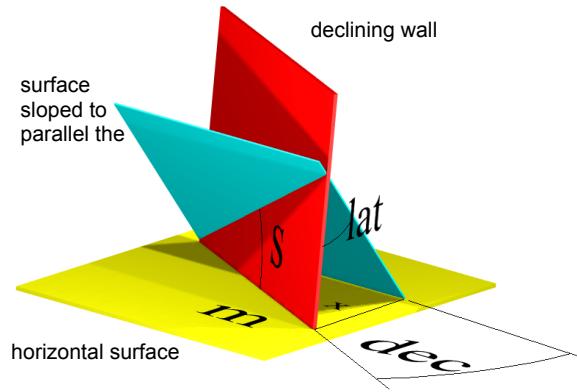
## Proof of Decliner/Great Decliner SD & SH angle (Tables A5.1 and A5.2)

In the proof and pictorial that follows, please note where the right angle is. Also, in the math that follows, "dec" (wall declination) is the normally defined wall declination using the SxxxW terminology.

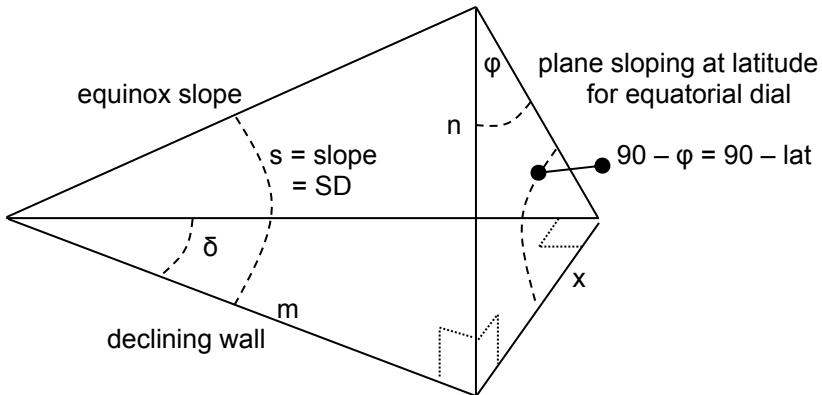


<b>[SH]</b>	$\sin(SH) = QB / TB$	Style Height
thus	$SH = \text{style height} = \text{asin}(QB / TB)$	
and	$\cos(lat) = CB / TB$ thus $TB = CB / \cos(lat)$	
and	$\cos(dec) = QB / CB$ thus $QB = CB * \cos(dec)$	
and given	$SH = \text{style height} = \text{asin}(QB / TB)$	
then	$SH = \text{asin}(CB * \cos(dec) / (CB / \cos(lat)))$	
so	$SH = \text{asin}(\cos(dec) * \cos(lat))$	(formula A.23)

## VERTICAL DECLINER ~ alternative proof of Style Distance ~ SD



$$\begin{array}{lll} \delta & = & \text{dec} \\ \psi & = & \text{lat} \end{array} \quad \begin{array}{lll} & = & \text{wall declination} \\ & = & \text{latitude} \end{array}$$



$$\sin(\delta) = x / m$$

$$\tan(90-\varphi) = n / x$$

$$\tan(s) = n / m$$

$$m = x / \sin(\delta)$$

$$n = x * \tan(90 - \varphi)$$

$$s = \tan(n / m)$$

$$s = \tan\left(\frac{x * \tan(90 - \varphi)}{x / \sin(\delta)}\right)$$

$$s = \tan(\sin(\delta) * \tan(90 - \varphi))$$

$$s = \tan(\sin(\delta) / \tan(\varphi))$$

$$s = \tan(\sin(\text{dec}) / \tan(\text{lat}))$$

This is the same formula for SD, style distance. This makes sense since the sub-style is perpendicular to the equinox line, and thus angularly displaced by an amount equal to the slope.

$$SD = \tan(\sin(\text{dec}) / \tan(\text{lat}))$$

## Proof of Decliner/Great Decliner DL angle (Tables A5.1 and A5.2 and A5.3)

The preceding pages developed SD and SH for a vertical decliner. In this context, SD is the angle from the gnomon base to local noon, and thus relates to "DL" which is an equivalent difference in longitude that would generate such a dial plate shift, as discussed in the big book.

SD is in essence the place where a surrogate horizontal dial's local noon line would be aligned. And as such is an offset for that surrogate horizontal dial's hour lines so that the hour lines of the surrogate horizontal dial would match those of the vertical decliner. Along with that procedure comes the correct alignment of declination or calendar curves, and also the analemma, this is discussed in the big book.

Given that for a vertical decliner:-

$$\begin{aligned} \text{SD} &= \tan(\sin(\text{dec}) / \tan(\text{lat})) \\ \text{and} \\ \text{SH} &= \sin(\cos(\text{dec}) * \cos(\text{lat})) \end{aligned} \quad (\text{formula A.22})$$

where SH is the latitude for a surrogate horizontal dial, and SD is an angle between the surrogate horizontal dial's local noon and the desired noon placement.

The surrogate horizontal dials hour lines are thus:

$$\text{shdhr} = \tan(\sin(\text{SH}) * \tan(\text{hour angle}))$$

and for local noon on the vertical decliner, which is some unknown hour on the surrogate horizontal dial, remember that SD is where the surrogate dial's noon lies, thus:-

$$\text{SD} = \tan(\sin(\text{SH}) * \tan(\text{hour angle}))$$

substituting SD we get:-

$$\tan(\sin(\text{dec}) / \tan(\text{lat})) = \tan(\sin(\text{SH}) * \tan(\text{shd hr angle}))$$

thus we can remove atan on both sides, thus:-

$$\sin(\text{dec}) / \tan(\text{lat}) = \sin(\text{SH}) * \tan(\text{shd hr angle})$$

substituting SH we get:-

$$\sin(\text{dec}) / \tan(\text{lat}) = \sin(\sin(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{shd hr angle}))$$

thus:-

$$\begin{aligned} \tan(\text{shd hr angle}) &= \frac{\sin(\text{dec}) / \tan(\text{lat})}{\sin(\sin(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{lat}))} \\ &= \frac{\sin(\text{dec})}{\sin(\sin(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{lat}))} \end{aligned}$$

thus:

$$\text{shd hr angle} = \tan^{-1} \left( \frac{\sin(\text{dec})}{\sin(\sin(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{lat}))} \right)$$

given:-

$$\text{shd hr angle} = \text{atan} \left\{ \frac{\sin(\text{dec})}{\sin(\text{asin}(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{lat}))} \right\}$$

and that the hour angle (shd hr angle) is equal to the difference in longitude that would cause this surrogate horizontal dial to displace the local noon so it would match the original vertical decliner's local noon. In other words, one hour (15 degrees) of hour angle is also the same as 15 degrees of longitude difference:-

$$\text{DL} = \text{atan} \left\{ \frac{\sin(\text{dec})}{\sin(\text{asin}(\cos(\text{dec}) * \cos(\text{lat})) * \tan(\text{lat}))} \right\}$$

$$\text{DL} = \text{atan} \left\{ \frac{\sin(\text{dec})}{\cos(\text{dec}) * \cos(\text{lat}) * \tan(\text{lat})} \right\}$$

$$\text{DL} = \text{atan} \left\{ \tan(\text{dec}) / (\cos(\text{lat}) * \tan(\text{lat})) \right\}$$

but since

$$\sin / \cos = \tan$$

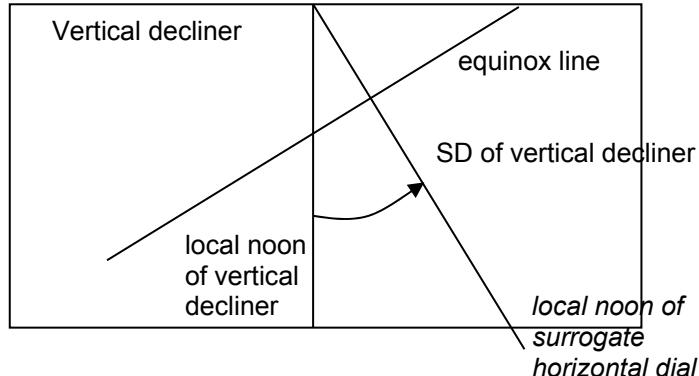
then

$$\tan(\text{lat}) = \sin(\text{lat}) / \cos(\text{lat})$$

thus

$$\cos(\text{lat}) * \tan(\text{lat}) = \cos(\text{lat}) * \sin(\text{lat}) / \cos(\text{lat}) = \sin(\text{lat})$$

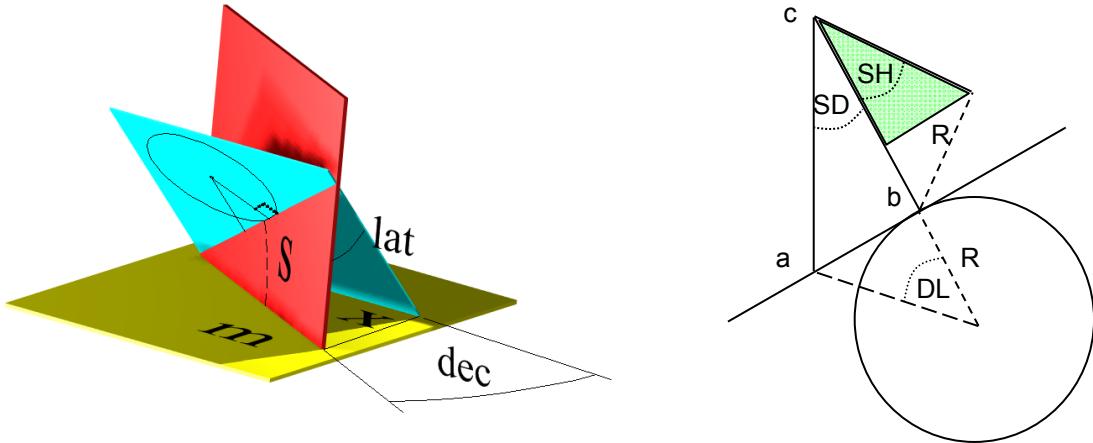
$$\text{so } \text{DL} = \text{atan}(\tan(\text{dec}) / \sin(\text{lat}))$$



**NOTE:** The above formula "DL" will ensure that local noon on a surrogate dial will be overlay local noon on the vertical decliner. If legal noon is desired, then the vertical decliner and the surrogate horizontal dial must consider dial longitude, so, the dial longitude difference from the legal meridian would be considered in both cases, as discussed in the big book.

**NOTE:** The DeltaCAD macro that draws analemmas for a dial has a limited range of analemmas, so if that DeltaCAD macro is used, you should reduce the absolute value of DL so that DL is:  $15 < \text{DL} > -15$  and this is only to ensure the DeltaCAD produced analemmas are usable, and for no other purpose. In other words it manages a weakness in our DeltaCAD analemma macro. Again, this is discussed in the big book.

VERTICAL DECLINER ~ **alternative proof** of Difference in Longitude ~ DL



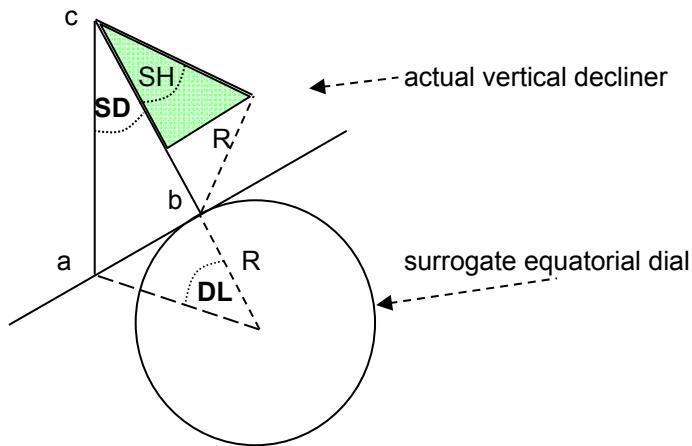
"DL" is the "Difference in Longitude" and is the degrees by which local apparent noon is displaced from the perpendicular from the equatorial dial to the equinox line. In essence, this is the longitude at which a non-longitude corrected h-dial would match this resulting vertical decliner.

$$\begin{aligned}
 \sin(SH) &= R / cb & \text{thus} & \quad cb = R / \sin(SH) \\
 \tan(SD) &= ab / cb & \text{thus} & \quad \tan(SD) = ab * \sin(SH) / R & \text{and} \\
 ab &= cb * \tan(SD) & \text{and} & \quad ab = \tan(SD) * R / \sin(SH) & \text{also} \\
 \tan(DL) &= ab / R & \text{thus} & \quad DL = \tan^{-1}(ab / R) = \tan^{-1}(ab * 1/R) \\
 && \text{and} & \quad DL = \tan^{-1}(R * \tan(SD) / \sin(SH)) * 1/R \\
 && \text{so} & \quad DL = \tan^{-1}(\tan(SD) / \sin(SH))
 \end{aligned}$$

being the radial for Local apparent noon

$$\begin{aligned}
 \text{but} \quad SD &= \tan^{-1}(\sin(dec) / \tan(lat)) & \text{from earlier} \\
 \text{and} \quad SH &= \sin^{-1}(\cos(dec) * \cos(lat)) & \text{from earlier} \\
 \text{so since} \quad DL &= \tan^{-1}(\tan(\tan^{-1}(\sin(dec) / \tan(lat))) / \sin(SH)) \\
 \text{hence} \quad DL &= \tan^{-1}((\sin(dec) * \cot(lat)) / \sin(SH)) & [\cot = 1/\tan] \\
 \text{then} \quad DL &= \tan^{-1}((\sin(dec) * \cot(lat)) / \sin(\sin^{-1}(\cos(dec) * \cos(lat)))) \\
 \text{hence} \quad DL &= \tan^{-1}((\sin(dec) * \cot(lat)) / (\cos(dec) * \cos(lat))) \\
 DL &= \tan^{-1}\left(\frac{(\sin(dec) * \cot(lat))}{(\cos(dec) * \cos(lat))}\right) \\
 DL &= \tan^{-1}\left(\tan(dec) / (\sin(lat) * \tan(lat) / \tan(lat))\right) & [\cos = \sin/\tan] \\
 \boxed{DL = \tan^{-1}(\tan(dec) / \sin(lat))}
 \end{aligned}$$

VERTICAL DECLINER ~ **alternative perspective** of Difference in Longitude ~ DL



"DL" is the "Difference in Longitude" and is the degrees by which local apparent noon is displaced from the perpendicular from the equatorial dial to the equinox line. In essence, this is the longitude at which a non-longitude corrected h-dial would match this resulting vertical decliner.

However, another perspective is that "DL" is, as shown above, a solar hour angle (on an equatorial dial) that produces a vertical decliner dial's hour line angle equal to "SD".

	B	C	D	E	F	G	H
2							
3			$sd = \text{atan}(\tan(dl) * \sin(sh))$				using the standard h-dial formula
4							
5			$\tan(sd) = \tan(dl) * \sin(sh)$				we removed the ATAN
6							
7			$\tan(dl) = \tan(sd) / \sin(sh)$				we switched sides
8							
9			$dl = \text{atan}(\tan(sd) / \sin(sh))$				we removes the TAN from the tan(DL)
10							
11	DL	sd	sh	dial longitude from legal meridian			
12							
13							$\text{DEGREES}(\text{ATAN}(\text{TAN}(\text{RADIAN}(D14)) / \text{SIN}(\text{RADIAN}(E14))))$
14	61.58	47.70	36.50				
15	33.94	28.00	52.20				
16							
17							
18							$\text{DEGREES}(\text{ATAN}(\text{TAN}(\text{RADIAN}(D19)) / \text{SIN}(\text{RADIAN}(E19)))) - F19$
19	58.38	47.70	36.50	3.20			
20	30.74	28.00	52.20	3.20			

Obviously, if the dial is not on the legal meridian, then the longitudinal difference between the dial's location and the legal meridian must be factored in, as shown in rows 18 et seq.

thus

$$DL = \text{atan}(\tan(sd) / \sin(sh)) - (\text{dial longitude} - \text{legal meridian})$$

## Declination (calendar) lines (for horizontal) dials using trigonometry.

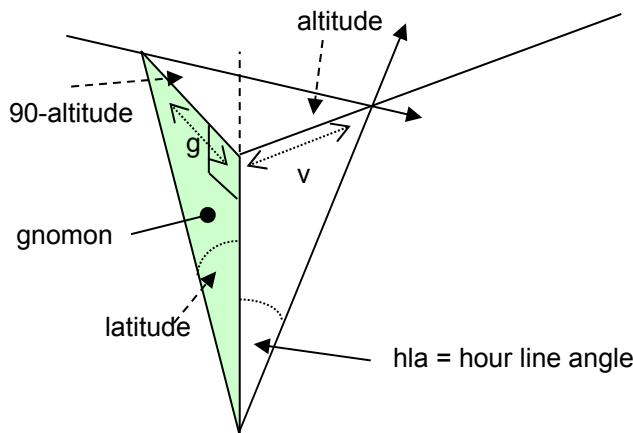
The declination line is nothing more than a depiction on the dial plate that shows the sun's declination. This varies by about plus or minus 23.5 degrees. It relates to the calendar date. Certain dates are significant, namely the winter and summer solstices, shortest day and shortest night respectively. The equinox has two dates and is when the day length is the same as the night time, however every day on the equator is an equinox. The solstice and equinox lines are helpful for other purposes, such as the Italian and Babylonian hours.

NOTE: Any flat hour angle dial plate whether horizontal or not can use these techniques.

This section on trigonometric methods was the basis for appendix tables A4.4 and A4.5

### Using the sun's altitude, certain standard declinations, and a unit gnomon linear height

Given a gnomon linear height, how far from the nodus base (nodus dropped perpendicular to dial plate) on a horizontal or flat dial is the declination point?



$$\tan(\text{alt}) = \text{gnomon linear height} / \text{distance from gnomon base} = g / v$$

$$\text{thus } v = g / \tan(\text{alt}) \quad \text{or} \quad v = g * \cot(\text{alt})$$

or in spreadsheet (Excel) terms which has the formula for altitude and where the declination is a fixed number such as 23.5, 0, or -23.5, or some other value

$$=\text{gnomonlinearheight} / \\ (\text{TAN}(\text{ASIN}(\text{SIN}(\text{RADIANS}(\text{decl})) * \text{SIN}(\text{RADIANS}(\text{lat})) + \text{COS}(\text{RADIANS}(\text{decl})) * \\ \text{COS}(\text{RADIANS}(\text{lat})) * \text{COS}(\text{RADIANS}(15 * (12 - \text{hh}))))))$$

This method works for all angles, and assumes a known perpendicular distance from the nodus to the dial plate. Thus this works for all style heights (latitudes including 0).

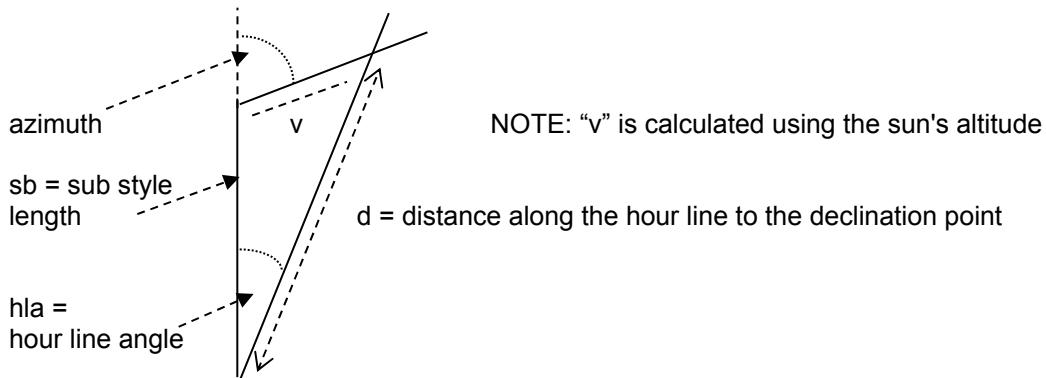
The tables for this method are in appendix **A4.4** for latitudes 0 to 65 for declinations of +23.5, 0, and -23.5

NOTE: The azimuth may be used as a vector from the nodus base, or using the law of sines, a distance from dial center along the hour line may be derived. However, the azimuth method fails when the azimuth is 0 or solar noon, it also fails at the equator. If using the dial center, then that dial center must be accessible which is often not true for great decliners.

## Declination (calendar) lines (for horizontal) dials using trigonometry (contd)

Using the sun's altitude and azimuth, certain standard declinations, and a unit gnomon linear height

Given a gnomon's linear height, how far along an hour line from the dial center is the declination point on a horizontal dial (with certain standard declinations)

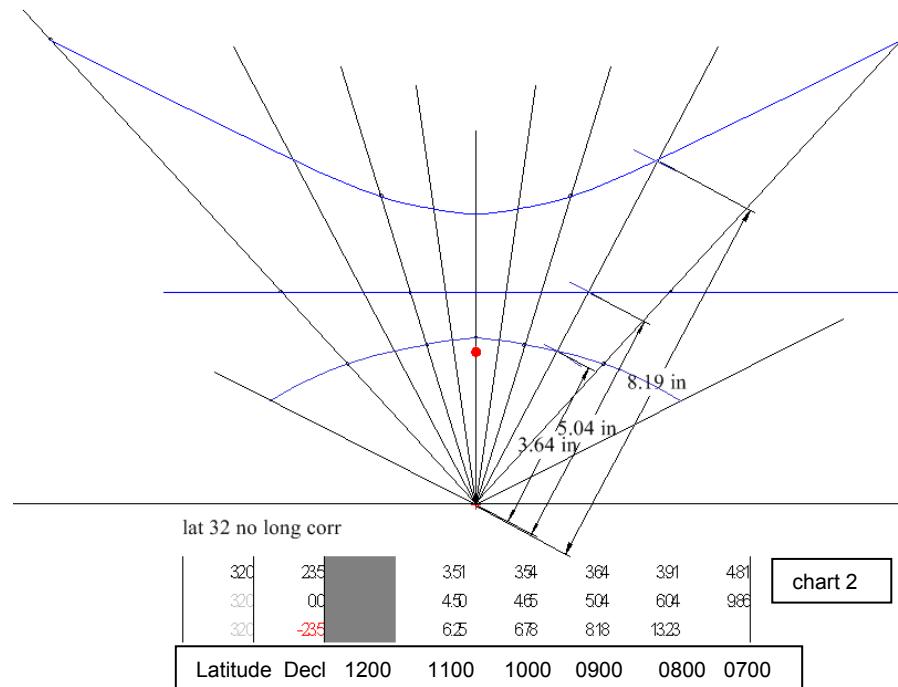


using the law of sines:  $d / (\sin(180-\text{azimuth})) = v / (\sin(\text{hla}))$   
 $d = v * \sin(180-\text{azimuth}) / (\sin(\text{hla}))$

This method fails when the azimuth is 0 or solar noon, it does not work on the equator, and is only practical if the dial center is accessible. This is often not true for great decliners. For this reason, this method is not common.

The azimuth method can be used from the gnomon base as an angle which when extended intersects the appropriate hour line, or using the law of sines can be used to develop the distance along an hour line from dial center which is the declination point. which is discussed here.

To the right is an example for latitude 32° using the distance from the dial center to the calendar point on the hour line. The gnomon linear height is 2 units.



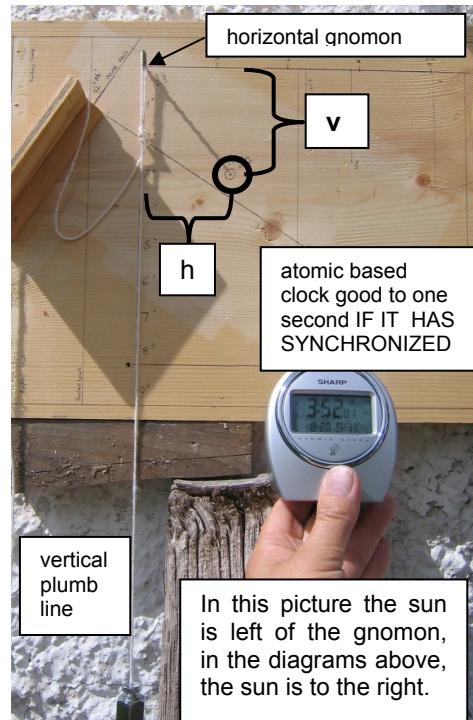
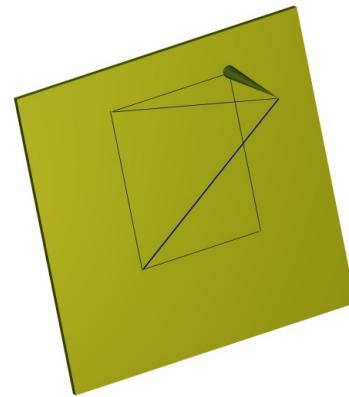
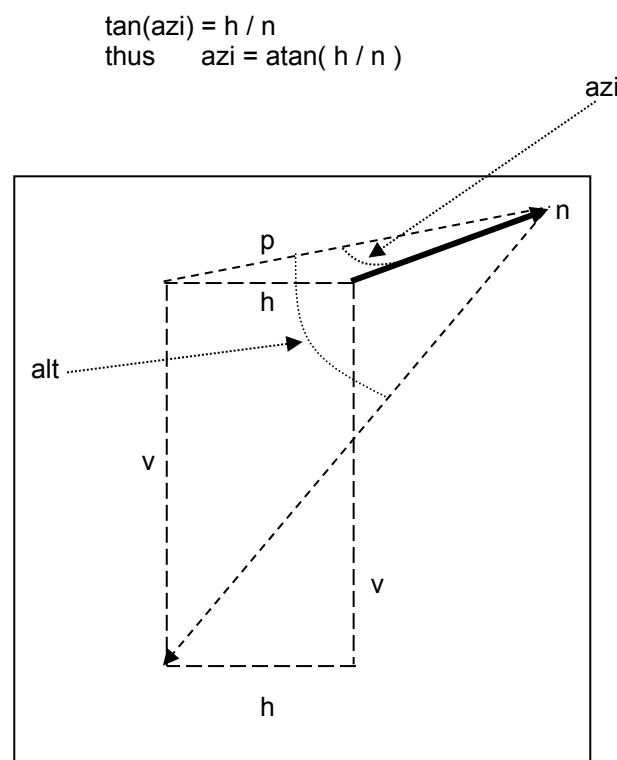
## PROOF OF ALTITUDE AND AZIMUTH ON A VERTICAL PLATE

This is the proof of the formulae for the azimuth-at-any-time method of deriving a wall's declination from true north or south. In essence actual azimuths are noted, and for their times, they are calculated also, and the difference between actual and calculated azimuths is the wall's declination.

Lines  $v$  and  $h$  are vertical and horizontal coordinates of a shadow on a plane that is affixed to a wall whose declination is to be measured.

Length  $n$  is the length of a rod perpendicular to the plane affixed to the wall whose shadow is cast on the plane generating coordinates  $v$  and  $h$  from where the rod meets the plane.

### AZIMUTH



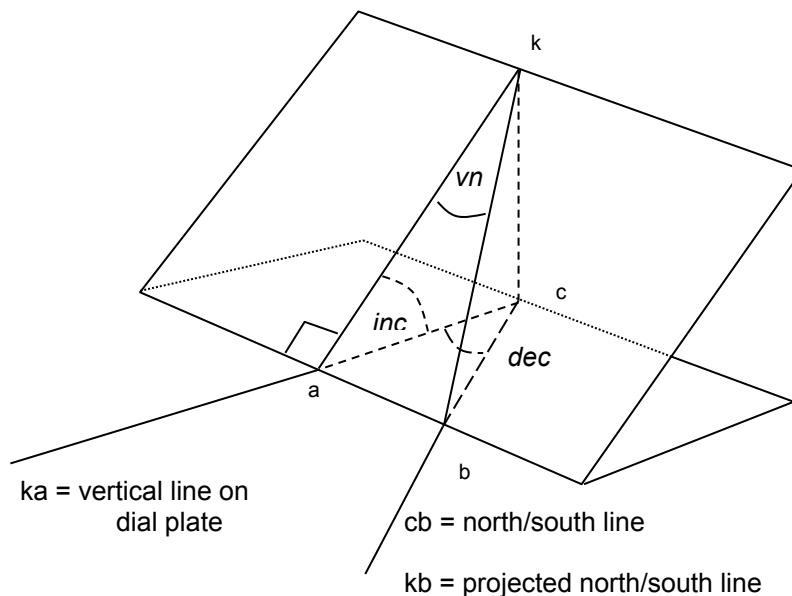
### ALTITUDE

$$\begin{aligned}\tan(\text{alt}) &= v / p \\ \text{and we already have } &\quad \tan(\text{azi}) = h / n \\ \text{and } &\quad \sin(\text{azi}) = h / p \\ \text{thus } &\quad p = h / \sin(\text{azi}) \\ \text{thus } &\quad \tan(\text{alt}) = v / p = v * \sin(\text{azi}) / h \\ \text{thus } &\quad \text{alt} = \text{atan} (\sin(\text{azi}) * v / h)\end{aligned}$$

## PROOF OF THE VERTICAL TO THE NOON LINE in an inclined decliner

Below is a picture, of the general dial in the celestial sphere, but in 3d.

$$\text{Vertical line to noon angle akb} = \tan(\text{dec}) * \cos(\text{inc fr hrz})$$

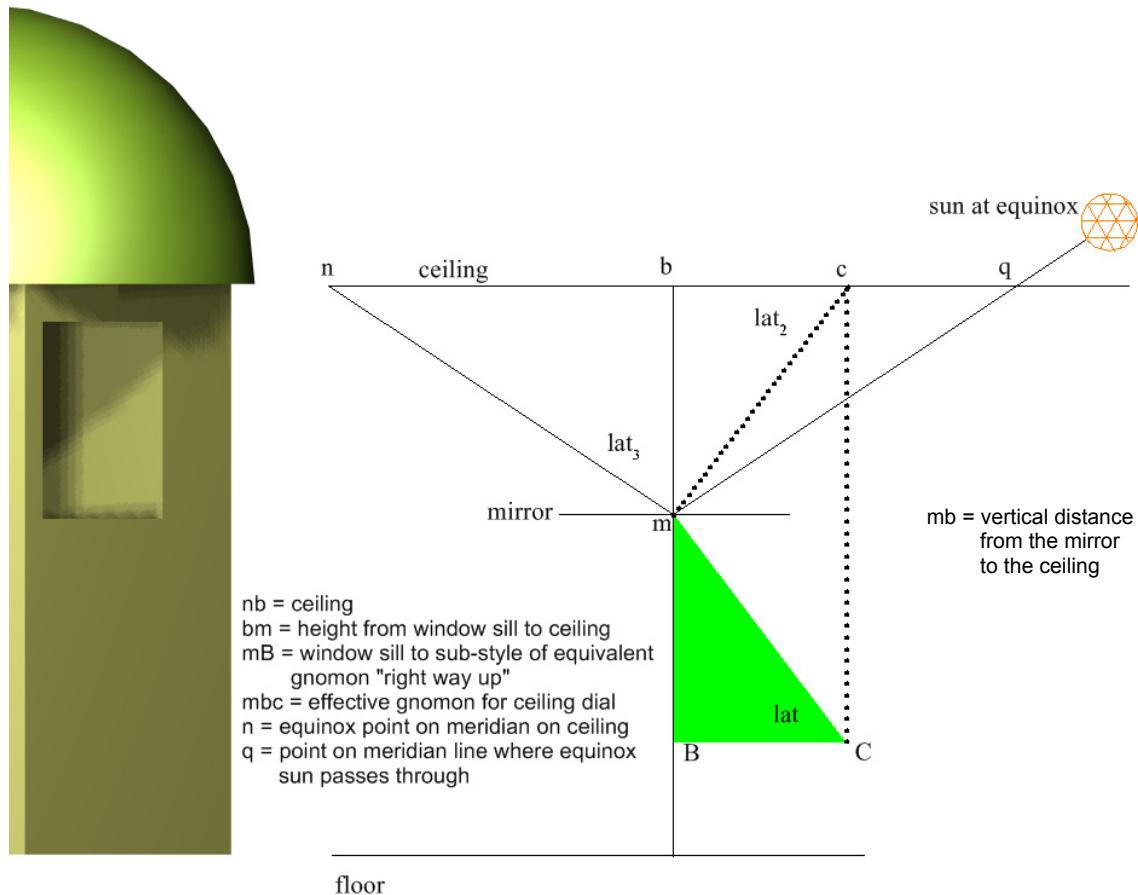


$$\begin{aligned} \tan(\text{dec}) &= ab / ac & \text{thus } ab &= ac * \tan(\text{dec}) \\ \cos(\text{inc}) &= ac / ak & \text{thus } ac &= ak * \cos(\text{inc}) \\ \tan(vn) &= ab / ak & \text{thus } vn &= \text{atan}(ab / ak) \end{aligned}$$

$$\begin{aligned} vn &= \text{atan}(ac * \tan(\text{dec}) / ak) & &= \text{atan}(ak * \cos(\text{inc}) * \tan(\text{dec}) / ak) \\ &= \text{atan}(\tan(\text{dec}) * \cos(\text{inc})) \end{aligned}$$

## REFLECTING DIALS (ALSO KNOWN AS CEILING DIALS)

With a reflecting dial, or ceiling dial, a mirror is placed on a surface such as a window sill, of course it is set level. The sun is reflected from the mirror at point "m", and for this example, the equinox is assumed. The reflected ray displays a reflection of the sun on the ceiling, a spot, and this is at point "n". Because the following uses the equinox, angle "Cmq" is a right angle.



The sun comes through "q" to "m" and is reflected to "n". If this were a true horizontal dial, the gnomon would be "BmC", where "Bm" is the style linear height, and this matches the height of the ceiling above the little mirror, "mb". (NOTE: lat = lat<sub>2</sub> = lat<sub>3</sub>, subscripts are order of derivation).

Angle "BCm" is the latitude, so also is "bcm" and "nmb".

$$\text{Since } nmb = \text{latitude}, \quad \tan(\text{lat}) = nb / bm \quad \text{thus} \quad nb = mb * \tan(\text{lat}) \quad [1]$$

$$\text{Since } bcm = \text{latitude}, \quad \tan(\text{lat}) = mb / bc \quad \text{thus} \quad bc = mb / \tan(\text{lat})$$

thus, distance from equinox point on meridian to the dial center "c" is nc = nb + bc

$$\text{hence } nc = mb * \tan(\text{lat}) + mb / \tan(\text{lat})$$

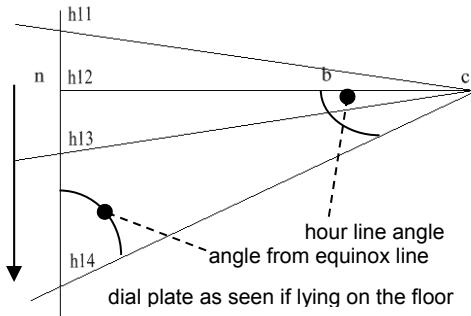
$$= mb * (\tan(\text{lat}) + 1/\tan(\text{lat})) \quad [2]$$

[The conventional formula is = 2\*mb /sin(2\*lat) ]

Thus we have a dial plate, with a meridian line (L.A.T. noon) and an equinox point on it, thus the equinox line can be drawn.

Similarly, we have the dial center, it is located from the equinox point "n" at a distance:-

$$= mb * (\tan(\text{lat}) + 1/\tan(\text{lat})) \quad \text{derived formula} \quad [2]$$



The dial plate on the ceiling is reversed, otherwise it is a normal horizontal dial. The morning hours are to our right as we look up at the ceiling, whereas looking down on a normal horizontal dial, they are to the left. With this in mid, the dial layout is a normal horizontal dial for the design latitude and longitude, except we reverse the sense of the angles.

Ceiling dials have a dial center that is not in existence, but it is used to locate the hour line angles. For example, 2pm, the "h14" line will have an hour line angle of some value "x". Since the dial center "c" is not in physical existence, we use the distance from point "n", hour point on equinox line h14 from "n"

$$\begin{aligned} nh_{14} &= \tan(\text{hour line angle}) * nc \\ &= \tan(\text{hour line angle}) * mb * (\tan(\text{lat}) + 1/\tan(\text{lat})) \end{aligned}$$

and at point h14, a line is drawn of 90-hour line angle. Hence the hour lines can be drawn. Calendar curves can also be drawn using the same techniques covered elsewhere. A key point to remember is that these dials can get rather large, reducing the mirror's height from the ceiling will make the dial smaller.

#### REFLECTING OR CEILING DIALS

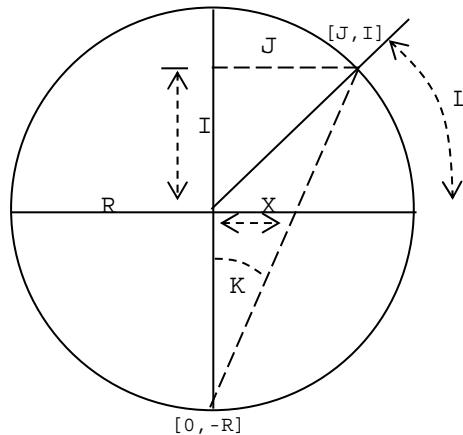
	1 nb = mb * tan(lat) [1]	nc=mb*(tan(lat)+1/tan(lat)) [2]
Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center
30	0.5774	2.3094
31	0.6009	2.2651
32	0.6249	2.2252
33	0.6494	2.1893
34	0.6745	2.1571
35	0.7002	2.1284
36	0.7265	2.1029
37	0.7536	2.0806
38	0.7813	2.0612
39	0.8098	2.0447
40	0.8391	2.0309
41	0.8693	2.0197
42	0.9004	2.0110
43	0.9325	2.0049
44	0.9657	2.0012
45	1.0000	2.0000

Distance from mirror to ceiling (style linear height)

Latitude	Distance from point above mirror to equinox point on the meridian line	Distance from meridian equinox point to dial center
46	1.0355	2.0012
47	1.0724	2.0049
48	1.1106	2.0110
49	1.1504	2.0197
50	1.1918	2.0309
51	1.2349	2.0447
52	1.2799	2.0612
53	1.3270	2.0806
54	1.3764	2.1029
55	1.4281	2.1284
56	1.4826	2.1571
57	1.5399	2.1893
58	1.6003	2.2252
59	1.6643	2.2651
60	1.7321	2.3094
61	1.8040	2.3584

## FORMULA DERIVATION FOR PLANISPHERIC ASTROLABES

The geometry of the astrolabe can be converted to trigonometry, as follows.



$J$  is the "x" distance from the vertical to the angle "L" intercept point

$I$  is the "y" distance for that intercept point

$L$  is an angle (in real life it is latitude plus as well as minus 90-alt where alt is the sun's altitude)

$R$  is the "Earth radius" used for the projection

$K$  is an intermediate working angle

$X$  is the distance along the horizontal for the intercept between point  $J$ ,  $I$  and  $0, -R$

so... given  $R$  and  $L$ , find  $X$

clearly:

$$X = R * \tan(K)$$

similarly:

$$J = (R+I) * \tan(K)$$

and also:

$$I = R * \sin(L)$$

similarly:

$$J = R * \cos(L)$$

also:

$$\tan(K) = J / (R + I)$$

but:

$$\begin{aligned} X &= R * \tan(K) \quad \text{which thus makes } X = (R * J) / (R + I) \\ &= (R * R * \cos(L)) / (R + R * \sin(L)) \\ &= (R * \cos(L)) / (1 + \sin(L)) \end{aligned}$$

The above is the basis for almucanter projections. In which case angle  $L$  is the latitude plus and also minus the co-altitude (sun's altitude from 90 degrees). The above is also used for one of the two points of the ecliptic circle, namely the one where  $L = 23.44$  degrees above the horizontal line. So, angle  $L$  is derived twice for each altitude of the sun:-

$$\begin{aligned} L &= \text{latitude} + (\text{90-altitude}) \quad \text{and} \\ L &= \text{latitude} - (\text{90-altitude}) \end{aligned}$$

And this generates two  $X$  values, from whence we derive a center and a radius for the almucanter (for that altitude at that latitude), namely  $X_1$  and  $X_2$ . The almucanter center and radius are also simply found by:-

the center "C" is at

$$\begin{aligned} y &= 0 \text{ and} \\ x &= 0.5 * (x_1 + x_2) \end{aligned}$$

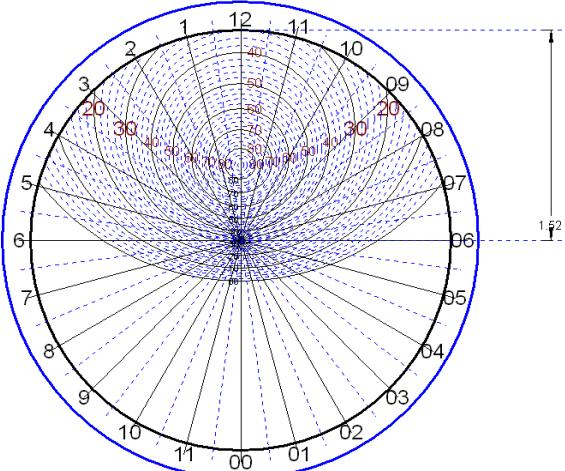
the radius is

$$r = x_2 - C$$

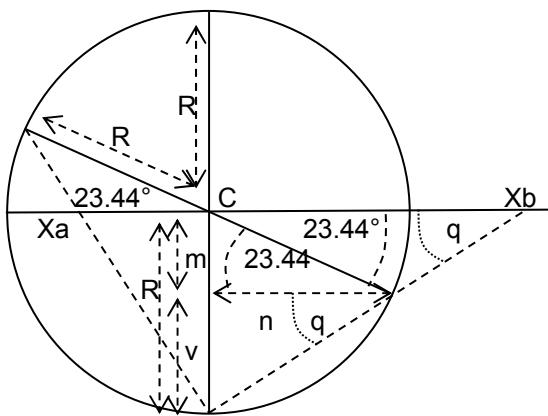
Since we have the latitude, we run a loop from 0 to 90 altitudes and draw those circles.

The detailed logic and coding is contained in the DeltaCAD macro for the astrolabe that is on the CD and web site associated with this book.

A boundary circle is defined by the rete's maximum dimension,  $XB$ , see next page.



The ecliptic circle is projected to the rete.



The rete is a circle, that has a rotational center at C but whose circle center is between points Xa and Xb. The rotational center of the rete fits on the almucanter that matches the dial's latitude, i.e. for a dial for latitude 32° the rete's rotational center would be on the 32° almucanter.

Point Xa is found using the formula on the preceding page, using an angle "L" of 23.44°

The same radius used for almucanders must be used for this ecliptic rete also.

"q" is some intermediate angle.

The length from C to point Xb, is found as follows. For simplicity we shall say Xb is the length in question. The angle "q" exists in two places, the vertical drop "m" is in two places also, and radius "R" is on the four radials, the replication of "m" and "R" is not shown to de-clutter the pictorial.

$$\begin{aligned} \text{clearly: } & \sin(23.44) = m / R \text{ thus: } m = R * \sin(23.44) \\ & \cos(23.44) = n / R \text{ thus: } n = R * \cos(23.44) \\ & \tan(q) = R / Xb \text{ thus: } Xb = R / (\tan(q)) \\ & \text{also: } \tan(q) = v / n \\ & \text{thus: } v = R - R * \sin(23.44) \end{aligned}$$

$$\text{because: } \tan(q) = v / n \quad \text{then: } \tan(q) = \frac{R - R * \sin(23.44)}{R * \cos(23.44)}$$

$$\text{since: } Xb = R / \tan(q) \text{ then: } Xb = \frac{R}{(R - R * \sin(23.44) / R * \cos(23.44))}$$

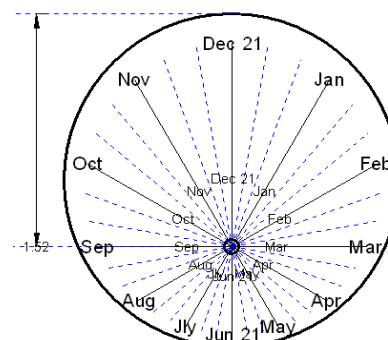
$$\text{or: } Xb = \frac{R * R * \cos(23.44)}{R - R * \sin(23.44)}$$

$$\text{or: } Xb = \frac{R * \cos(23.44)}{1 - \sin(23.44)}$$

All that is then left is the derivation of the rete's rotational point (0,0), which is also the center from which the month lines radiate, and the rete's physical center for a boundary circle. Xb is the rotational maximum radius, which defines the limiting circle of movement for the dial plate and its almucanders.

It is critical that the "earth radius" be the same for the climate plate of almucanders and rete ecliptic profiles. The resulting final climate plate of almucanders should also match the final rete, which it did.

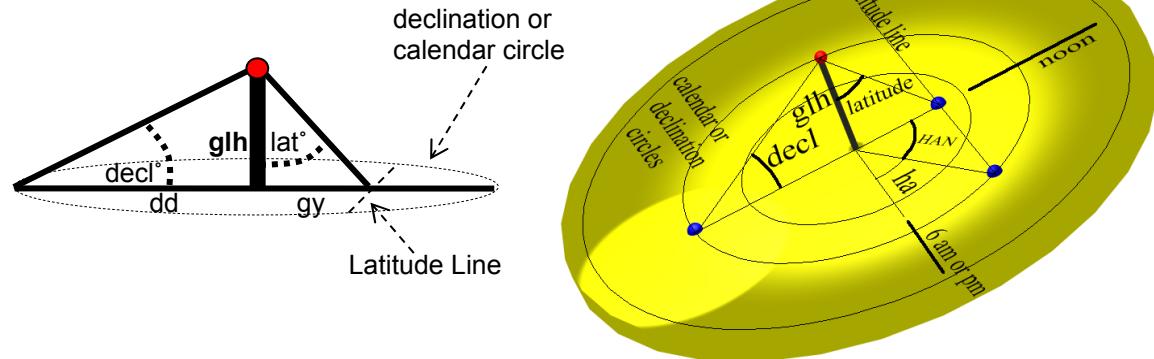
The DeltaCAD macros are included on this book's CD and website to show the mathematical methods involved. They show how to correctly locate the rotational points and the perimeter circles. They also demonstrate code to contain the month radials within a constraining circle, and, albeit unused, code to contain almucanter arcs within a circle. That logic alone is worthy of study.



## FORMULA DERIVATION FOR SUNRISE AND SUNSET HOUR ANGLE FROM 0600 OR 1800, OR, FROM NOON LOCAL APPARENT TIME.

Refer to the nomogram in appendix 6 for sunrise/set based on the equatorial dial. The basis of this proof is that a gnomon's nodus on an equatorial dial casts a sunrise/set shadow on a fixed straight line for the latitude regardless of the sun's declination. The shadow's point varies along that line depending on the sun's declination, and that indicates the hour of sunrise/set. Here, "glh" is the gnomon linear height which controls the radius of the day's calendar circle as well as the location's latitude line. The solution is where the latitude line intersects the day's calendar circle.

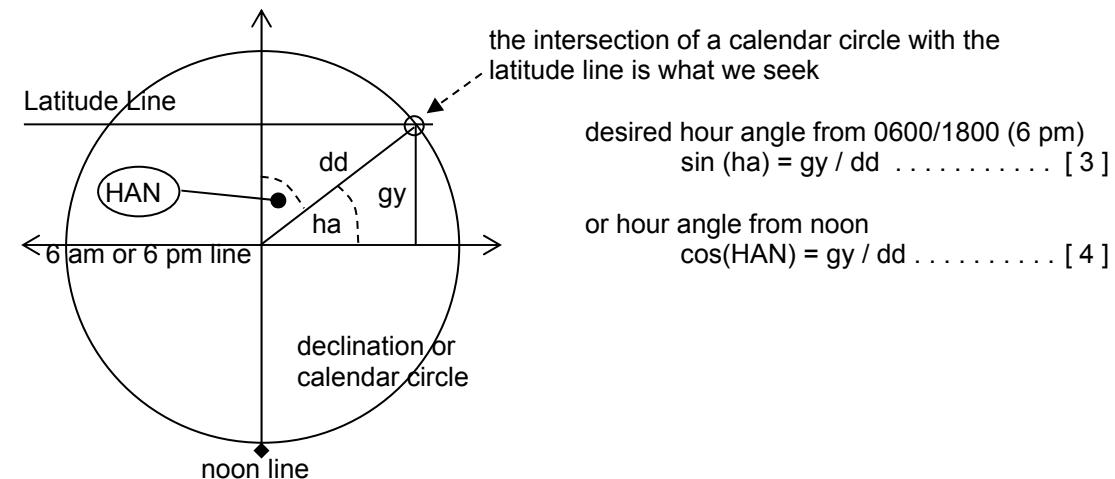
**VIEW FROM THE SIDE**



$$\tan(\text{lat}) = \text{gy} / \text{glh} \dots [1]$$

$$\tan(\text{decl}) = \text{glh} / \text{dd} \dots [2]$$

**VIEW FROM ABOVE**



so	$\text{ha}$	$=$	hour angle from 0600 or 1800 (6 pm)	
		$=$	$\arcsin(\text{gy} / \text{dd})$	from 3
			$\arcsin(\text{glh} * \tan(\text{lat}) / \text{dd})$	from 1
			$\arcsin(\text{glh} * \tan(\text{lat}) / (\text{glh} / \tan(\text{decl})))$	from 2
			$\arcsin(\tan(\text{lat}) * \tan(\text{decl}))$	canceling out

or	$\text{HAN}$	$=$	hour angle from noon	
		$=$	$\arccos(\text{gy} / \text{dd})$	from 4
		$=$	$\arccos(\text{glh} * \tan(\text{lat}) / \text{dd})$	from 1
		$=$	$\arccos(\text{glh} * \tan(\text{lat}) / (\text{glh} / \tan(\text{decl})))$	from 2
		$=$	$\arccos(\tan(\text{lat}) * \tan(\text{decl}))$	canceling out

## APPENDIX 8 - COLLECTION OF FORMULAE INCONSISTENCIES AND APPARENT INCONSISTENCIES FORMULAE CONVERTED FOR A SPREADSHEET

COMMON ABBREVIATIONS ARE:- Solar declination: "dec", latitude: "lat", the sun's local hour angle: "lha" being 15 times the hour from noon, "hla" is the hour line angle on a dial, "sh" is angular style height, "sd" is style distance. Check if degrees or radians, in doubt check .XLS files.

### SUNS DECLINATION FOR ANY GIVEN DAY OF THE YEAR

Source: <http://eande.lbl.gov/Task21/C2/algo1/1-11.html>

Day number, J J=1 on January 1, 365 on 31 December, February has 28 days, "pi" is 3.14159

Jan	Feb	Mar	Apr	May	Jun
0	31	59	90	120	151
Jly	Aug	Sep	Oct	Nov	Dec
181	212	243	273	304	334

Day angle:  $da = 2 * \pi * (j - 1) / 365$  (is an intermediate figure in radians)

A8.1

Sun Declination:  $dec = \text{degrees} (0.006918 - 0.399912 * \cos(da) + 0.070257 * \sin(da) - 0.006758 * \cos(2 * da) + 0.000907 * \sin(2 * da) - 0.002697 * \cos(3 * da) + 0.001480 * \sin(3 * da))$

A8.2a

alternative formula:  $dec = 23.45 * \sin((0.9678(j - 80)))$  source: Claude Hartman  
multiply by  $2 * 3.1416 / 360$   
to get radians from degrees

A8.2b

### SUNS ALTITUDE AND AZIMUTH ON ANY GIVEN HOUR GIVEN THE SUNS DECLINATION

The sun's declination is "dec", and the latitude is "lat", and the sun's local hour angle is "lha".

ALTITUDE: The sun's altitude is its angle when looked at face on  
 $alt = \text{ASIN}(\text{SIN}(dec) * \text{SIN}(lat) + \text{COS}(dec) * \text{COS}(lat) * \text{COS}(lha))$

A8.3

AZIMUTH:  $azi = \text{ATAN}(\text{SIN}(lha) / ((\text{SIN}(lat) * \text{COS}(lha)) - (\text{COS}(lat) * \text{TAN}(dec))))$

A8.4

note: Some authors present two different formulae. They agree in all aspects except for 6am and thus also 6pm, when using the author's spreadsheet.

### SUNRISE AND SUNSET TIME FORMULA

The sun's declination is "dec", and latitude is "lat". The sun's local hour angle is "lha".  
For the setting or rising sun, the azimuth and the sun's hour angle from noon are:-

A8.5

$$\text{aziRiseSet} = 180 - \arccos(\text{sin}(dec) / \text{cos}(lat))$$

A8.6

$$\text{lhaRiseSet} = \arccos(\text{tan}(lat) * \text{tan}(dec))$$

## EQUATORIAL DIAL CALENDAR AND SUNRISE/SUNSET LINE DATA

Calendar lines are arcs whose radius = gnomon linear height / tan( dec )  
 Horizontal sunrise/set line from gnomon distance = gnomon linear height \* tan( lat )

A8.7

## HORIZONTAL DIAL HOUR LINE ANGLE

$$hla = \text{atan}(\sin(\text{lat}) * \tan(\text{lha}))$$

also the reverse:

$$\text{lha} = \text{atan}(\tan(hla) / \sin(\text{lat}))$$

A8.8

## POLAR DIAL Times are from noon. "sh" = style linear height.

$$\begin{aligned} \text{from style to hour line} &= sh * \tan(\text{lha}) \\ \text{distance up an hour line to calendar line} &= sh * \tan(\text{dec}) / \cos(\text{lha}) \end{aligned}$$

A8.9

A8.10

## MERIDIAN DIAL ~ EAST/WEST VERTICAL NON DECLINING DIAL

"lha" time here is from 6am or 6pm, not from noon. "sh" = style linear height.

$$\begin{aligned} \text{from style to hour line} &= sh * \tan(\text{lha from 6pm or 6am}) \\ \text{distance up an hour line to calendar line} &= sh * \tan(\text{dec}) / \cos(\text{lha}) \end{aligned}$$

A8.11

A8.12

## SOUTH VERTICAL NON DECLINER HOUR LINE ANGLE

$$hla = \text{atan}(\tan(\text{lha}) * \sin(90 - \text{lat}))$$

angle hour line makes with  
12 o'clock line

A8.13

$$hla = \text{atan}(\tan(\text{lha}) * \cos(\text{lat}))$$

since  $\cos 0 = \sin 90$  etc

## ANALEMMATIC DIAL

"lat" is latitude, "lha" is the sun's local hour angle from noon (15 times the hour from local noon)

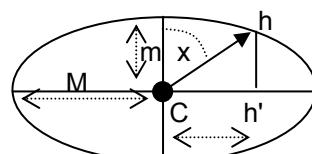
$$m = M * \sin(\text{lat})$$

$m$  = semi minor axis, you select the "M" value

A8.14

an hour point's horizontal distance:-

$$Ch' = M * \sin(\text{lha})$$



A8.15

to get an hour point's vertical distance:-

$$hh' = M * \sin(\text{lat}) * \cos(\text{lha})$$

A8.16

or to get an hour point by angle from C assuming an ellipse has been drawn

$$x = \text{arctan}(\tan(\text{lha}) / \sin(\text{lat}))$$

A8.17

to get the analemmatic points for the gnomon up or down the "m" scale

$$z = M * \tan(\text{dec}) * \cos(\text{lat})$$

A8.18

### STANDARD TIME FROM L.A.T. (Local Apparent Time, do not confuse L.A.T. with "lat")

$$\begin{array}{lll} \text{legal time} & = \text{L.A.T.} & + \text{EOT.corr} + \text{west.long.corr} + 1 \text{ if summer} \\ \text{or standard} & & - \text{east.long.corr} \\ \text{time} & & \end{array}$$

A8.19

### STANDARD TIME TO MARK AN HOUR LINE (as in calibrating hour lines using empirical dialing)

$$\begin{array}{lll} \text{dial hour point} & = & \text{clock time for that hour point} + \text{EOT i.e.} \\ \text{legal time to} & = & \text{desired L.A.T.} + \text{EOT} \\ \text{mark} & & \end{array}$$

This is not inconsistent. If the EOT were  $-10$  minutes, then when the dial reads 1400, the legal time would be 1350. So, at 1350, with an EOT of  $-10$ , the sun's shadow will indicate the 1400 hour point. This may appear inconsistent with the rules of algebra, however it is correct. Because of this apparent inconsistency, the dialist is advised to draft a table of times and the hour point they would thus indicate before marking a dial empirically.

### SOLAR TIME FROM STANDARD TIME (as in finding true north)

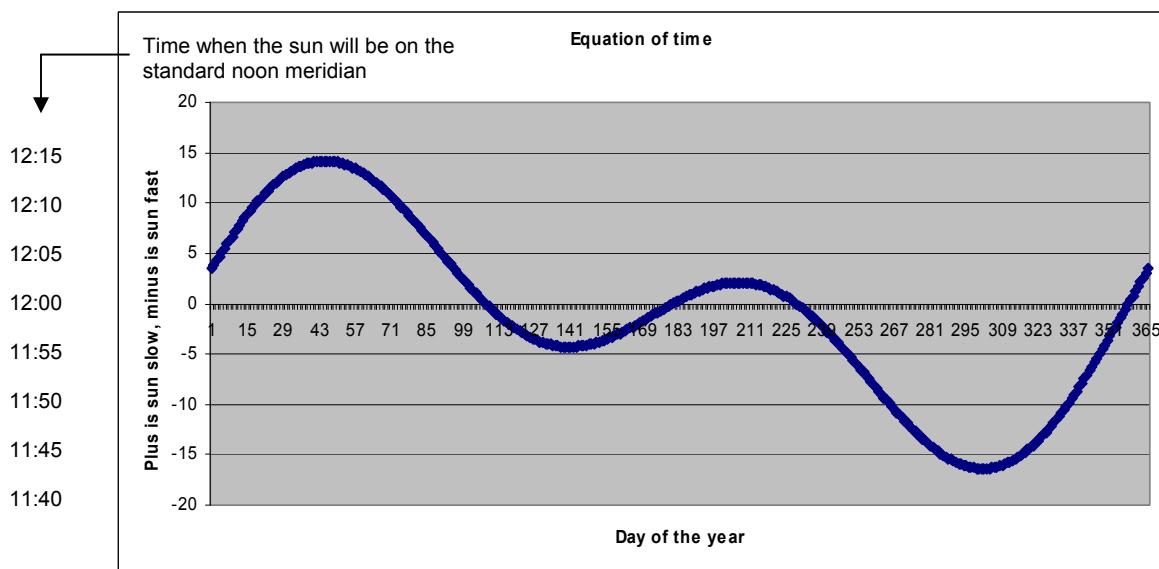
$$\text{Legal time} = \text{L.A.T.} + ( \boxed{\begin{array}{l} + \text{west.long.corr} + \text{EOT.corr} + 1 \text{ if summer} \\ - \text{east.long.corr} \end{array}} )$$

Solar noon indicates true north because the sun is at its highest point. Thus the shadow produced at solar noon will point to true north. Solar noon happens at the standard time adjusted as follows:-

$$\begin{array}{ll} \text{legal time} & = 12:00:00 + \text{EOT} + 1 \text{ (if summer)} + \text{longitude correction} \\ \text{for solar} & \\ \text{noon} & \end{array}$$

A8.20

This is not inconsistent. It may appear that signs should be reversed, however we are in fact achieving the correct arithmetic rules.



## VERTICAL DECLINER ~ HOUR LINE ANGLES & GNOMON ANGLES

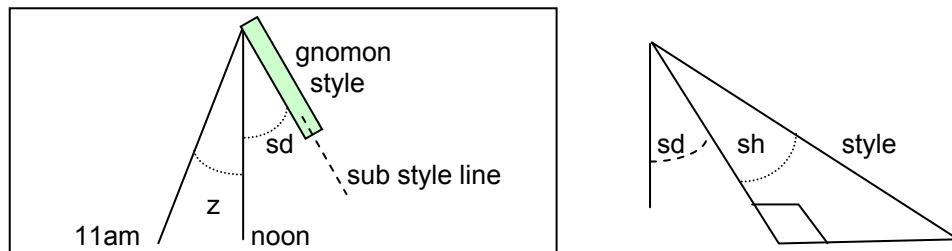
The hour line angles are:  $hla = \text{atan}(\cos(\text{lat}) / (\cos(\text{dec}) * \cot(lha) + \sin(\text{dec}) * \sin(\text{lat})))$  A8.21

or, see appendix 7:  $hla = \text{atan}(\sin(hs) / \sin(90 - hs - \text{dec})) * \tan(\text{lat})$   
where "hs" is a surrogate h-dial's hour line angle A8.21a

Gnomon rotation simplifies calendar and analemma drafting, and uses the following formula:-

Gnomon offset from vertical is:  $sd = \text{atan}(\sin(\text{dec}) / \tan(\text{lat}))$  **Style Distance** A8.22

Style and sub style angle is:  $sh = \text{asin}(\cos(\text{lat}) * \cos(\text{dec}))$  **Style Height** A8.23



A design for South xx degrees East provides figures usable for the other three quadrants. The afternoon NxxW uses SxxE pm hours, and the morning NxxE uses SxxW am hours. If longitude correction is applied, care must be applied as hour lines shift. The North facing decliner gnomons are inverted, and the vertical is midnight.

**DL**, "Difference in Longitude" derived for vertical decliners, is a "longitude" for a surrogate horizontal dial of latitude SH, which then enables this surrogate dial's analemma and other features to be placed on the original vertical decliner.

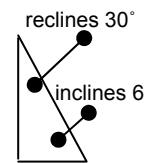
$$DL = -(\text{Atan}(\tan(\text{dec}) / \sin(\text{lat})) - (\text{Ing} - \text{ref}))$$
 A8.24

$$DL = \text{atan}(\tan(sd) / \sin(sh)) - (\text{dial longitude} - \text{legal meridian})$$
 A8.24a

## VERTICAL EAST OR WEST DIAL THAT SLOPES OR INCLINES/RECLINES

The latitude of the "design dial" is:-  $= 90 - \text{the latitude of the actual dial}$   
and

The declination of the "design dial" is:-  $= 90 - \text{the reclinaton of the actual dial}$   
or  $= \text{the inclination of the actual dial}$


A8.25
A8.26

## BIFILAR SUNDIAL

There are variations on this dial design that make it universal. However, the following two formulae are for the bifilar dial designed for a specific latitude.

The north south wire can be any height, the east west wire height is equal to:-  
east west wire height  $= \text{height of the north south wire} * \sin(\text{lat})$

A8.27

While the north south wire is placed over the noon line, the east west wire is placed at a distance from the dial center that is equal to:-

dist from dial center  $= \text{height of the north south wire} * \cos(\text{lat})$

A8.28

reference: <http://www.de-zonnewijzerkring.nl/eng/index-bif-zonw.htm>

Beware that the shadow of the cross-hair can be off the dial plate for early hours or winter hours.

## EQUATION OF TIME

A formula derived from Frans Maes from data by Savoie producing the EOT in minutes and using two sine waves is used for some spreadsheets. The values in the  $\sin(\dots)$  function result in radians, so the formula is spreadsheet ready as-is. Value d = 1 to 365

**A8.29a**

$$E = 7.36 * \sin(2 * 3.1416 * (d - 4.21) / 365) + 9.92 * \sin(4 * 3.1416 * (d + 9.9) / 365)$$

Another formula using the sum of three sine waves is used for some other spreadsheets. The  $\sin(\dots)$  values result in degrees hence the required indicated radian conversion.

**A8.29b**

$$\begin{aligned} E = & -1 * (9.84 * \sin(\text{RADIANS}(2 * (360 * (\text{mm1} + \text{dd} - 81) / 365))) - \\ & 7.53 * \cos(\text{RADIANS}(360 * (\text{mm1} + \text{dd} - 81) / 365)) - \\ & 1.5 * \sin(\text{RADIANS}(360 * (\text{mm1} + \text{dd} - 81) / 365))) - 0.3 \end{aligned}$$

where: mm1 is the number of days prior to this month's day 1, So Jan is 0, Feb is 31, Mar is 59, April is 90, etc, assuming a non leap year. For leap years add 1 for March to December.

Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec
0	31	59	90	120	151	181	212	243	273	304	334

dd is the day of the month, being 1 to 31

i.e. mm1+dd is the Julian day of the year

Another three wave formula is:

**A8.29c**

$$\begin{aligned} E = & 7.5 * \sin(\text{RADIANS}(d - 5)) - 10.2 * \sin(\text{RADIANS}(1.93 * (d - 80))) + \\ & 0.5 * \sin(\text{RADIANS}(1.5 * (d - 62))) \end{aligned}$$

Another formula derived from the work of Frank Cousins uses the sum of seven sine waves, produces the EOT in seconds, however this book does not use it in any programs:-

**A8.29d**

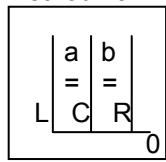
$$E = -(-97.8 * \sin(SL) - 431.3 * \cos(SL) + 596.6 * \sin(2 * SL) - 1.9 * \cos(2 * SL) + 4 * \sin(3 * SL) + 19.3 * \cos(3 * SL) - 12.7 * \sin(4 * SL))$$

where "SL" is the solar longitude, being  $SL = (-1 * ((356 / 365.2422) * 360 - 270)) + \text{ Julian day of year}$   
the values in  $\sin(\dots)$  result in degrees, so the  $\text{RADIANS}(\dots)$  function (not shown) is required for a spreadsheet.

Every approximation is just that, and this book uses several methods for the EOT to demonstrate the real world of approximations, with their benefits as well as drawbacks. Even established published tables vary by almost a minute. Part of this is explained by the year within a leap year cycle, part by the decade the table was printed, and so on. **The most accurate formula** uses the astronomical Julian day, this is involved and discussed in detail in **chapter 5** of the main book. Please refer to appendix 10 which has some other sources of formulae.

## NOMOGRAM STYLES AND ASSOCIATED FORMULAE (refer to chapter 32)

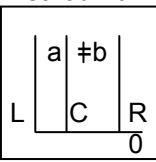
method 1a



$$C = L + R$$

can multiply and divide with logs

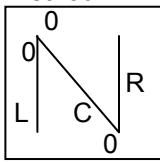
method 1b



$$C = L + R$$

can multiply and divide with logs

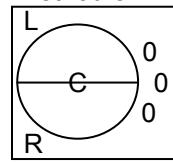
method 2



$$C = L / R$$

logs not needed

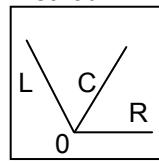
method 3



$$C = L * R$$

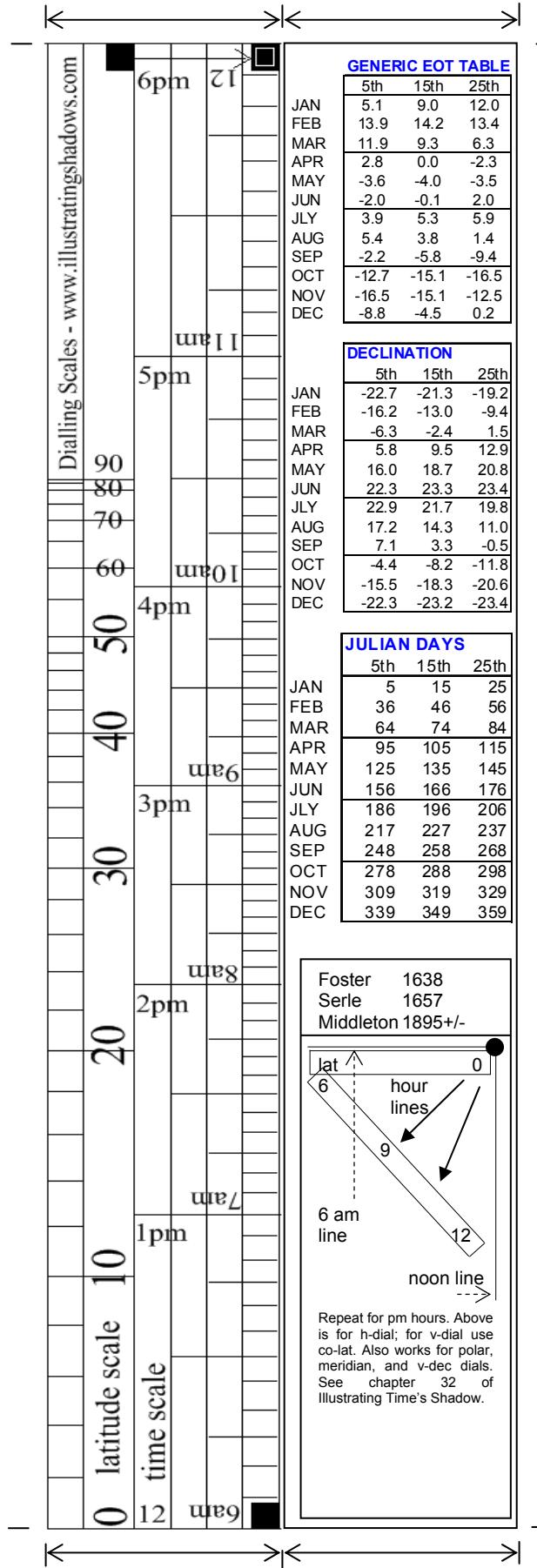
logs not needed

method 4



$$1/C = 1/L + 1/R$$

logs not needed



### DIALING SCALES (Foster, Serle, Middleton)

For a given length on the hours scale of say 10, the distances from 9 am or 3 pm (1500), the mid point hours, are:-

$$\text{dist} = \text{scale}[\text{i.e. } 10] * \tan(15 * \text{hours})$$

A8.30

And the distances for latitude are:

$$\text{dist} = \text{scale}[\text{i.e. } 10] * \sin(\text{lat}) / \sqrt{1 + \sin^2(\text{lat})}$$

A8.31

A spreadsheet to calculate the scales is available in:-

*illustratingShadows.xls*

A picture of the scales is in a file called:-

*Dialling Scales Serle Foster Middleton.JPG*

A TurboCAD model is called:-

*Dialling Scales Serle Foster Middleton.tcw*

Note:  $\sin^2(\text{lat})$  means:-  $\sin(\text{lat}) * \sin(\text{lat})$

Note: Dialing scales can be used not only for horizontal, vertical, and vertical decliner, but also for polar and meridian dials.

Note: The image to the left can be cut out, folded, and laminated.

## INCLINED DECLINERS ~ HOUR LINE ANGLES & GNOMON ANGLES

Style Distance from noon

$$SDn = \text{atan}((\sin(\text{dec}) * \sin(\text{inc}) * ((\sin(\text{inc}) * \cos(\text{dec}) - \tan(\text{lat}) * \cos(\text{inc})) / (\cos(\text{inc}) + \tan(\text{lat}) * \cos(\text{dec}) * \sin(\text{inc})))))) \quad \boxed{\text{A8.32}}$$

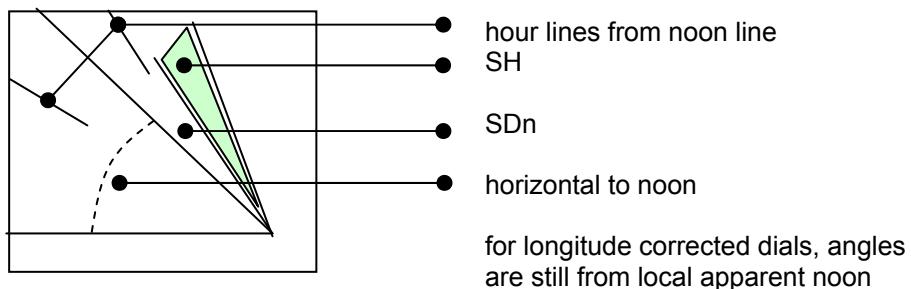
$$\text{Style Height SH} = \text{asin}((\cos(\text{lat}) * \sin(\text{inc}) * \cos(\text{dec})) - (\sin(\text{lat}) * \cos(\text{inc}))) \quad \boxed{\text{A8.33}}$$

The SD and hour line angles are from the noon line. They are adjusted with the angle from the horizontal to noon distance, and the resulting number subtracted from 90 degrees provides the same data with the vertical as a reference.

$$\text{Horizontal to noon} = \text{atan}(\tan(\text{dec}) * \cos(\text{inc})) \quad \boxed{\text{A8.34}}$$

The hour line angles from noon are:-

$$= \text{atan}(((\cos(\text{lat}) * \sin(\text{inc})) - (\sin(\text{lat}) * \cos(\text{inc}) * \cos(\text{dec})) * \tan(\text{lha})) / (\cos(\text{inc}) * \sin(\text{dec})) / (\cos(\text{dec}) + \sin(\text{dec}) * \sin(\text{lat}) * \tan(\text{lha}))) \quad \boxed{\text{A8.35}}$$



The formulae used in the spreadsheet are:-

Style Distance from noon

$$SDn = \text{DEGREES}(\text{ATAN}((\sin(\text{dec}) * \sin(\text{inc}) * ((\sin(\text{inc}) * \cos(\text{dec}) - \tan(\text{lat}) * \cos(\text{inc})) / (\cos(\text{inc}) + \tan(\text{lat}) * \cos(\text{dec}) * \sin(\text{inc})))))) \quad \boxed{\text{A8.36}}$$

Style Height SH

$$SH = \text{DEGREES}(\text{ASIN}((\cos(\text{lat}) * \sin(\text{inc}) * \cos(\text{dec})) - (\sin(\text{lat}) * \cos(\text{inc})))) \quad \boxed{\text{A8.37}}$$

$$\text{Vertical to noon} = \text{DEGREES}(\text{ATAN}(\tan(\text{dec}) * \cos(\text{inc}))) \quad \boxed{\text{A8.38}}$$

The hour line angles from the vertical are:-

$$= \text{DEGREES}(\text{ATAN}(((\cos(\text{lat}) * \sin(\text{inc})) - (\sin(\text{lat}) * \cos(\text{inc}) * \cos(\text{dec})) * \tan(\text{ha})) + \cos(\text{inc}) * \sin(\text{dec})) / (\cos(\text{dec}) + \sin(\text{dec}) * \sin(\text{lat}) * \tan(\text{ha}))) \quad \boxed{\text{A8.39}}$$

## CALENDAR LINE/CURVE FORMULAE (Declination curves, Date curves)

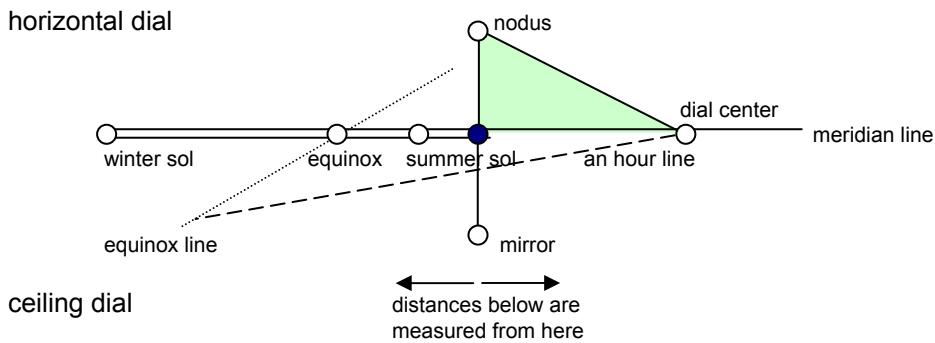
see the page preceding table A4.4  
see the page preceding table A4.5

these pages have the descriptive text  
and pictorials

**A8.40**

CEILING DIALS but also usable for large horizontal dials (even with inaccessible dial center)

Assume that:  $vd$  = vertical distance from the mirror to the ceiling  
or  
 $vd$  = gnomon linear height, from the nodus to the dial plate immediately below



Then from a point on the ceiling immediately above the mirror, or a point on the dial plate immediately below the nodus of a horizontal dial to a point on the north south meridian line would be:-

$$\text{ceiling above mirror to winter solstice or below nodus to winter solstice} = vd * \tan(\text{lat} + 23.44) \quad \boxed{\text{A8.41a}}$$

$$\text{ceiling above mirror to equinox or below nodus to equinox} = vd * \tan(\text{lat}) \quad \boxed{\text{A8.41b}}$$

$$\text{ceiling above mirror to summer solstice or below nodus to summer solstice} = vd * \tan(\text{lat} - 23.44) \quad \boxed{\text{A8.41c}}$$

$$\text{ceiling above mirror to dial center or below nodus to dial center} = vd * (\tan(\text{lat}) + 1/\tan(\text{lat})) \quad \boxed{\text{A8.41d}}$$

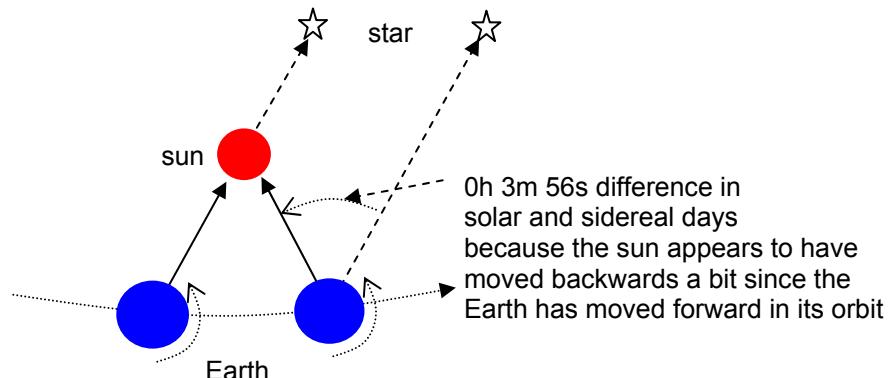
And the distance from the north south meridian line, along the equinox line, for an hour line, given an hour line angle is:-

$$\text{hr.dist} = \tan(\text{hour line angle}) * vd * (\tan(\text{lat}) + 1/\tan(\text{lat})) \quad \boxed{\text{A8.41d}}$$

NOTE: distance "vd" is depicted at distance "mb" in the appendix of proofs and derivations.

## CONSTANTS AND VARIABLES OF THE SOLAR SYSTEM

Earth tilt:	23.5 degrees or more exactly 23 degrees 47 minutes
Moon tilt	the moon's orbit is tilted by 5 degrees from the Earth's orbit
Object size	both the sun and the moon, as observed from Earth, are about 0.5 degrees wide
Precession	the Earth wobbles and its axis rotates once every 25800 years
Sidereal day:	on rotation of the Earth compared to a star, 23h 56m 4s of a mean solar day
Solar day:	one rotation of the Earth plus a bit because the Earth moves in relation to the sun compared to a sidereal day

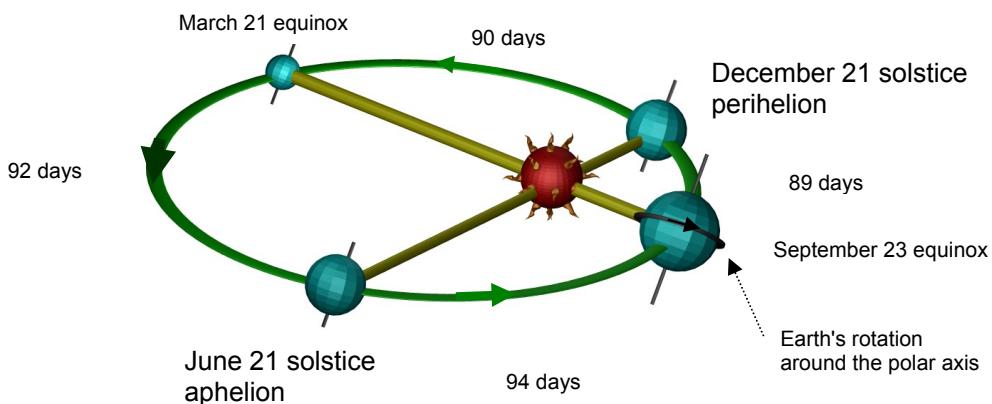


While sun is a long way away, the Earth moves enough so that each day the sun has moved "back a bit" compared to a star which is much further away, thus the solar day is a bit longer than the sidereal day based on the stars.

Sidereal year	366.25 rotations of the Earth
Solar year	365.25 rotations of the Earth
Days EOT=0:	There are four days when the EOT is effectively zero, they are roughly April 15, June 15, September 1, and December 25.
Solstice	Shortest and longest days, December 21 and June 21 approximately
Equinox	Daytime equals night time. March 21 and September 23 approximately except on the equator when every day is an equinox

Solstice to solstice EOT sine wave: because the Earth orbit slows and speeds on the solstices, solar apparent time varies in one full wave which is 7.64 minutes peak to peak

Intra-solstice-equinox EOT sine wave: because the sun appears to move north and south, then except for the equinoxes and the solstices, the sun's eastward travel appears to go faster or slower in two waves which are 9.86 minutes peak to peak



### IMPORTANT DATES FOR THE DIALIST

February 11th	EOT max: sun is slow and the EOT is +14 minutes 12 seconds
March 21	vernal equinox
April 15	EOT = 0
May 13th and 14th	an EOT peak: sun is fast, so the EOT is -3 minutes 39 seconds
June 15	EOT = 0 (some use June 14)
June 21	summer solstice
July 25th and 26th	an EOT peak: sun is slow with an EOT of +6 minutes 30 seconds.
September 1	EOT = 0
September 23	autumnal equinox (some use September 21 by convention)
early November	EOT max: sun is fast and the EOT is then - 16 minutes 22 seconds
December 21	winter solstice (some use December 22)
December 25	EOT = 0 (some use December 24)

The use of the 21st for the solstices and equinoxes is often used as an approximation, and easy to remember. However the actual date may vary based on the year, leap year, and so on. In the year 1246 the EOT smaller deviations were both 4 minutes 58 seconds, and the maxima were both 15 minutes 39 seconds. The February minimum is shrinking about 12 seconds a century, the May maximum is shrinking by about 9 seconds a century, the July minimum is growing about 13 seconds a century, and the November maximum is growing by about 5 seconds a century.

### LIMITING, UNUSABLE AND USABLE HOURS ON A DIAL PLATE

There are limiting or unusable hours, so when designing a sundial there is no point in marking up lines that can't be used. If the dial is large then time and materials are wasted as is real estate on the dial plate because the shadow of the style or nodus will never touch those unusable hours or calendar information. For all dials, it is better to eliminate unusable space and increase the size of the remaining hour lines and calendar information, thereby increasing their accuracy.

Displayable hours depend on latitude and dial type. For example, on the equator every day is an equinox thus the most a dial can display is 6 am to 6 pm local apparent time (L.A.T.). At the poles, night lasts months, day last months, or somewhere in between. Thus a horizontal dial can display 24 hours. Software with shadow simulators can help visualization of usable shadow, one such example is SHADOWS, a software program sometimes used by the author but not affiliated with this book. Alternatively, a CAD solar-travel mesh that can help visualize hours of use may be used, and is on the CD and web site for this book. In summary form, with times being local apparent time, the rules of thumb are:-

Armillary	{	can display from sunrise to sunset, however, an armillary dial plate can interfere with itself near the equinox, and equatorial dials may indicate nothing at the equinox
Equatorial		6 am to 6 pm
Horizontal		6 pm to 6 am
Vertical facing the equator		but not after the latest sunset nor before the earliest sunrise
Vertical facing the pole		12 hours maximum
Vertical decliner		not hours above the horizontal line for the nodus roughly one hour shift for each 15 degrees of wall declination
Recliners		requires deep thought unless this is a polar dial
Meridian		east facing – sunrise to noon, west facing – noon to sunset
Polar dials		6 am to 6 pm, unless you also use the underneath part.

Astrolabe, Shepherd, Capuchin, Ogee, and other altitude dials: sunrise to sunset

## **FORMULAE CONVERTED FOR SPREADSHEETS**

Several chapters cover the detailed use of formulae in spreadsheets. This page merely shows some selections of the formulae discussed in those chapters without significant comment. The objective being solely as a quick reference.

**ANGULAR MEASURE:** Angular measures are commonly in degrees, however, spreadsheets use radians, which are different to degrees. There are  $2 * \pi$  radians in a circle of 360 degrees. Thus, every formula that uses degrees needs the measure to be converted to radians.

=RADIANS(360) would return 6.283

The trigonometric functions SIN, COS, TAN, COTAN, and so on, need the radian conversion first.

=TAN(RADIANS(45)) would return 1.0

Any function that returns an angular measure, such as ATAN, ACOS, ASIN, also printed as  $\text{TAN}^{-1}$ ,  $\text{COS}^{-1}$ ,  $\text{SIN}^{-1}$ , would need to be converted back to degrees using the DEGREES function.

=DEGREES(6.283) would return 359.984 or 360.0 depending on precision

The number of significant digits on the fractional side not only determines accuracy, it also controls rounding. The above function can return 360.0 rather than 359.9

**ROUNDING:** The number of significant digits after the decimal point is controlled by the FORMAT, CELLS, NUMBER option. Some functions do rounding of their own.

=INT(5.1)	=INT(5.9)	=INT(-5.1)	=INT-(5.9)	returns
5	5	-6	-6	

The INT (integer) function returns the integer part of a number on the left side of the decimal point, rounds down. And the number below 5.9 is 5, however the number below -5.1 is -6 because this is already in the negative scale. However, the ABS (absolute) function which removes the sign may be used if the above rounding down on the negative side is not desired.

=INT(ABS(5.5))	=INT(ABS(-5.5))	returns
5	5	

If the sign must be retained, then the SIGN function can be used. Assume that cell B3 has a value of 5.5 and cell B4 has value of -5.5 then the following results can be obtained:

=SIGN(B3)*INT(ABS(B3))	=SIGN(B3)*INT(ABS(B3))	returns
5	-5	

**TIME CONVERSION:** Second, they show the use of methods to convert "hh.mm" for example to "hh.hh". Assume cell B3 has a value of 4.30 meaning a time but in a decimal cell, meaning to us humans, four thirty, and we wish the time in hours and decimals of hours then:

=((100\*b3-INT(100\*b3/100)\*100)/60)+INT(100\*b3/100) returns 4.5

If the time is negative then remember that INT will provide erroneous results, thus the ABS and SIGN function can be used, or a conditional IF as shown below.

=IF(B3<0, -1\*( INT(ABS(B3))+(ABS(B3)-INT(ABS(B3)))\*0.6 ),INT(B3)+(B3-INT(B3))\*0.6 )

CONVERTING DECIMAL MINUTES TO MINUTES AND SECONDS: Always test functions that you use. For example, the integer function =INT(value) is not what you might expect. The integer of 10.1 and -10.1 is commonly held to be 10, and -10 however the actual INT function in some spreadsheets is the integer after rounding down. Thus the integer of -10.001 may be -11 and not -10. For example, assume cell C7 is m.mm or -m.mm (such as the plus or minus EOT values in decimal):-

=IF(C7<0,-1\*(INT(ABS(C7))+(ABS(C7)-INT(ABS(C7)))\*0.6), INT(C7) + (C7-INT(C7))\*0.6)

3.40 ( 3.40 minutes)	results in	3.24 (3 minutes 24 seconds)
-3.40 (-3.40 minutes)	results in	-3.24 (3 minutes 24 seconds)

This formula tests the cell's sign, uses the integer of the absolute (positive) value to get around the round down, and thus converts plus or minus minutes in decimal to minutes and seconds.

And if you wished the absolute value then you could use =ABS(. . .) or =SIGN(C7)\*( . . .), yes it is permissible to code:

=ABS(IF . . . )      or      =SIGN(C7) \* ABS(IF . . . )

BLANKS IN FORMULAE: Some spreadsheets sometimes get upset on long formulae if blanks are used, sometimes they do not. It may say they are erroneous when all that is problematic is those blanks.

#### TRIGONOMETRY IN A CELL

hour line angle  $H = \text{atan}(\sin(\text{lat}) * \tan(\text{ha}))$   
 hour line angle =DEGREES(ATAN(TAN(RADIANS(15\*time))\*SIN(RADIANS(lat))))

INTERMEDIATE VALUES: Sometimes the cell formula can get very large and out of hand. If you make an error on entering a formula, the spreadsheet will offer a correction. Be very careful of accepting their suggestion. It is best to match the parentheses yourself. If a formula gets too large, consider using intermediate columns, or cells, that hold intermediate data. Intermediary values are usually kept off to one side, the right side in this case, and used as working accumulators for the final value.

One can hard code cell references using the \$column\$row syntax, however the benefits may outweigh the drawbacks.

AND FINALLY, IN REVIEW: When you choose a formula, from whatever source, always test it at extreme as well as in between values. For example, there are two versions of the formulae for azimuth in publication.

Using the graph function of a spreadsheet can visually identify formulae inconsistencies. The graph function can also generate hour lines and altitude curves.

There is a free spreadsheet from Open Office. It is mostly Excel compatible. The web site is something like:  
<http://www.openoffice.org/>

The spreadsheets in this book are available on:- [www.illustratingshadows.com](http://www.illustratingshadows.com)

## **FORMULA OR GEOMETRIC ACTUAL OR APPARENT INCONSISTENCIES**

### **SUNS ALTITUDE AND AZIMUTH**

ACTUAL INCONSISTENCY: Waugh presents two different formulae. The one on page 139 agrees with Folkard and Ward, and the one on page 92 does not, but agrees in all aspects except for 6am and thus also 6pm using my spreadsheet version. Similarly, Mayall's formula on page 243 has the same 6am and 6pm anomaly with my spreadsheet version. The Waugh page 139, and Folkard & Ward page 74 works best with my spreadsheet.

### **DECLINING VERTICAL DIALS**

ACTUALLY NOT INCONSISTENT: The method presented by Rohr (chapter three, page 59) and by Dolan (pages 104 of chapter five) is simple and uses an auxiliary horizontal dial's hour lines, and the angle between the vertical and projected horizontal dial is the wall's declination. Waugh (chapter nine page 76) or Mayall (page 112 of chapter seven) use a very similar appearing model, but they use 15 degree lines for the horizontal dial and a calculated angle for the separation of the vertical dial and the horizontal projection. This is not a mistake. Their method differs from Rohr's and Dolan's in that they first develop the gnomon's rotation (SH style height, SD style distance), deriving an angle that separates the real vertical dial from the horizontal construction dial, and that angle is not the wall's declination. Because their method derives a new angle, it also provides for using 15 degree distances on that construction horizontal dial. The choice of method is up to the dialist. Also, Mayall and Waugh differ slightly in explaining their geometric method for the gnomon rotation.

Also, the derived formulae for hour line angles in this book appears different from the more common formulae. The formulae all produce the same results for all data ranges.

### **CEILING DIALS**

ACTUALLY NOT INCONSISTENT: the derived formulae for hour line angles in this book appears different from the more common formulae. The formulae all produce the same results for all data ranges.

### **HOUR ANGLE, LATITUDE, ELEVATION, AND DECLINATION**

ACTUAL INCONSISTENCY: Rohr chapter eight page 109 on the left side of an equation simplification, converts " $\cos(90-h)$ " to " $\cos(h)$ " which is probably an error. On the right side of the simplification however, the conversions appear correct.

### **BASIC ARITHMETIC AND THE EQUATION OF TIME**

Sometimes we go from clock time to solar, so why do we not reverse the signs for the EOT, etc. Because we are still using the solar indicated local apparent time and the EOT to deduce a clock time needed to get the sun's shadow in the right place. Thus signs are not reversed because we are using the standard formula as is, the EOT is still operating on solar apparent time.

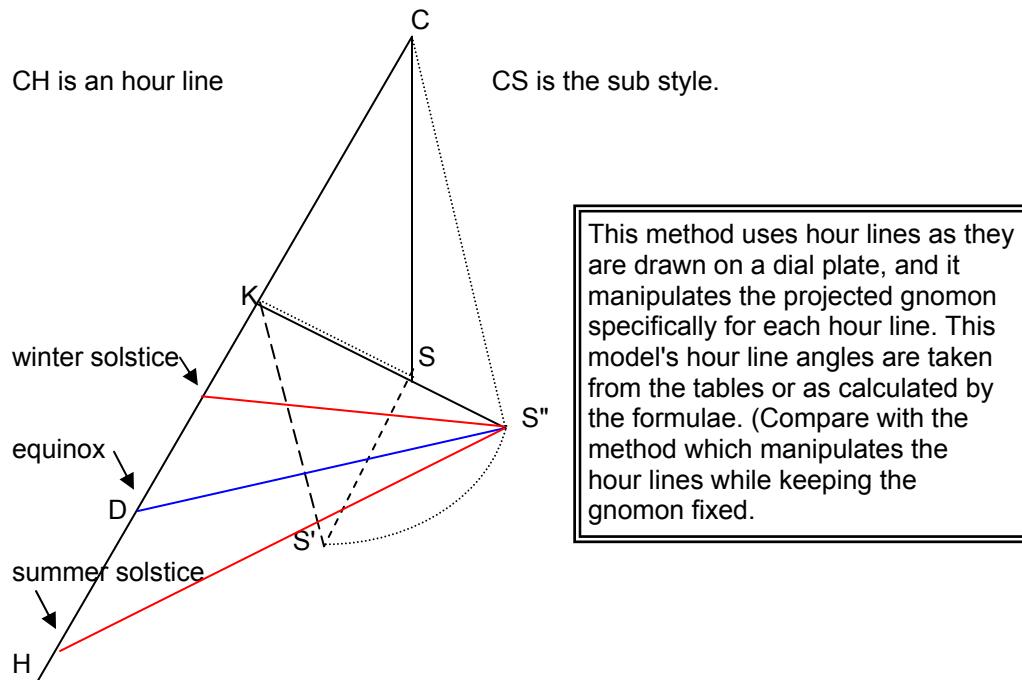
### **EAST OR WEST VERTICAL (MERIDIAN) DIAL FORMULA APPARENT INCONSISTENCY**

This book uses 6am or 6pm as the baseline for calculating the time difference for hour lines, many other books use noon, which results in radically different looking formulae. This is not an error, it is the result of selecting 6 o'clock versus noon as the base from which hour lines are calculated.

## DECLINATION LINES (OR CURVES) FOR THE HORIZONTAL DIAL USING GEOMETRY.

ACTUALLY NOT INCONSISTENT:

One common method for drawing declination or calendar lines or curves is described below. To find the declination for any date you might use a table such as in appendix 2, 4 or appendix 5. It may also be calculated using a formula discussed in appendix 8 covering formulae. This graphical approach is used by some of the well known authors on sun dials.



From S, drop a perpendicular to the hour line, creating point K. Then, create line SS' parallel to the hour line CH, the length SS' is the height of the style.

KS' is the true distance from the hour line (point K) to the tip of the style. Rotate S' to S" so that KS" is not only the true distance from the hour line (point K) to the tip of the style, but also it is now at right angles to it. So, CS" is, from the perspective of point K on the hour line CH, the projection of the style of the gnomon.

Draw a perpendicular from CS" to the hour line at point D, in other words CS"D is 90 degrees, this point D is the equinox point for this hour line.

With point S" as the center point, and the line S"D as a equinoctial baseline, declinations are drawn for whatever declination is desired, just the solstices are shown, being plus and minus 23 degrees 27 minutes, or basically 23.5 degrees.

Other authors use a similar picture but the hour lines are not hour line angles as is the case above. Is this inconsistent? No. In the above model the hour lines are fixed and the gnomon is manipulated, it is redrawn for each hour line. In other methods, the gnomon is fixed, and the hour line angles are manipulated.

## APPENDIX 9 – TEMPLATES FOR DIAL DESIGN

A DRAFTING SHEET FOR VERTICAL DECLINING DIALS TO FIND STYLE DISTANCE< STYLE HEIGHT, AND TO FACILITATE LOCATING THE HOUR LINES.

A9.1

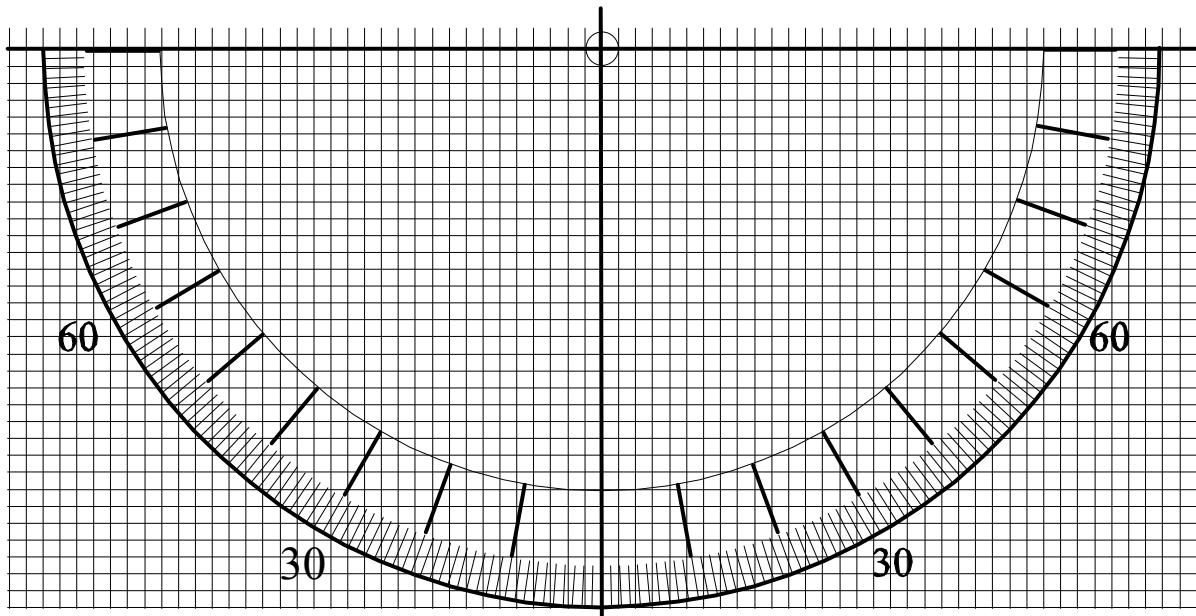
Draw gnomon here for south dials declining east

Draw wall declination here for south dials declining east

Draw wall declination here for south dials declining west

Draw gnomon here for south dials declining west

East and west decliners have their hour line angles reversed. By following the rules above for the gnomon and the declination, the final hour lines drawn will be correct, with am on the left (west), and pm on the right (east).

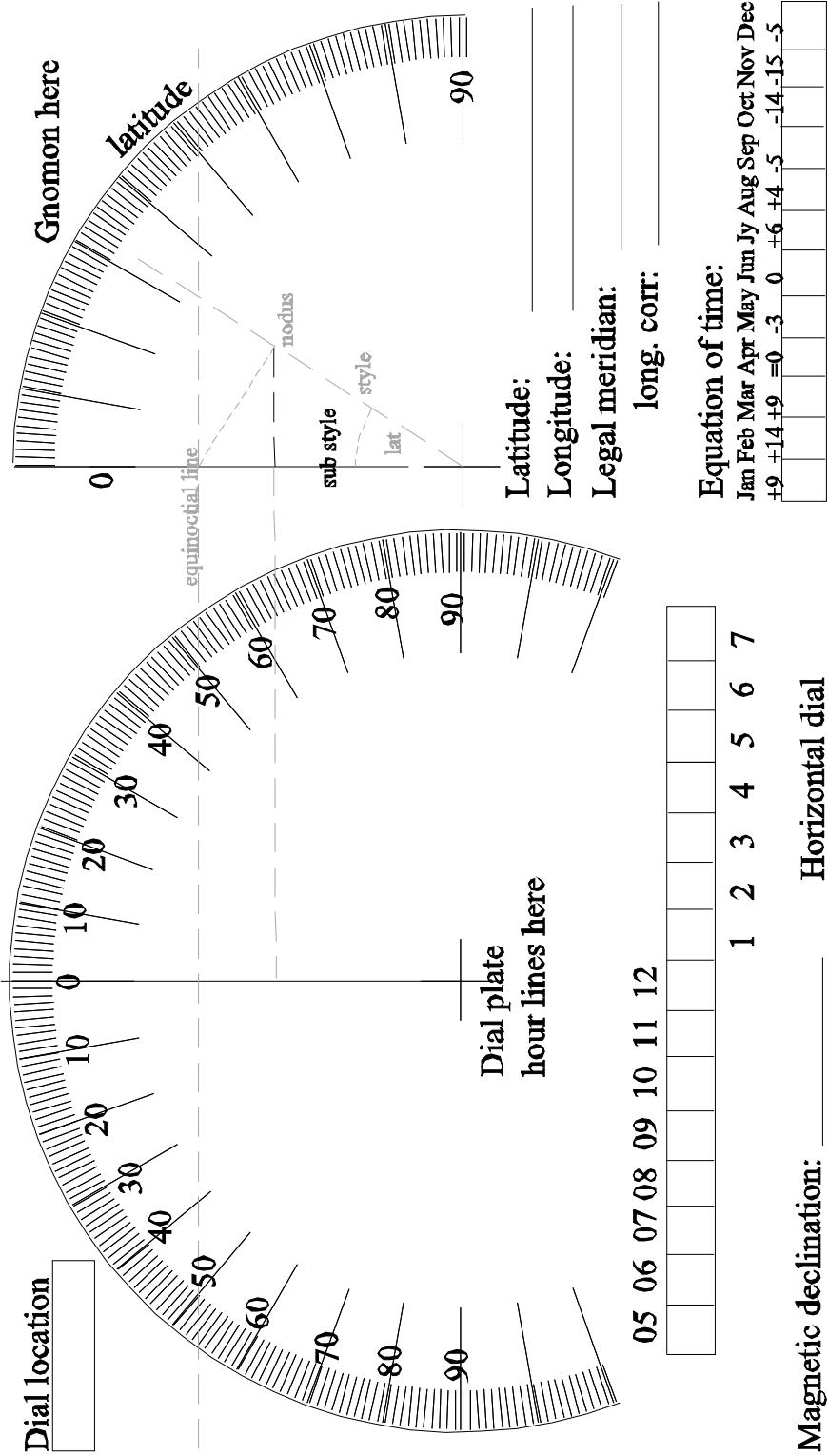


This template may be copied provided credit is given to Illustrating Shadows, [www.illustratingshadows.com](http://www.illustratingshadows.com) and Simon Wheaton-Smith



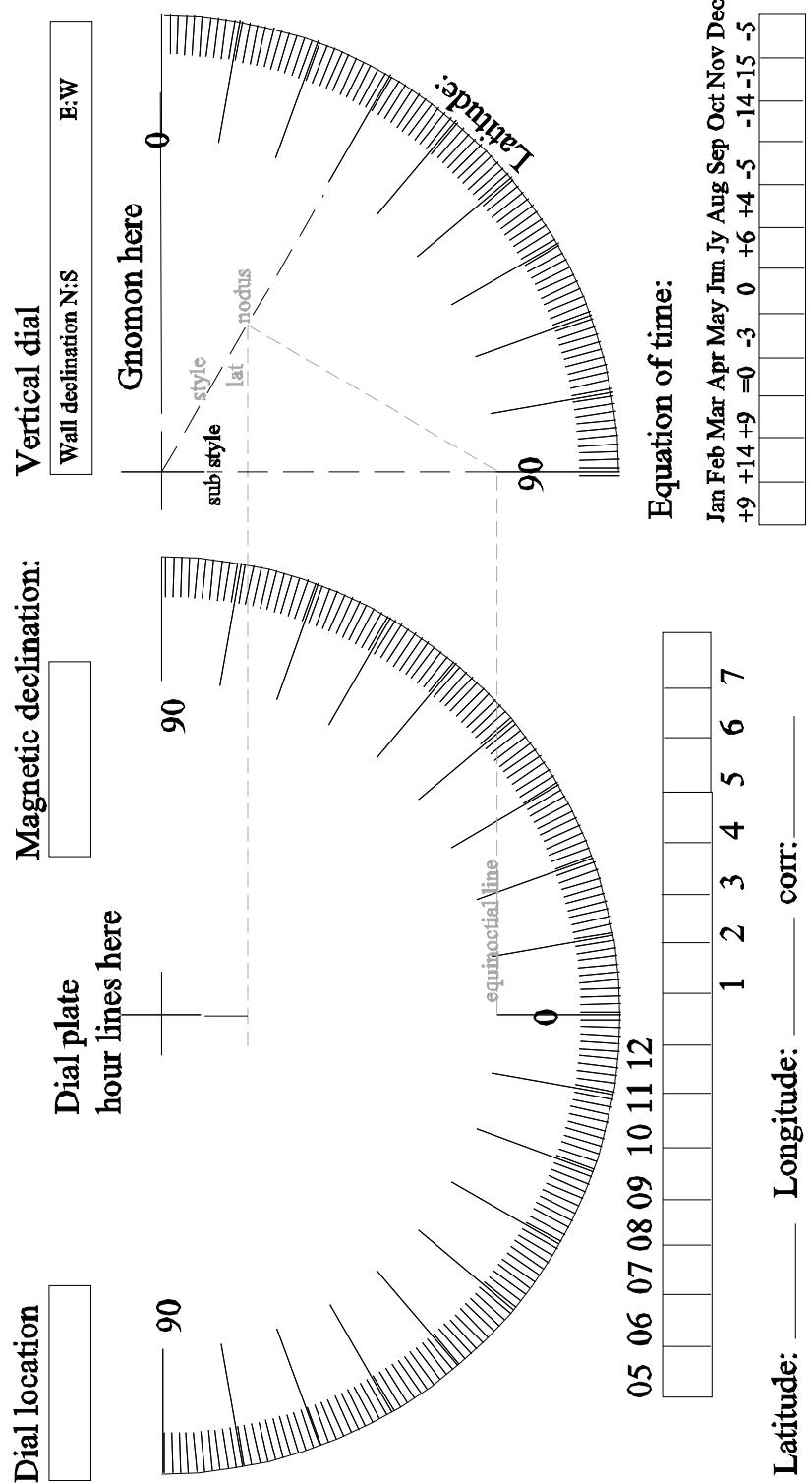
A DRAFTING SHEET FOR HORIZONTAL DIALS

A9.2



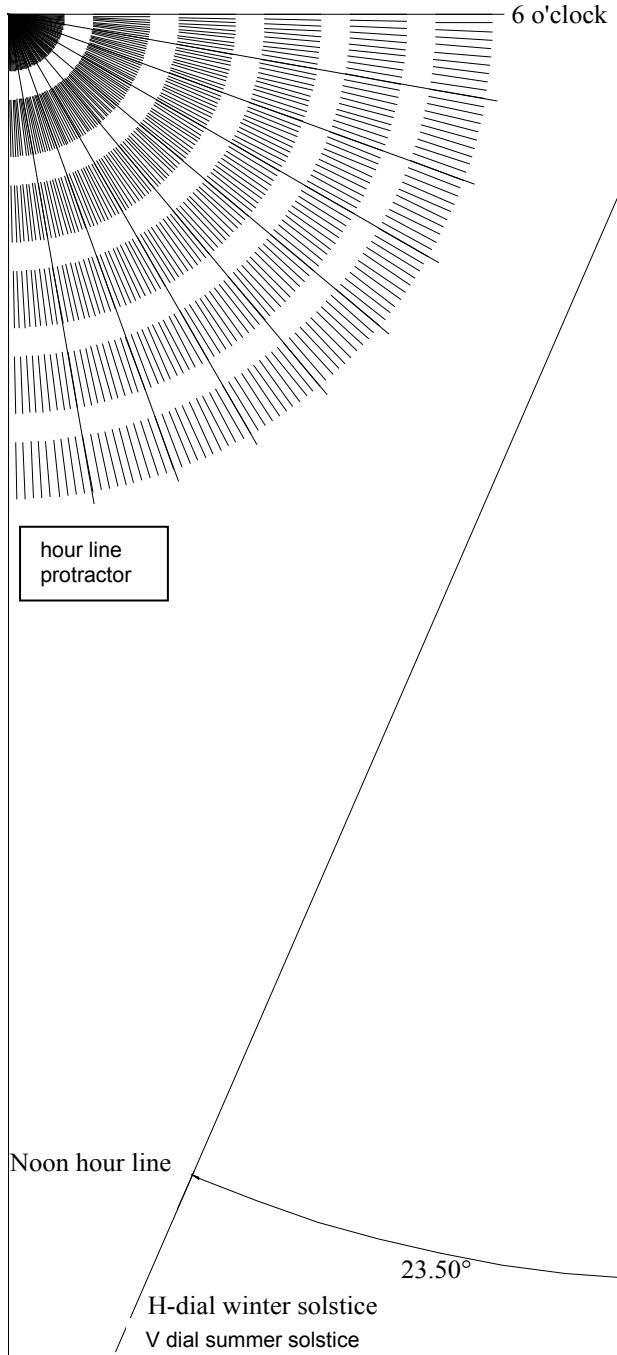
A DRAFTING SHEET FOR VERTICAL DIALS

A9.3

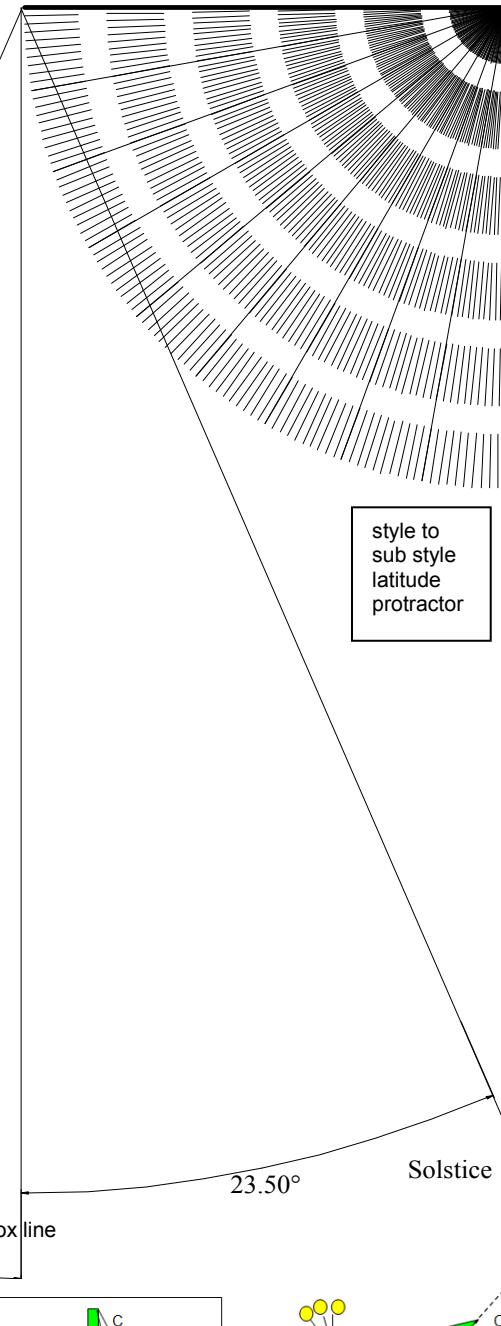


This template may be copied provided credit is given to Illustrating Shadows, [www.illustratingshadows.com](http://www.illustratingshadows.com) and Simon Wheaton-Smith

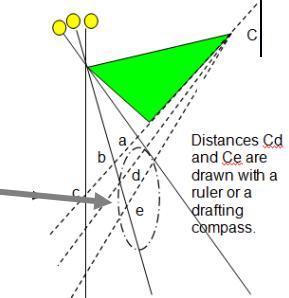
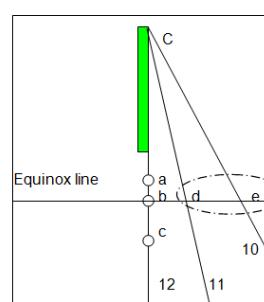
Protractors for declination lines/curves



Nodus      Style

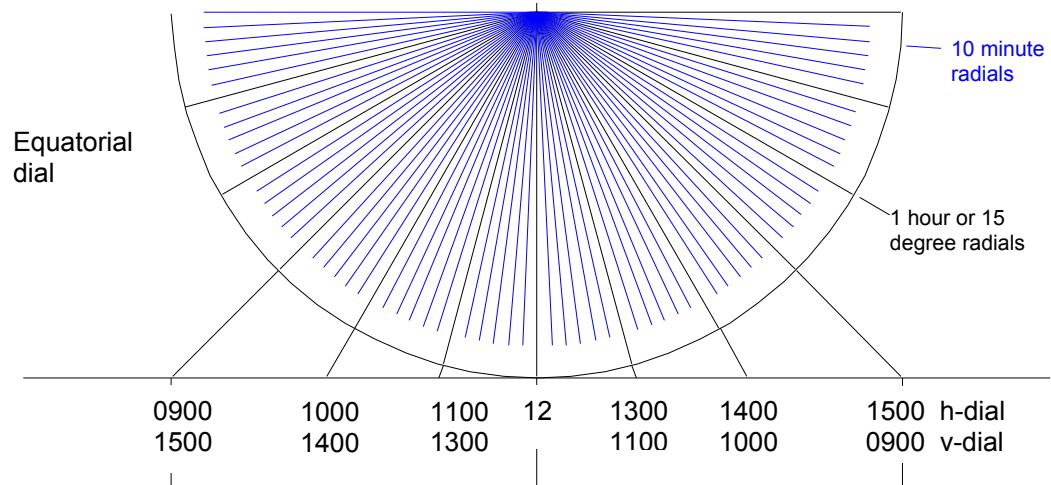


This template may be freely copied, please retain a reference to [www.illustratingshadows.com](http://www.illustratingshadows.com)



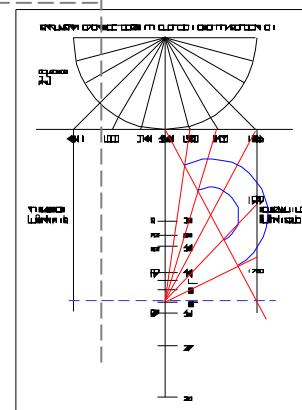
A TEMPLATE FOR HORIZONTAL (AND VERTICAL DIAL DESIGN). SEE CHAPTER 12

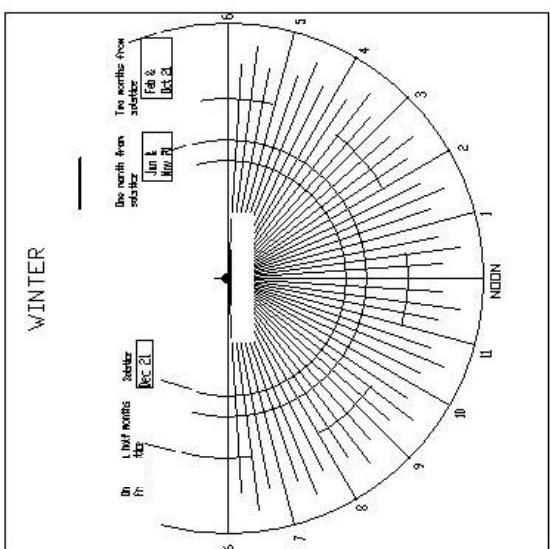
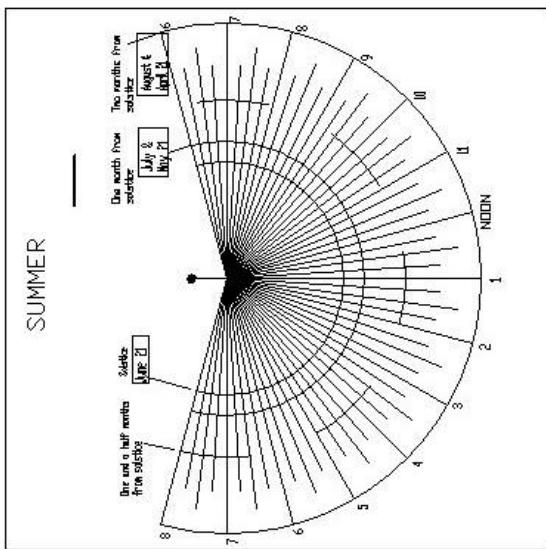
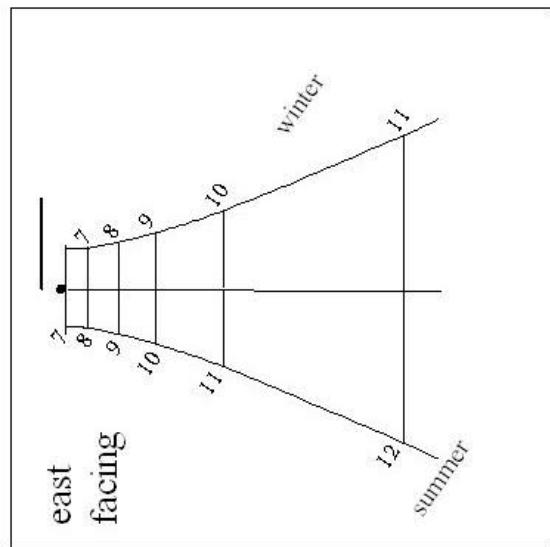
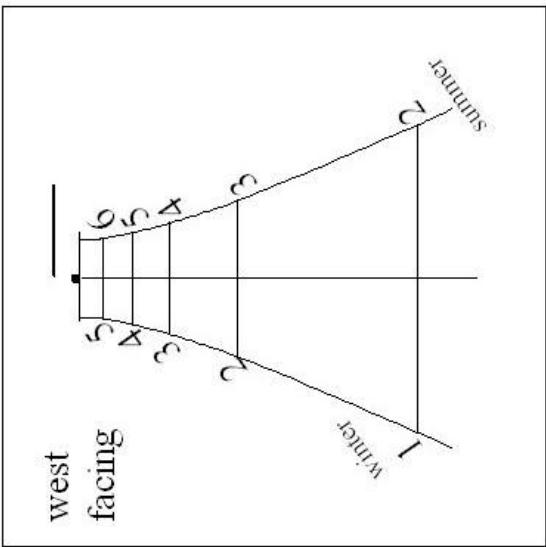
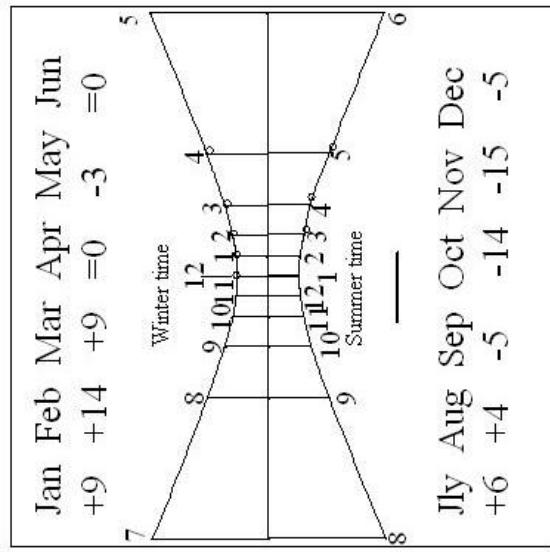
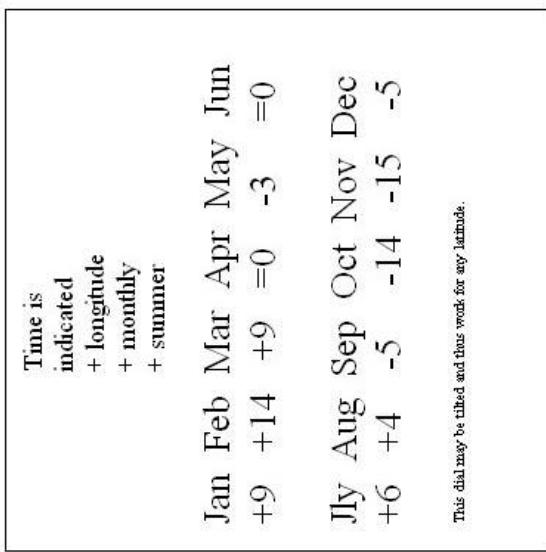
THIS SHOWS EQUATORIAL AND H-DIAL & V-DIAL RELATIONSHIPS



Connect a line from the vertical latitude line to an hour angle of the equatorial dial. For a vertical dial, use 90-latitude, and reverse the hours left to right. For hours before 0900 or after 1500, drop a line from 0900 or 1500 down to meet a line drawn horizontally from dial center (latitude, marker). Connect a diagonal from noon to the outer bottom of the oblong and 1600 from 1500 = 1400 from 1500, 1700 from 1500 = 1300 from 1500, etc. See chapter 12 of *Illustrating Time's Shadow* for these techniques, and the pictorial insert to the right.

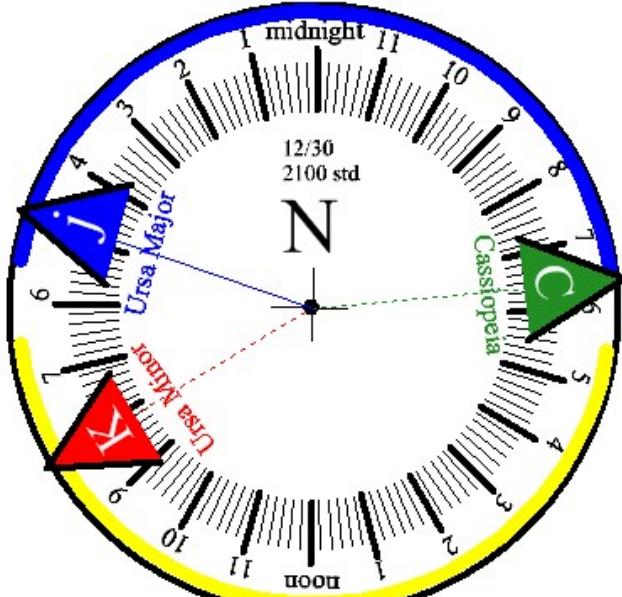
70      20



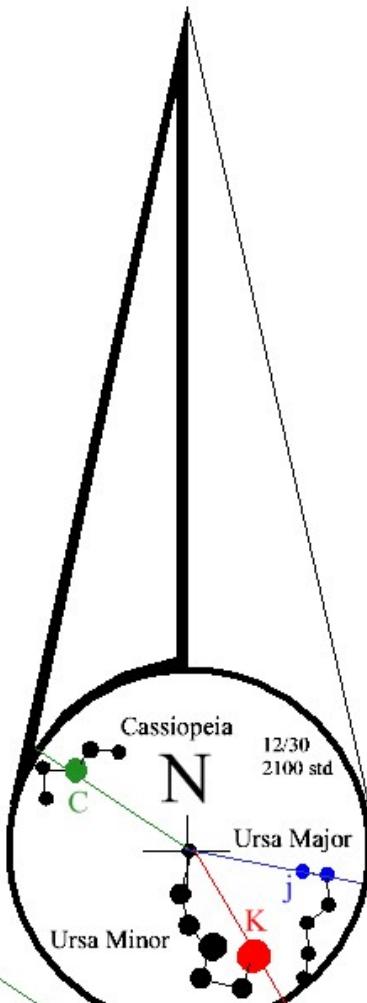
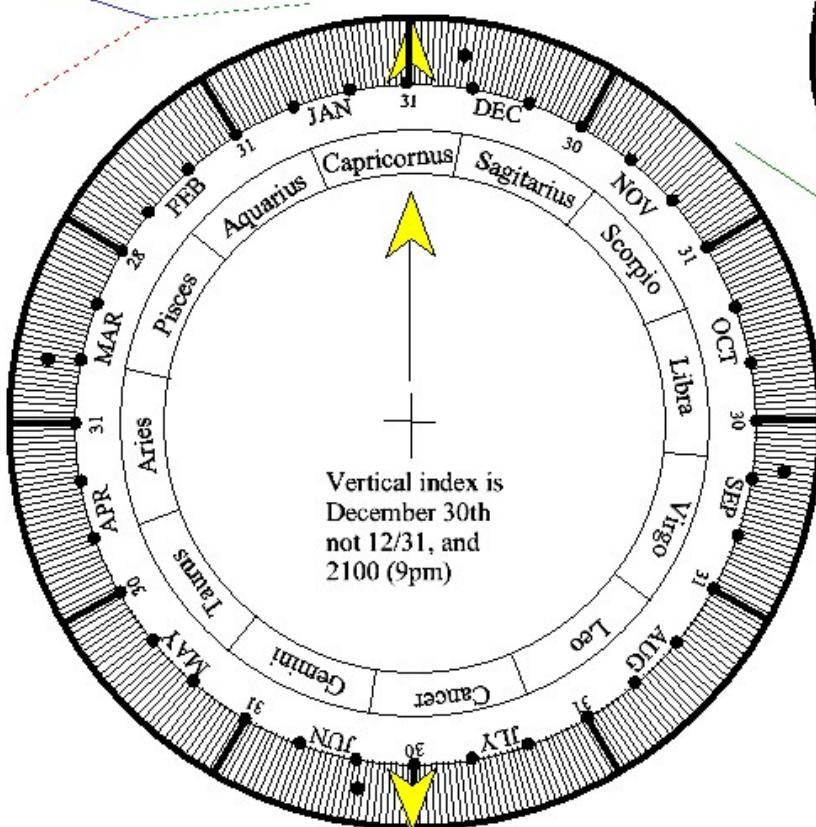


Cassiopea Nocturnal dial

Little Dipper Nocturnal dial Big Dipper Nocturnal dial



The K, CA and j pointers are the mirror image of the actual star map layout (i.e. rotated 180 on its axis)



Outer disk is  
CD size

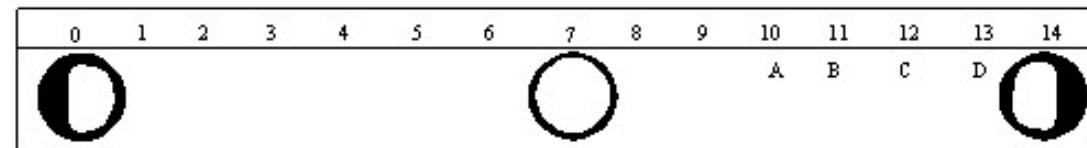
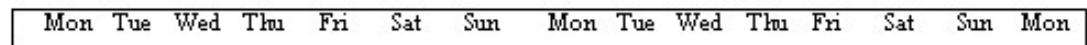
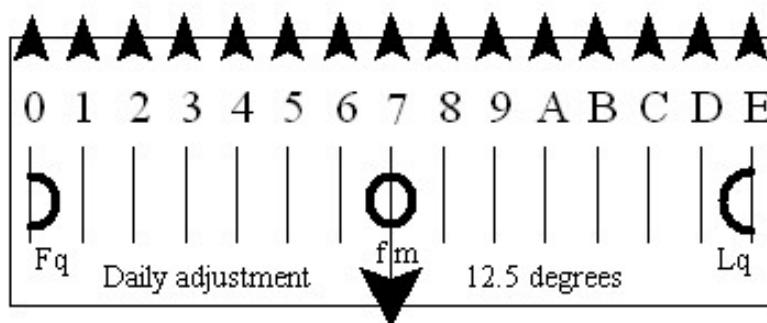
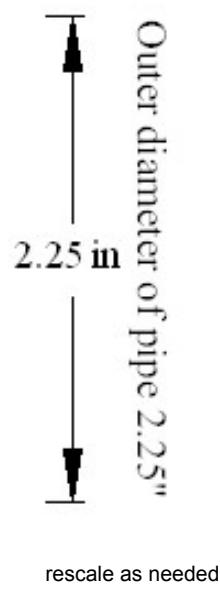
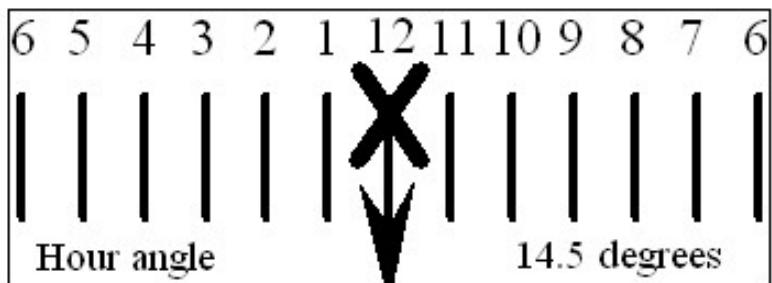
Star	Correction
C	.....
K	.....
j	.....

Correction for star chart variance and observer longitude

Rotate until a  
null shadow

LUNAR DIAL

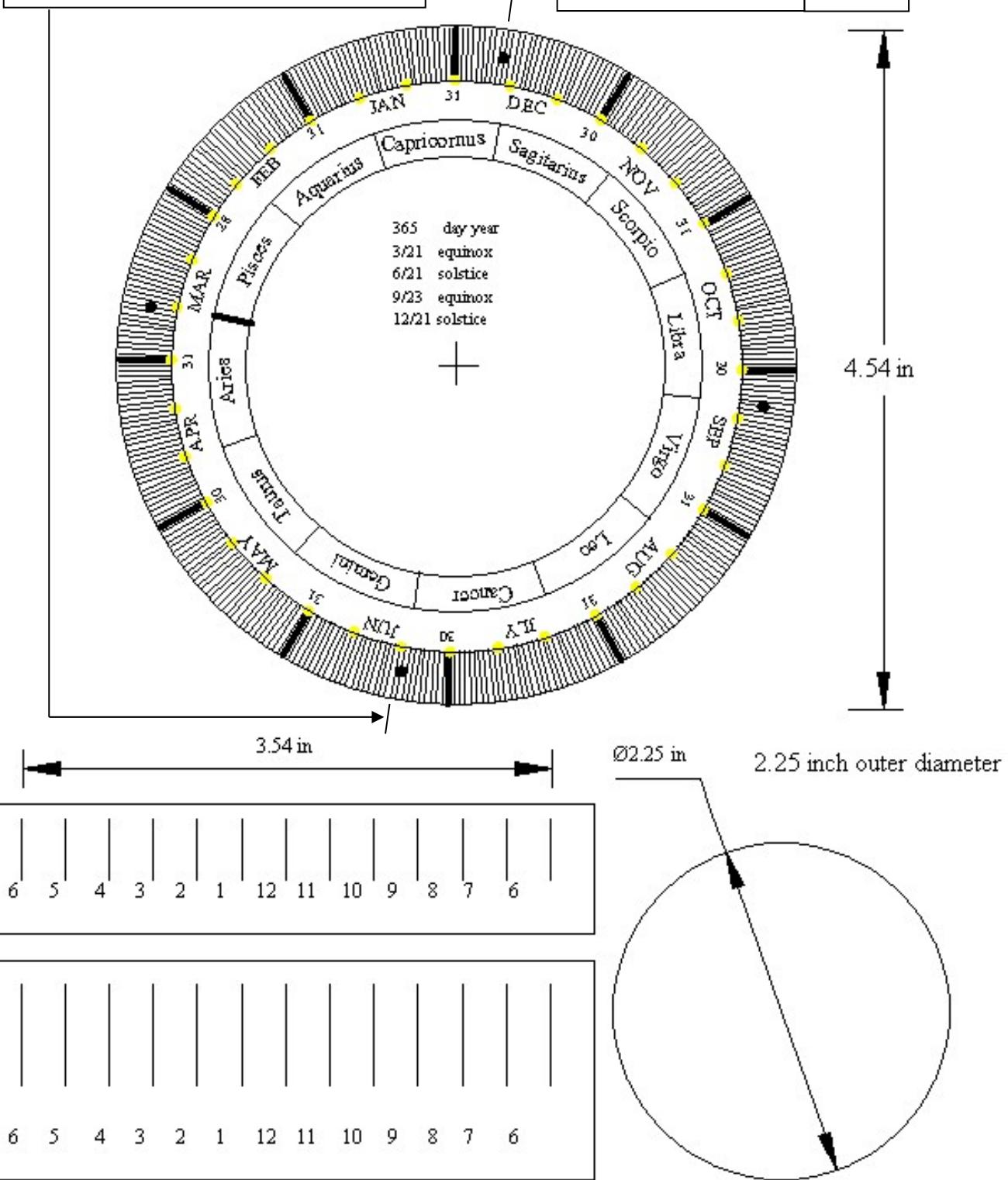
A9.7



December 21 and June 21 are placed at the extremes of the canted section.

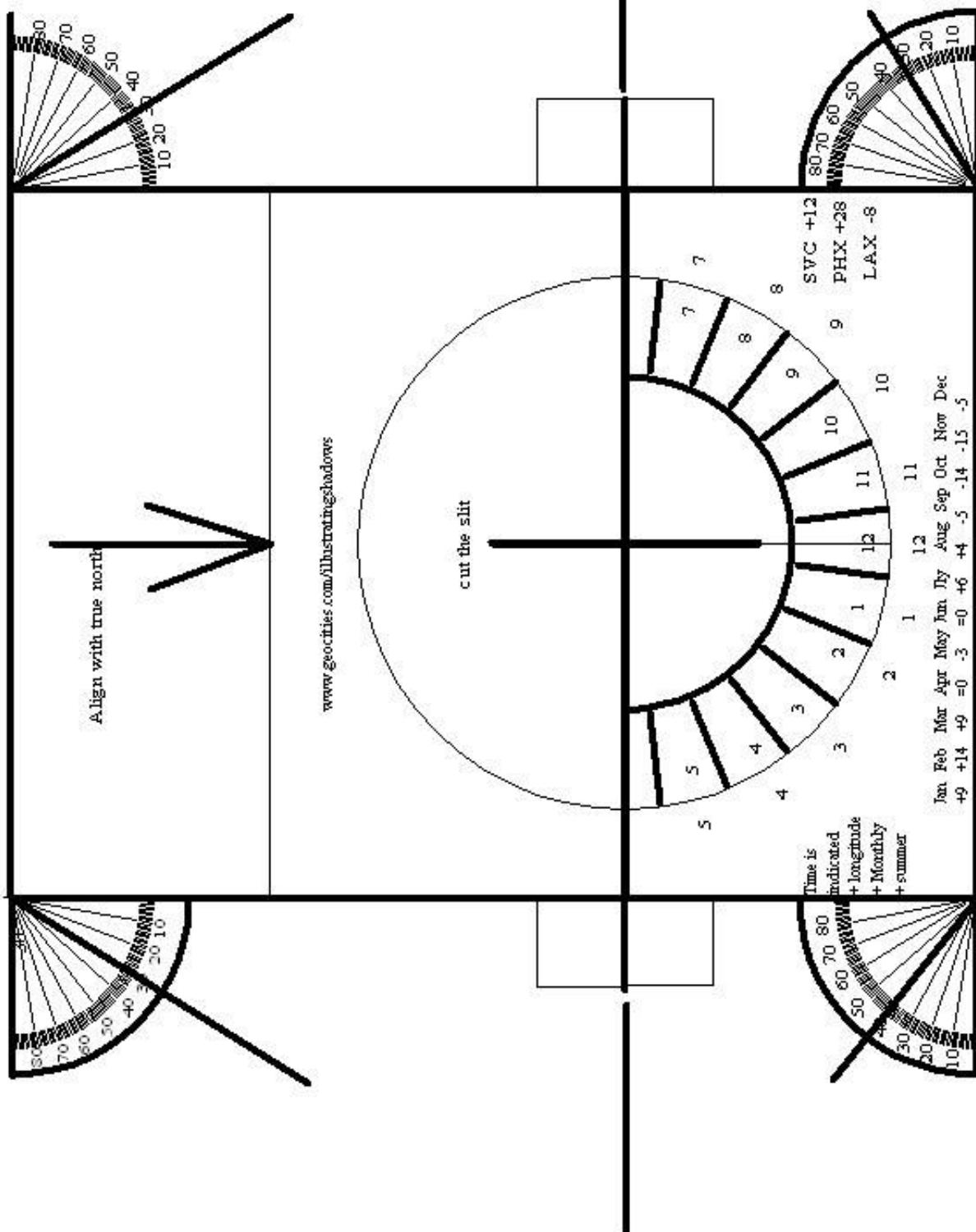
## ECLIPTIC DIAL

A9.8



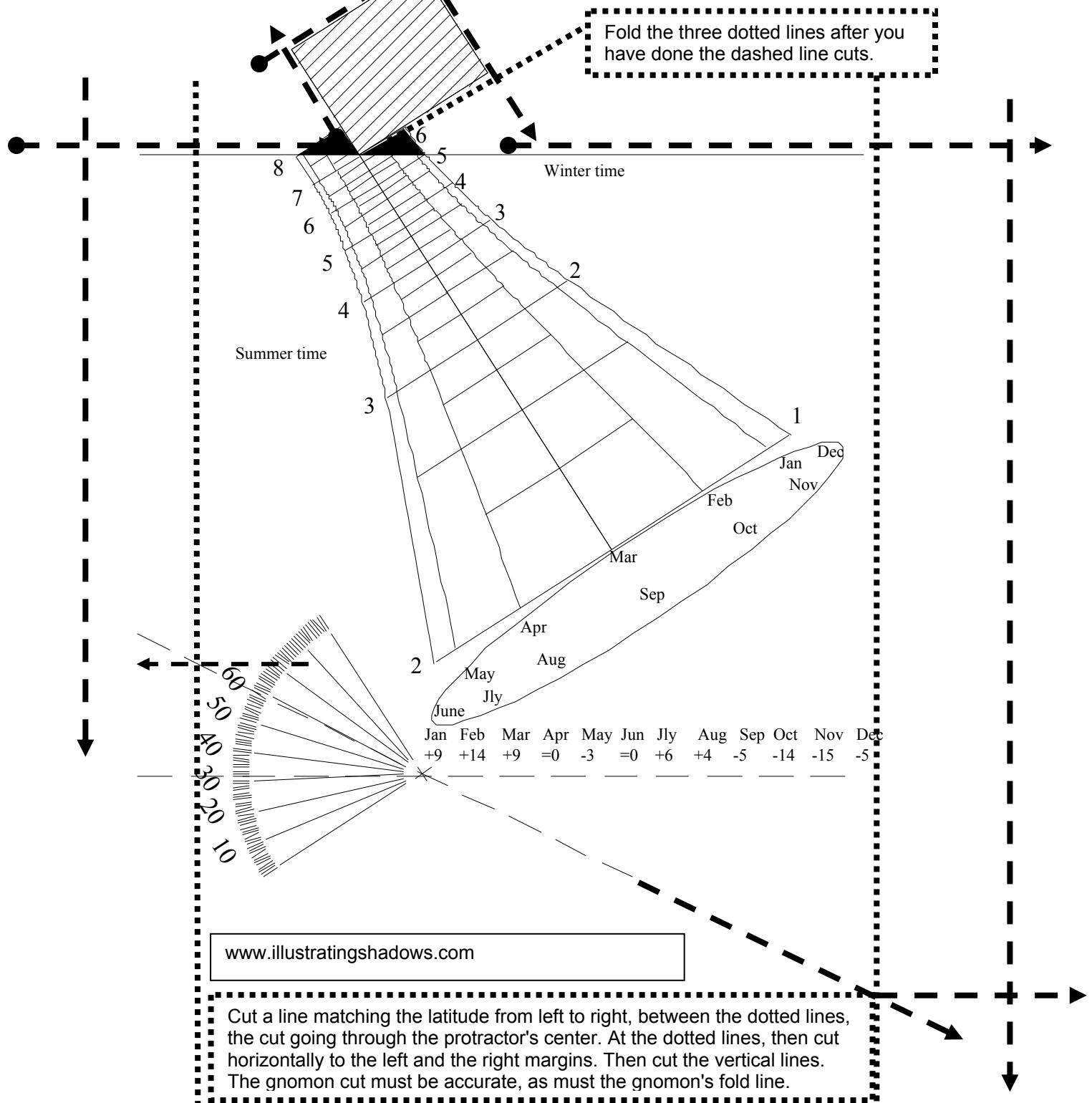
## PAPER CUTOUT SUNDIALS

### ARMILLARY DIAL & EQUATORIAL DIAL IN SUMMER

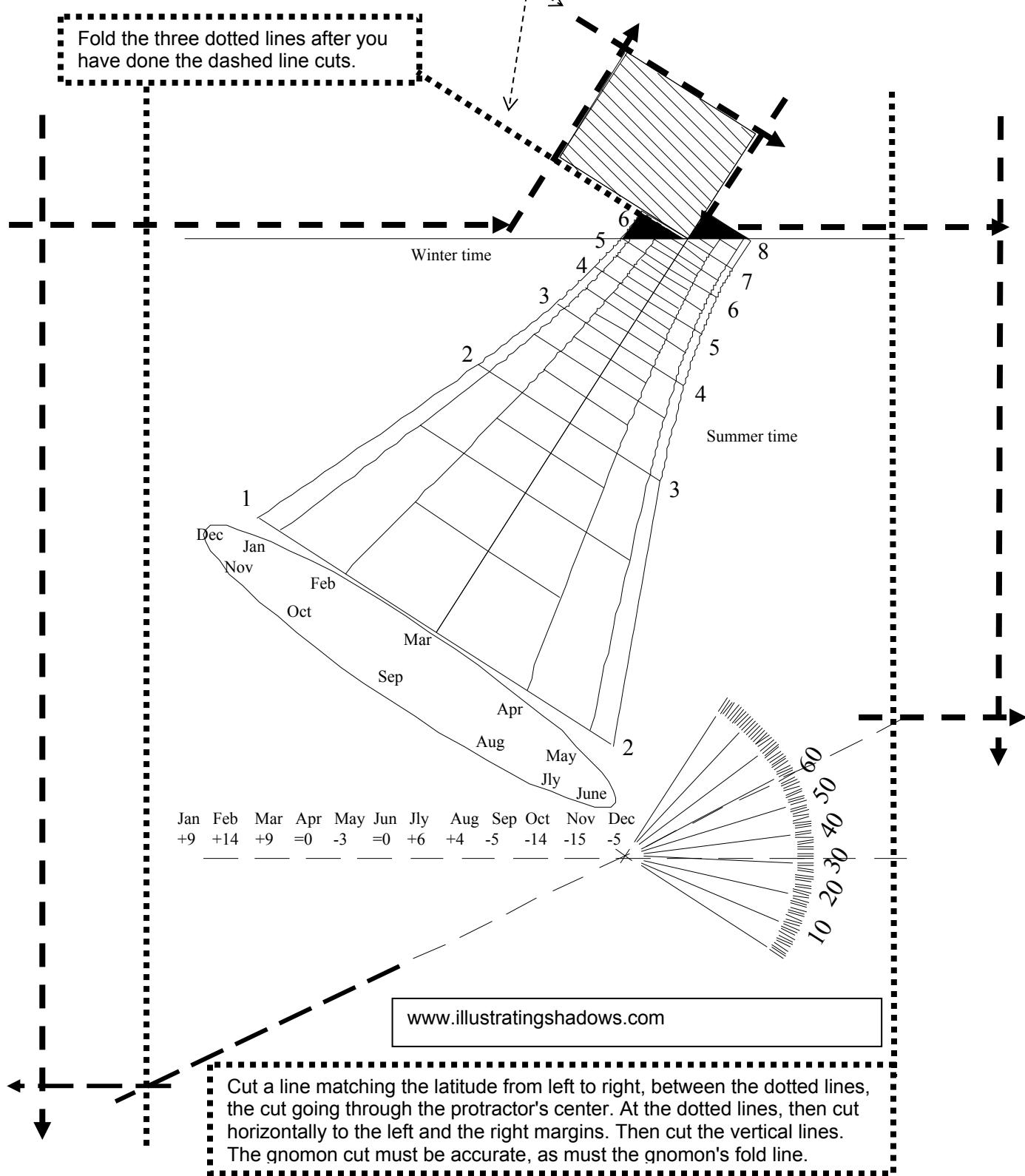


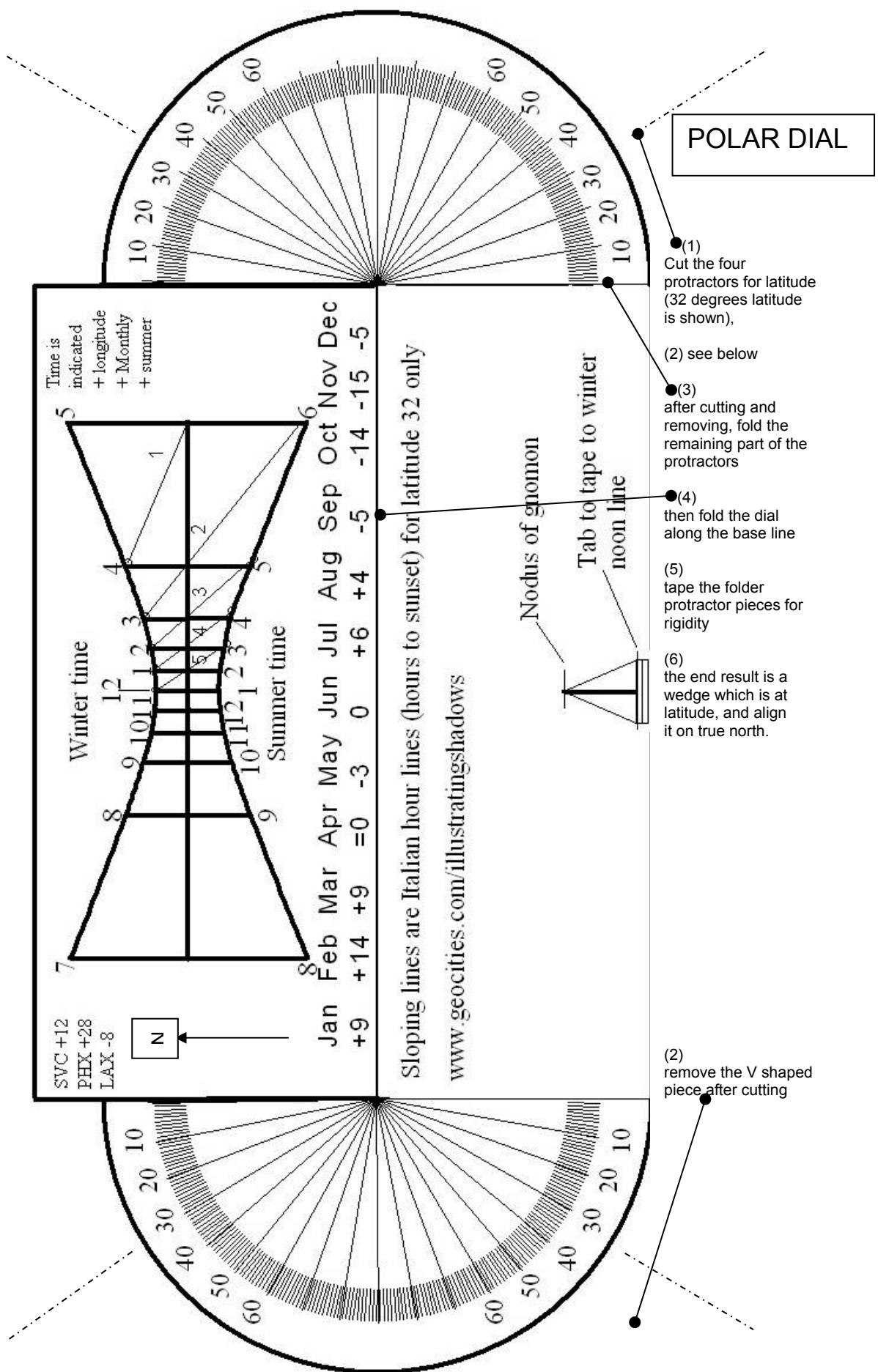
## WEST MERIDIAN DIAL

The gnomon cut and fold must be accurate.  
The reverse side of dial plate must face true west.  
Gnomon folded away from viewer because dial is  
between viewer and the sun (as in stained glass).

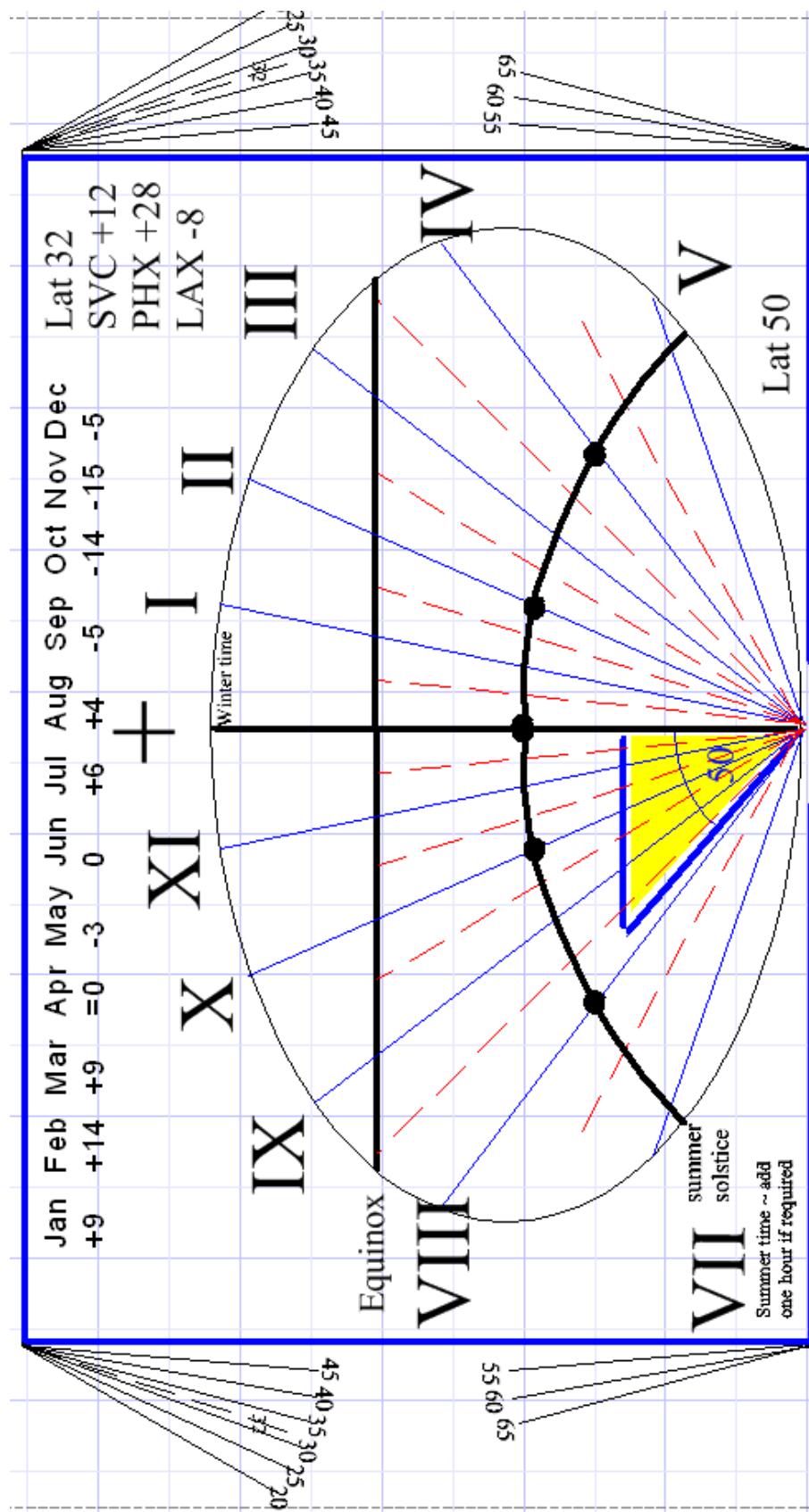


## EAST MERIDIAN DIAL



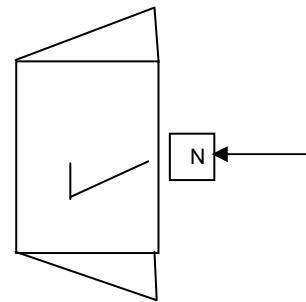


## HORIZONTAL DIAL

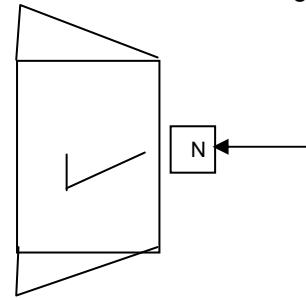


If latitude of the location is 50 degrees, cut out the dial and ignore the protractors off to the side.

If latitude of the location is less than 50 degrees, locate the latitude on the scale 20-45 and extend the angled line out on both sides, then cut out the dial. And fold those two triangles.



If latitude of the location is more than 50 degrees, locate the latitude on the scale 55-65 and extend the angled line out on both sides, then cut out the dial. And fold those two triangles.

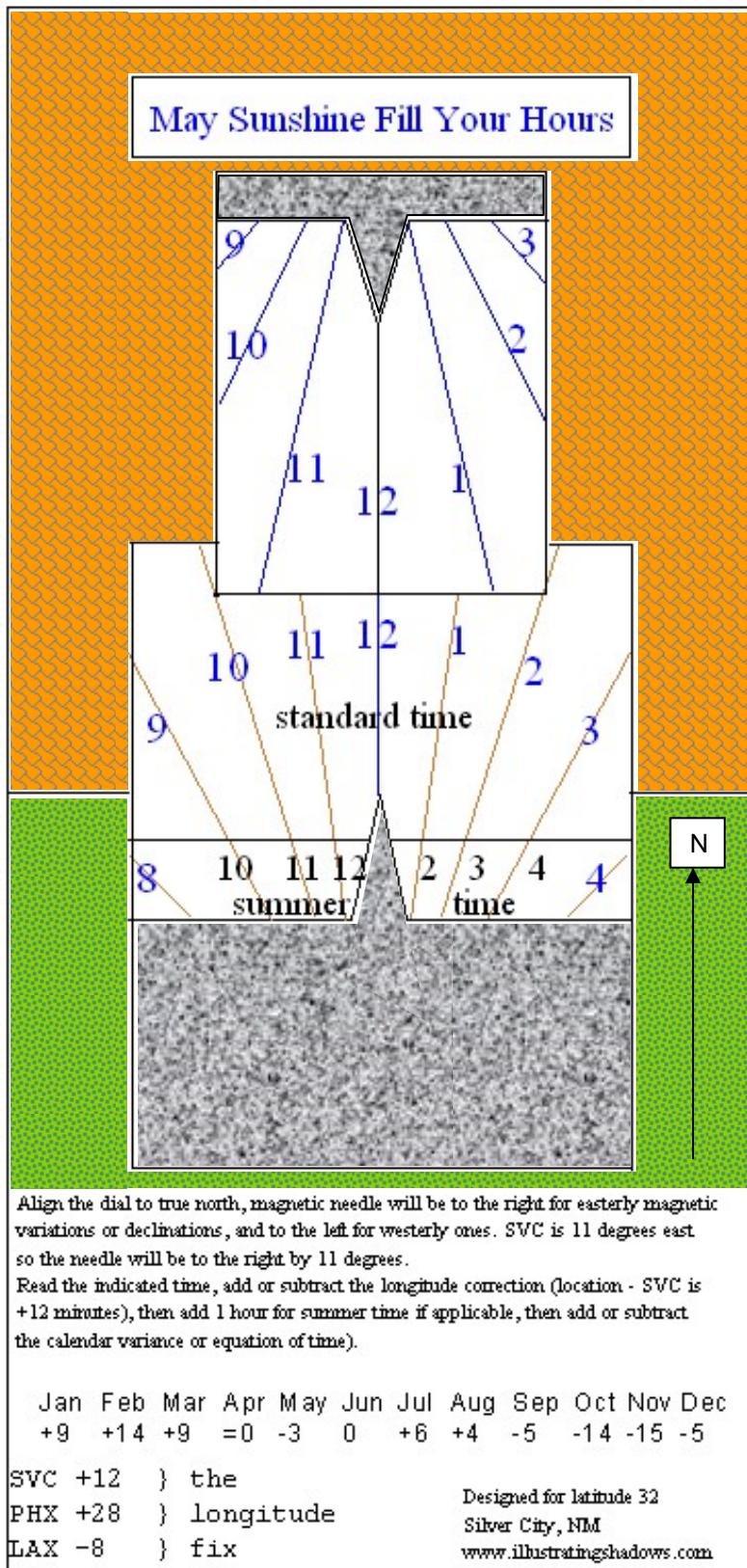


Cut out the gnomon's two sides and rotate it up.

Align to true north.

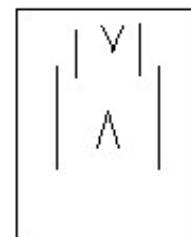
Strengthen the middle of the dial so it will not sag.

BASE LAT IS  $50^\circ$  so this works from  $20^\circ$  to  $65^\circ$

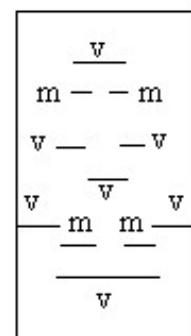


Cut out and pop up  
dial for latitude 32

### Cuts



### Folds



v = valley fold  
m = mountain fold



## APPENDIX 10 – BOOKS, SOFTWARE, AND OTHER REFERENCES

### BOOKS

Amazon.com and Abebooks.com networks have many books. Here is a partial book list of useful books on sundials.

Readily available in online bookstores, uses logs and anti-logs since written some time ago:-

- Sundials: History, Theory, and Practice By: Rene R. J. Rohr, 1965, translated 1970  
A good grounding in the history and theory of most sundial types.
- Sundials: Their Theory and Construction By: Albert Edmund Waugh, 1973  
A good book, with good reference material
- Sundials: Their Construction and Use By: R. Newton Mayall, Margaret W. Mayall  
A good book, with good reference material - first published 1938, second pr 1951

Less readily available, often found in on-line sites such as ebay:-

- Sundials, Roy K. Marshall, 1963
- Sundials Old and New, A. P. Herbert 1967
- Sundials - A Simplified Approach by Means of the Equatorial Dial, by Frank W Cousins  
A good book, good graphical methods, 1968, also known as
- Sundials – The Art and Science of Gnomics, by Frank W Cousins, 1970
- A Choice of Sundials, Winthrop W. Dolan, 1975  
A good book, good graphical methods for some dials not in other books
- Sundials Australia. Margaret Folkard, and John Ward.  
Good work on shadows and on formulae.
- The Art of Sundial Construction, Peter Drinkwater  
Somewhat quaint language, yet with helpful insights
- Sundials And Roses Of Yesterday, Earle, Alice Morse  
A somewhat nostalgic book with interesting pictures and other trivia,  
also a good collection of poems relating to dials.
- Hartmann's Practika - John Lamprey
- Astronomical Algorithms, Jean Meeus, also (uses 2000 as the epoch)
- Astronomical Formulae for Calculators, Jean Meeus (uses 1900 as the epoch)  
Has formulae for the equation of time, Julian day, and a general sundial formula
- Greek and Roman Sundials, Sharon L. Gibbs, 1976
- The Book of Sundials, Mrs Alfred Gatty, 1872

Related books:-

- The Sundial and Geometry, Lawrence E. Jones, NASS publication
- The Great Sundial Cutout Book, Robert Adzema and Mable Jones
- The Earth Is A Sundial – Mitsumasa Anno (also printed as Anno's Sundial)
- Sundials and Spheres, by Kenneth Lynch, architectural focus
- Sundials – Incised Dials or Mass Clocks, Arthur Robert Green
- Stars and Planets, Ian Ridpath
- 
- The Techniques of Decorative Stained Glass, Paul San Casciani
- The Art of Painting on Glass, Albinas Elskus
- Answers to Potters' Questions II, The American Ceramic Society, Ruth C. Butler editor
- The Pop-Up Book,, Paul Jackson.

## VERY USEFUL SPREADSHEET SOFTWARE

<a href="http://www.illustratingshadows.com">http://www.illustratingshadows.com</a>	Excel page ~ MS Office Excel
illustratingShadows.xls	All purpose set of sheets – all dialling needs, wall declination, determining a dial's latitude, astrolabe, EOT, etc
simpleShadows.xls	h-dial and simple altitude/azimuth
cubicShadows.xls	covers lots of dialing needs, simplified versions of: illustratingShadows.xls
<a href="http://www.openoffice.org/">http://www.openoffice.org/</a>	Open Office is free and MS Office compatible
<a href="http://www.kingsoftstore.com/">http://www.kingsoftstore.com/</a>	free & MS Office compatible
<b>Android:</b>	There are spreadsheets available for the Android and other cell phones
e-droid-cell	is excellent and there is a free version
Kingsoft	<a href="http://www.kingsoftstore.com/">www.kingsoftstore.com/</a>
Also there are bubble level tools and compasses also available	
<b>Sol Et Umbra</b>	is an App with much useful data

## SOFTWARE OF INTEREST TO THE DIALIST

SHADOWS	<a href="http://www.shadowspro.com">www.shadowspro.com</a>	most comprehensive, license is reasonable, and a free version covers many common needs
 		NASS (North American Sundial Society) BSS (British Sundial Society) has links to software such as the popular ZW2000, and SUNDI

## PROGRAMMABLE CAD, AND VRML SOFTWARE

<a href="http://www.deltacad.com/">http://www.deltacad.com/</a>	DeltaCAD (uses BASIC macros)
<a href="http://www.nanocad.com">http://www.nanocad.com</a>	NanoCAD (Java Script, VBS)
<a href="http://www.freecadweb.org/">http://www.freecadweb.org/</a>	FreeCAD (Python)
<a href="http://www.powerdraw.software.informer.com">http://www.powerdraw.software.informer.com</a>	Powerdraw (Pascal like)
<a href="http://www.imsisoft.com">http://www.imsisoft.com</a>	TurboCAD Deluxe (can export vrml)
<a href="http://www.progesoft.com/en/products/progecad-smart/">http://www.progesoft.com/en/products/progecad-smart/</a>	ProgeCAD free (LISP)
<a href="http://www.parallelgraphics.com/products/isb/">http://www.parallelgraphics.com/products/isb/</a>	ISB virtual world builder
<a href="http://www.cortona3d.com/cortona">http://www.cortona3d.com/cortona</a>	vrml browser plug-in
<a href="http://www.blender.org/">http://www.blender.org/</a>	Blender free CAD (Python)

## WEB RESOURCES US and UK Societies have excellent web sites

<a href="http://sundials.org/">http://sundials.org/</a>	NASS North American Sundial Society
<a href="http://www.sundialsoc.org.uk/">http://www.sundialsoc.org.uk/</a>	BSS British Sundial Society
<a href="http://www.mysundial.ca">http://www.mysundial.ca</a>	Sundial Primer – excellent site
<a href="http://www.stainedglasssundials.com">http://www.stainedglasssundials.com</a>	Stained Glass dials of the world
<a href="http://www.infraroth.de/slinks.html">http://www.infraroth.de/slinks.html</a>	An excellent links page

## RESEARCH AND SOURCES OF FORMULAE

<http://www.lib.berkeley.edu/ENVI/Sundials.html>

## PROGRAMMING LANGUAGES AND SYSTEMS ~ notes and urls

### LEGACY LANGUAGES:-

**ADA:** A system is available at: <http://libre.adacore.com/libre/>

**ALGOL:** A system is available at: <http://www.xs4all.nl/~jmvdveer/algol.html>

The downloaded executable is in a "tgz" form but may download as a ".tar" file. If so, you must change the ".tar" suffix back to ".tgz" otherwise Winzip will fail to unzip the files.

**PASCAL:** There are several Pascal compilers available. One is an 8mb downloadable fully integrated development environment (IDE) with a modern look and feel, another is about 28mb, has an IDE, but its windows are older style, yet it has the best documentation. Also, LAZARUS is an Open Source system Pascal GUI forms based system, similar to Delphi.

8mb, looks like Windows	<a href="http://www.bloodshed.net/devpascal.html">http://www.bloodshed.net/devpascal.html</a>
28mb, good notes	<a href="http://www.freepascal.org/download.var">http://www.freepascal.org/download.var</a>
rtl.pdf	Run time library: functions and required "uses"
ref.pdf	Reference: language structures
Lazarus (GUI Pascal)	<a href="http://www.lazarus.freepascal.org/">http://www.lazarus.freepascal.org/</a>

<b>FORTRAN:</b> compiler/linker is at::	<a href="http://gcc.gnu.org/fortran/">http://gcc.gnu.org/fortran/</a>
notes about it are at:	<a href="http://gcc.gnu.org/wiki/GFortran">http://gcc.gnu.org/wiki/GFortran</a>
the compiler download link is:	<a href="http://gcc.gnu.org/wiki/GFortranBinaries">http://gcc.gnu.org/wiki/GFortranBinaries</a>

<b>COBOL</b>	<a href="http://opencobol.org">http://opencobol.org</a>	not tested
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<b>BASIC:</b> An excellent system is at:	<a href="http://www.justbasic.com/">http://www.justbasic.com/</a>
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<b>C and C++ and IDE:</b> There is a free C or C++ compiler available	<a href="http://www.windows8downloads.com/win8-dev-c--wdoxnrt/">http://www.windows8downloads.com/win8-dev-c--wdoxnrt/</a>
	<a href="http://sourceforge.net/projects/orwelldevcpp/?source=dlp">http://sourceforge.net/projects/orwelldevcpp/?source=dlp</a>

<b>C#</b>	<a href="http://msdn.microsoft.com/en-us/vstudio/hh341490.aspx">http://msdn.microsoft.com/en-us/vstudio/hh341490.aspx</a>
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<b>PERL and IDE</b>	<a href="http://www.activestate.com/activeperl">http://www.activestate.com/activeperl</a>
	<a href="http://open-perl-ide.sourceforge.net/">http://open-perl-ide.sourceforge.net/</a>

### LEGACY COMPUTERS:-

**IBM 1401:** [www.illustratingshadows.com/stats-IBM1401.html](http://www.illustratingshadows.com/stats-IBM1401.html)

Written using Lazarus (open source), this assembles SPS and Autocoder programs, with H and V dial programs using "card" input as well as switches. Tracing and core dumps, are provided. The simulator and its source code are also Open Source.

**IBM 360:** [www.illustratingshadows.com/stats-IBM360.html](http://www.illustratingshadows.com/stats-IBM360.html)

Written using Lazarus (open source), this assembles BAL programs, with H and V dial programs using console dials. Tracing and core dumps, are provided. The simulator and its source code are also Open Source.

**IBM 7094:** There are several IBM 7094 simulators available, the one with a graphical depiction of the console and that also supports FORTRAN II is available at:

<http://www.members.optushome.com.au/intaemul/Emul7094.htm>

## **PROGRAMMABLE CAD SYSTEMS:-**

Illustrating Shadows scripts or programs exist for the following CAD systems. Programming Shadows has additional very helpful notes:-

**DELTACAD:** A 2D CAD system programmable in a BASIC like language.  
<http://www.deltacad.com>

**NANOCAD:-** Is free, programmable in VBS (Visual Basic Script), Java Script , and LISP.  
<http://www.nanocad.com>

**FreeCAD:-** Is free, powerful Python scripting with sundial scripts  
<http://www.freecadweb.org/>

**Powerdraw:** Pascal like programming. Documentation is limited.  
<http://www.powerdraw.software.informer.com>

**ProgeCAD:-** From ProgeSOFT, supports LISP. Free for non commercial use.  
<http://www.progesoft.com/en/products/progecad-smart/>

**TURBOCAD:-** TurboCAD offers VBS (Visual Basic Script), and parametric scripts  
<http://www.turbocad.com>

**Blender:-** A free 3D system (not really CAD), supports Python, sundial scripts available  
<http://www.blender.org/>

## **OBJECT ORIENTED PROGRAMMING SYSTEMS:-**

**Lazarus:** a free IDE with Pascal underpinnings, similar to Delphi, and Open Source, and it is available at:-  
[www.lazarus.freepascal.org/](http://www.lazarus.freepascal.org/)

**Visual Basic** Visual Studio products are at:-  
<http://msdn.microsoft.com/vstudio/express/downloads/default.aspx>

**VISUAL C#** is downloaded, first as a small installer, then as a large file of stuff, which it then installs if the installer likes your system. This is from Microsoft, and the registration process worked easily. Excellent IDE in the latest Visual Studio  
<http://msdn.microsoft.com/vstudio/express/downloads/default.aspx>

**JAVA:** (using the NetBeans IDE). There are free systems available from several organizations. First, there is a JDK (Java development kit), and it has the stuff of which JAVA is made. Second, there are several IDEs (integrated development environments), one is NetBeans, however, others exist. Some are graphical, some are not. NetBeans is graphical, and is used here. JPadPro is a non graphical IDE, not used here. NetBeans and the appropriate JDK can be downloaded together from the following download sites. (urls may change over time).

<http://www.netbeans.info/downloads/index.php>  
<http://java.sun.com/javase/downloads/index.jsp>

**JAVA SCRIPT:** Supported by many browsers, more procedural than object oriented, easy to write and test (especially using FireFOX), and can be run locally in a browser, as well as on the internet web, including smart phones. Also used in NanoCAD, a free CAD system.

## OBJECT ORIENTED PROGRAMMING SYSTEMS - continued:-

<b>PYTHON:</b> An alternative to JAVA:-	<a href="http://www.python.org/">http://www.python.org/</a>
Python itself	<a href="http://www.python.org/download/releases/2.5.6/">http://www.python.org/download/releases/2.5.6/</a>
Python Scripter	<a href="http://www.mmm-experts.com/">http://www.mmm-experts.com/</a>
Documentation	<a href="http://www.python.org/doc/">http://www.python.org/doc/</a>
Useful introduction	<a href="http://wiki.python.org/moin/BeginnersGuide">http://wiki.python.org/moin/BeginnersGuide</a>

Python is used in FreeCAD as well as in the free Blender CAD program

## SCIENTIFIC GRAPHICAL PROGRAMMABLE SYSTEMS:-

<b>SCILAB:-</b>	Home page and manuals are at:  <a href="http://www.scilab.org/">http://www.scilab.org/</a> <a href="http://www.scilab.org/product/man/">http://www.scilab.org/product/man/</a>
<b>EULER:-</b> Similar to SciLAB:	<a href="http://mathsrv.ku-eichstaett.de/MGF/homes/grothmann/euler/">http://mathsrv.ku-eichstaett.de/MGF/homes/grothmann/euler/</a>
<b>OCTAVE:-</b> Similar to SciLAB, at:-	<a href="http://www.gnu.org/software/octave/">http://www.gnu.org/software/octave/</a>

**NOTES:** Illustrating Shadows programs exist for all the above language systems. Many are discussed in Illustrating Time's Shadow, some are available along with copious notes, as supplements for each language's web page in: <http://www.illustratingshadows.com>

**NOTES:** Extensive Excel spreadsheet usage is documented in Illustrating Time's Shadow and two different methods of graphical depiction used, one used an algorithm to use a pie chart, the other derives x:y points and uses a line draw feature of INSERT CHART, X:Y SCATTER, and an option for connecting x:y pairs as lines. Most Excel spreadsheets run as-is on **Open Office** software, including the graphical depictions. Open Office software is free.

TurboCAD uses VBS, and as such is also object oriented. TurboCAD's parametric scripts which are very simple to use are not object oriented.

## REFERENCES THAT RELATE TO NOMOGRAMS:

The nomograms in Illustrating Time's Shadow were built using: [nomogram.bas](#) which is a DeltaCAD macro designed assuming that there were two parameters, one displayed on the left side and the other on the right. And that the solution was found in the center, mid way between the two data inputs. Of interest, the normal base 10 logarithm didn't appear to work, so Napierian logs (natural logs) were used.

Graphical and Mechanical Computation  
[http://www.myreckonings.com/wordpress/wp-content/uploads/Graphical\\_and\\_Mechanical\\_Computation.pdf](http://www.myreckonings.com/wordpress/wp-content/uploads/Graphical_and_Mechanical_Computation.pdf)

Python software <http://www.myreckonings.com/>  
JAVA online: <http://www.ece.rochester.edu/~jones/NomoDevel/nomogram.htm>

## OPERATING SYSTEM ISSUES – WINDOWS XP, VISTA, 7, 8: 32 AND 64 BIT

The software presented in this book was originally designed and tested on a 32 bit operating system called Windows XP. Windows XP has major updates called service packs. Much of the software runs on Windows XP with service pack (SP) 1. Some of the Microsoft software requires SP2, for example software using .NET support such as Visual Basic. After Windows XP came Windows Vista which has two flavors, one was a 32 bit system, the other a 64 bit system. NOTE: Windows 64 bit programs cannot run on a 32 bit system, thus for example:

Lazarus 64 bit code will only run on 64 bit systems  
Lazarus 32 bit code will run on 32 and 64 bit systems

**DELTACAD:** The Windows task bar no longer has a place for the MACRO and the DISPLAY AREA, there is now only one task bar for DELTACAD. It is harder to go back and forth between the drawing area and the macro. The aspect ratio is good for wide displays. The XP DeltaCAD syntax was more forgiving, DCCREATETEXT format is more rigid, the function syntax demands parameters even if not used. However it is easier to run macro.BAS from anywhere on a double click in VISTA.

**VRML and my virtual worlds:** On Windows Vista win64 the Cortona plugin has been known to crash, however that is the exception, the rule is that it runs well, so retry. It works with my Jaipur world of sundials and for my individual dials if VRML 2 (some of mine were VRML 1). However, in Vista, for standalone viewing offline, use a standalone viewer. The following work well in Windows XP as well as Vista, with XP being more flexible.

Use **Cortona** plugin for internet worlds and models      [www.cortona3d.com/cortona](http://www.cortona3d.com/cortona)  
some vrml 1.0 models do not work  
most vrml 2.0 models work well  
supports full blown worlds as well as models but...  
sometimes Windows Internet Explorer loops on asking for permissions

Use **Instant Reality** for worlds on your local hard drive    [www.instantreality.org](http://www.instantreality.org)  
supports full blown models  
less good for models

Use **Swirl** (Pine Coast) for models on your hard drive    [www.pinecoast.com](http://www.pinecoast.com)  
supports models  
supports X3D the successor to VRML which it also supports  
less good for worlds

## ALMANACS

<a href="http://www.almanac.com/rise/index.php">http://www.almanac.com/rise/index.php</a>	Old Farmer's
<a href="http://geomag.nrcan.gc.ca/index-eng.php">http://geomag.nrcan.gc.ca/index-eng.php</a>	geomagnetism
<a href="http://aa.usno.navy.mil/data/docs/RS_OneYear.html">http://aa.usno.navy.mil/data/docs/RS_OneYear.html</a>	sun/moon rise etc
<a href="http://www.ngdc.noaa.gov/geomag/declination.shtml">http://www.ngdc.noaa.gov/geomag/declination.shtml</a>	magnetic decl/var

NOTE: The term magnetic declination is used in the solar and sundial communities. It is exactly the same as magnetic variation that is used in the piloting and navigation communities.

## SOME POINTS TO CONSIDER WHEN READING THIS BOOK

### DEFINITIONS

The definitions used in this book are appropriate to this book's values and intent. Some in the sundial community use a more formal set of definitions. For more exhaustive research, the work of John Davis - BSS Glossary Editor, is considered a standard and can be accessed via the NASS (North American Sundial Society) or BSS (British Sundial Society) web sites. The standard is more rigid and more suited for mathematicians than those in this book. One such web link is:

<http://www.sundialsoc.org.uk/glossary.php>

### TEMPLATES, PICTORIALS, AND DIAL PLATE PORTRAYALS

Templates contained herein are intended to be usable as is, and accurate to within acceptable tolerances. However, some pictorials are not to scale and a note states that fact when it is not intuitively obvious. Also, some curves on some dials are not "perfect", however when they are drafted using software, the intent is to show what the software can do, and its limitations. Thus, software produced curves are not doctored to make their product match the higher standards of this book.

### CLAY TEMPERATURES, SLIPS, AND GLAZES. GLASS TEMPERATURES.

The clay used was a terra cotta red suitable for 2230°F (temperature) or cone 6 (heat), not cone 06, and a similar clay for cone 5. Clay slip additives were Mason 6363 sky blue, 6242 Bermuda green, 6406 buttercup yellow, 6310 Mediterranean blue, 6376 robin's egg blue, and titanium dioxide for a whitish slip. Slip additives were mixed with water and clay. Use caution with any powdered clay or pigment, use a face mask and rubber gloves. Bisque (slip) firing was slow to 250°F (lose trapped water), faster to 900°F (burn chemically bound water), then full speed to about 1800°F (burn carbon and sulfur), and soak there for a while. The clay glazes used were Duncan GL612 (gloss) and SN351 (satin) fired slow to 800°F, thence to 2230°F cone 6, or cone 5. Use caution when firing slip, or glaze as the vented gasses can be noxious. Liberal glazing and minimal clay incisions and max temperature soaking decrease freeze thaw winter moisture damage. Clay vitrification is the secret, and lower cone clays fired to their vitrification cone can last many years outdoors. Horizontal dials are the most susceptible to freeze thaw damage.

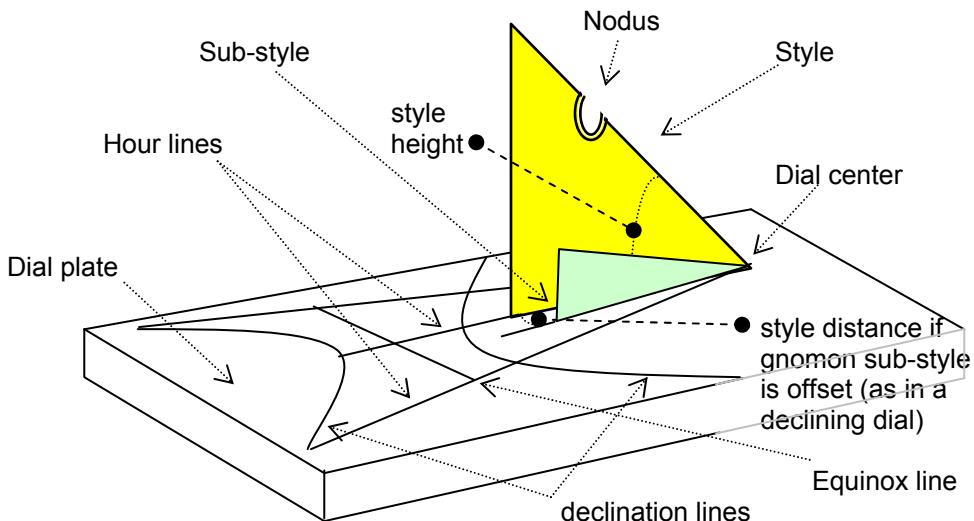
The glass used was double strength (DS), the black and brown pigments were fired at 750°C, and the golden yellow silver-nitrate stain was fired at 1250°F on the other (exterior) side.

### DIAL PLATE NUMERALS

Conventional Arabic	0	1	2	3	4	5	6	7	8	9	10
Hindu Arabic	•	।	২	৩	৪	০	৬	৷	৸	৯	১০
Persian Arabic	•	۱	۲	۳	۴	۵	۶	۷	۸	۹	۱۰
Roman	none	I	II	III	IV	V	VI	VII	VIII	IX	X
Greek		20 : XX		50 : L		100 : C		500 : D		1000 : M	
	none	A'	B'	Γ'	Δ'	E'	F'	Z'	H'	Θ'	Ι'
		11 : IA'		12 : IB'							

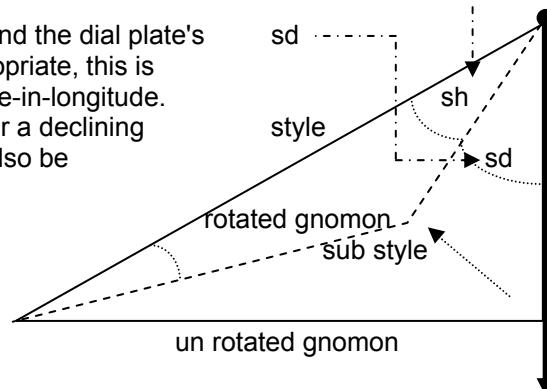
## DIAL PIECES AND ELEMENTS

gnomon	the entire contraption used to cause a shadow, the shadow of course tells the time, and can also tell the day of the year. From the Greek word "to know".
style	the line on the top of a gnomon that makes a fixed shadow line that indicates the time
nodus	either the tip of the style, or in some cases a notch or some other distraction in the style, used to indicate calendar information. The nodus controls the calendar lines, the style controls the hour line shadow. A nodus allows a larger gnomon that would otherwise work on a dial plate.
dial plate	the object upon which are drawn hour lines, calendar information, in other words the dial furniture
dial furniture	the whole contraption including the gnomon and the dial plate
dial center	marks of interest on the dial plate
hour lines	where the hour lines converge, in some cases they do not however
hour points	the lines each of which indicate time when the style's shadow aligns with it some dials (analemmatic) do not have a style nor hour lines, they have hour points and use a vertical movable gnomon.



style height      the angle made between the style and the sub-style      sh  
 note: in some cases where it is absolutely clear, this may refer  
 not to the angular distance, but rather to the linear distance from the dial  
 plate to the tip of the style, an example might be the polar and the true  
 east and west vertical dials.

style distance      the angle between the sub-style and the dial plate's  
 vertical or horizontal line as appropriate, this is  
 not the same as "dl", the difference-in-longitude.  
 The gnomon is always declined for a declining  
 dial, however the gnomon may also be  
 rotated for convenience which  
 (a) avoids chamfering  
 (b) simplifies calendar lines  
 (c) does other good things



## CONSTRUCTION AND SAFETY TIPS

For hour angle dials, the style should parallel any and all hour lines when viewed from any direction, if they do not, then something is not quite right.

Hour angle dials use the hour angle around the style and can easily be latitude corrected by tilting. Italian and Babylonian hours are not corrected when a dial is tilted because they depend on the Earth's curvature at a specific latitude.

To test small dials quickly, place on a movable surface with a known good dial, to act as a proxy. Rotate both dials and see if the sun's shadow matches on both of them.

NEVER look at the sun, nor close to it. All observations of the sun must be indirect. AND when using a laser to indicate lines for a dial, make sure it does not shine in anyone's eyes.

Wood can warp with temperature and humidity. Wooden gnomons thus may deflect.

Gnomons are frequently sculpted with holes or other cutouts to reduce wind resistance. Gnomons are subjects of vandalism, plan accordingly. Gnomons of copper plate can flex, and over time bend. Such gnomons should be strengthened.

Gnomons are pointed, and as such can inflict injury if you charge at them on purpose or by mistake, such as slipping while working on a vertical dial.

Lead was once used for dial plates, it being easy to score and bend. However, lead and the animal life forms do not co exist very well. Take care when using lead, or lead based solders. Fluxes used for solder work can also be toxic to the skin or the lungs.

Spray paints have fumes that can be injurious to the body.

Glass and wood, copper as well, when manipulated can cause splinters in the hands or the eye. Personal protective equipment should thus be employed.

When using a computer to draw dial plates, remember to also draw marks for the screws to bolt a protective cover such as plexi-glass over the paper and onto the dial surface. This will protect against covering up useful data such as the equation of time charts or tables, and the like.

Longitude correction can be built into the dial hour lines or hour points, or deferred to a modified equation of time. Portable dials should never have the longitude correction as it would limit their portability. Even larger dials may be moved when a family relocates. If a dial has no longitude correction built into it, it is easier to design, and can be relocated. The correction for the EOT is simply modified to include the longitude correction. There is nothing wrong with this solution.

When downloading spreadsheets, documents, files, or programs always use a virus checker.

When building spreadsheets, always double check formulae and functions. For example the DATEVALUE function provided questionable values. Use a spreadsheet's charting function to quickly detect inconsistencies, and to gain insights.

Use programs like SHADOWS to confirm your calculations, it also has a great shadow simulator.

When using any knife or snipers, always use eye protection.

When using a kiln, observe all safety precautions including venting.

Use ladder safety when working on ceiling dials, it is easy to lose balance.

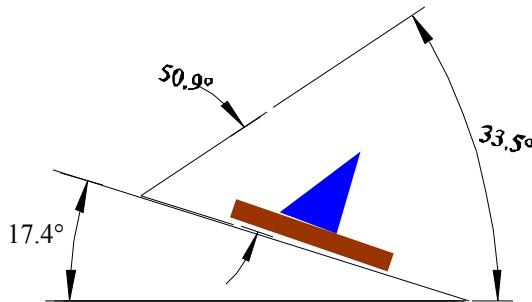
## TESTING A DIAL SOMEWHERE OTHER THAN DESIGN LOCATION

To use or to test a sundial in a location other than the dial's design location, the following two steps must be taken.

### Step 1

#### EXAMPLE

design lat:	50.9
used at Lat:	33.5
difference:	17.4



- ( 1 ) tilt the dial by the difference in latitude **in this case, 17.4 degrees**

Now the gnomon's style is correct, we must consider longitude differences

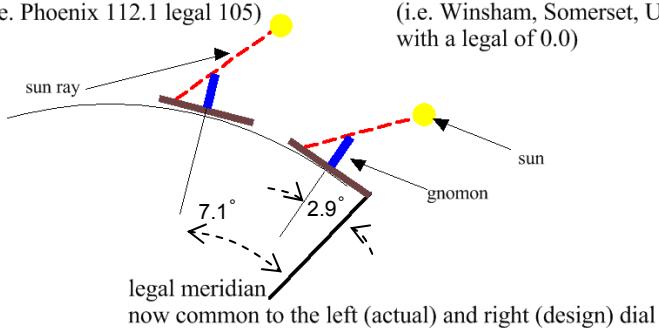
### Step 2

Next, with simple arithmetic "move" the actual dial towards the design dial so that they "share" the same "legal meridian". In this specific example, the actual dial ends up west of the design dial.

**NOTE:** For a dial being used at a location west of its design location, a later time is needed to achieve the same hour angle on the dial, and thus the same hour line. The opposite if the dial is east of the design location.

DIAL IN USE AT  
longitude 7.1 from legal  
(i.e. Phoenix 112.1 legal 105)

BUT DESIGNED FOR  
longitude 2.9 from legal  
(i.e. Winstan, Somerset, UK is 2.9  
with a legal of 0.0)



In the above example:-

Design lat:	2.9	legal: 0.0	<b>diff = 2.9</b> from a legal of 0.0
Used at Lat:	112.1	legal 105.0	<b>diff = 7.1</b> from a legal of 0.0
Difference (7.1 - 2.0)	4.2°	since 1° = 4 minutes, minutes = 16.8 mm.m	

- ( 2 ) correct the times by the difference in minutes **in this case, add 16.8 plus the EOT**

In other words, at the hour in question plus the difference in minutes (+ or -) plus the EOT, the shadow should be on the appropriate hour line.

**NOTE:** this works for hour lines, it does NOT work for Babylonian nor Italian lines. However, the declination curves (calendar lines and curves) will still be correct.

The main spreadsheet "illustratingShadows.xls" has a worksheet called "EOTandLONG" which provides the correction for longitude plus the EOT, both by the day, or by the 5<sup>th</sup>, 15<sup>th</sup>, and 25<sup>th</sup> of the month, see next page for an example.

## EOTandLONG worksheet of: illustratingShadows.xls

This worksheet allows you to enter the design longitude and legal meridian, and the "in use at" longitude and legal meridian. It produces a quick table, and below the example uses the information on the preceding page:-

Revised EOT for relocated dial			
Dial:	2.9	Used:	112.1
	5th	15th	25th
Jan	21.9	26.0	29.0
Feb	30.8	31.0	30.0
Mar	28.4	25.8	22.9
Apr	19.6	16.9	14.8
May	13.5	13.1	13.7
Jun	15.2	17.2	19.3
Jly	21.3	22.7	23.3
Aug	22.8	21.4	19.0
Sep	15.7	12.2	8.7
Oct	5.4	1.3	0.9
Nov	0.3	1.3	3.6
Dec	7.2	11.7	16.7

SMALLEST CORRECTION: 0.32  
LARGEST CORRECTION: 31.05

This chart can be used when a dial is designed for the legal time meridian but used at a different longitude. It can also be used when a dial is moved in longitude but the latitude is substantially the same.

EOT per se = 0
15-Apr
15-Jun
1-Sep
25-Dec

## EXAMPLE ONLY

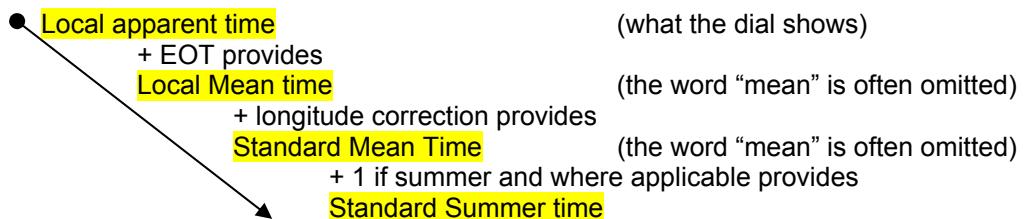
EQUATION OF TIME (EOT):	NON LEAP YEAR	LONGITUDE CORR	DIAL DESIGNED		BUT USED HERE							
			Long	2.9	new long	112.1						
generic EOT table			legal	0.0	legal	105.0						
Jan	Feb	Mar	Apr	May	Jun	Jly	Aug	Sep	Oct	Nov	Dec	
1	20.05	30.31	29.23	20.78	13.95	14.57	20.58	23.16	16.95	6.63	0.36	5.67
2	20.53	30.44	29.03	20.49	13.83	14.72	20.77	23.10	16.64	6.30	0.33	6.05
3	20.99	30.57	28.82	20.19	13.72	14.88	20.95	23.02	16.32	5.98	0.32	6.44
4	21.45	30.67	28.61	19.90	13.62	15.05	21.14	22.94	15.99	5.67	0.32	6.83

The above is an example of the first few days extracted from the "EOTandLONG" worksheet.

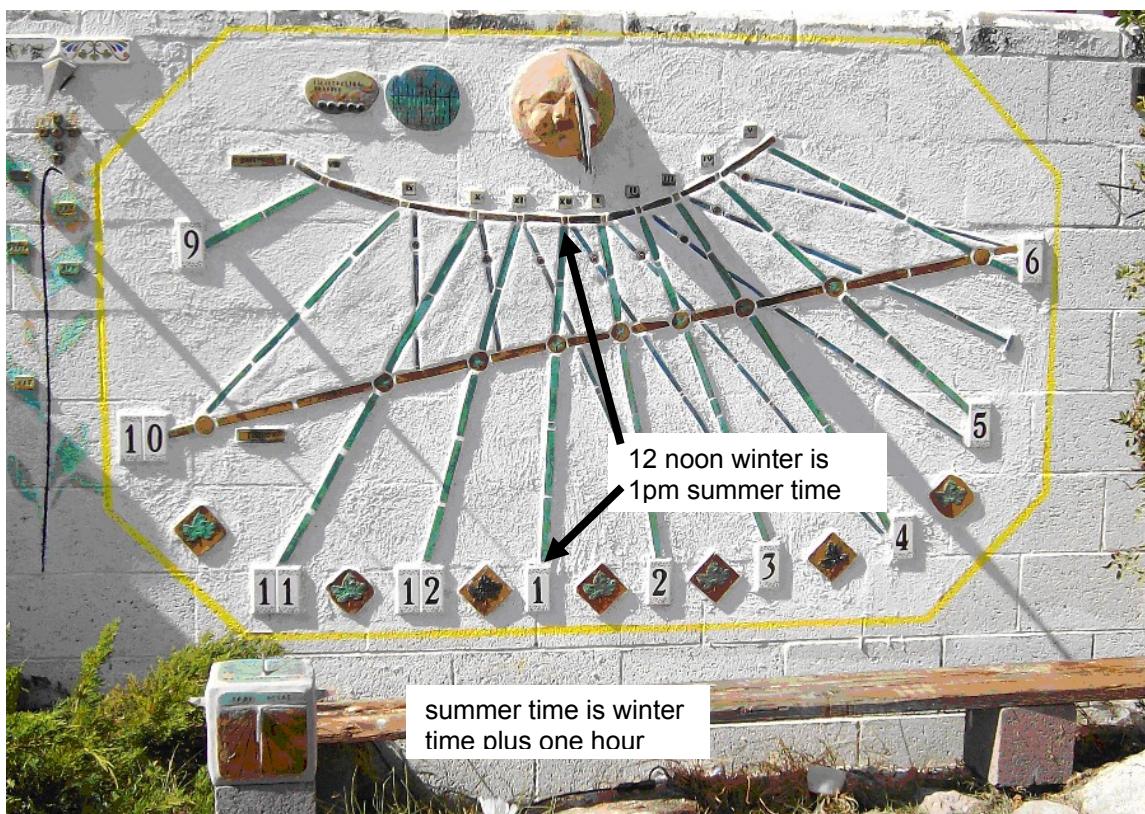
**NOTE:** some people design dials to be east about 17 minutes from the intended final destination so they always have a positive net EOT correction.

## DEPICTING DAYLIGHT SAVING TIME ~ OR SUMMER TIME

Below is a vertical declining dial, with winter time on the winter solstice curve, summer time on the summer solstice curve. It is that easy. The formula is simple, summer time is usually 1 hour later than winter time. So, on the noon line for winter time would be 12 noon, and summer time would then be 1pm, and so on. Summer time is winter time plus 1 hour in most places.



**NOTE:** Many private sundial designers build a longitude correction into their dials, a few also include the “analemma” to incorporate the EOT.



See also: summer time, daylight saving

Illustrating Time's Shadow chapter 5, page 32

Rohr	page 30, Sundials, History, Theory, and Practice
Waugh	page 17, Sundials, Their Theory and Construction
Mayall	page 94, Sundials, Their Construction and Use
Cousins	page 79, Sundials, The Art and Science of Gnomonics

## CASE STUDIES ~ <http://www.illustratingshadows.com/>...filename...

Below is a very partial list of the Illustrating Shadows case studies. Some of these case studies are on the web site, all of them are on the CD. The files with blanks in the file name are not on the web site, and not all case studies are on the web site either, only a select few. These are design or construction notes with helpful hints.

### ARIZONA

allDialPlatesPhoenixAZ.zip	meridian west facing, sandstone column 45° offset glass, copper s vertical, S51E vertical decliner, true north vertical, grey column cube dial true cardinal, armillary, inclined decliner heart shaped Saltillo dial with Persian lettering
caseStudyArabicVdial.pdf	floor tile as true south vertical dial
caseStudyFloorTileVdial.pdf	large 40" across true south vertical dial, calendar curves
caseStudyLargeVdial.pdf	true south middle east looking Saltillo tile dial
PHXvodial.pdf	see also: vDialArabianPatioOct6.pdf 45 degree from cardinal, cube dial, for a garden center park
phxGardenCenterDial.pdf	a polar dial using Saltillo tile
PHX polar dial.pdf	brick column horizontal dial, Italian, Babylonian hours also
PHXhdialMar21.pdf	a ten inch square column cube dial just off cardinal points
PHXvdec10inchColumn.pdf	

### NEW MEXICO

analemmatic-garden-dial.pdf	a large analemmatic dial
CaseStudy-S5W-v-clay.zip	S5W vertical decliner, New Mexico, clay
case-study-polar-dial.pdf	polar dial using copper tube and copper wire
case-study-openbook-dial.pdf	an open book dial with no separate gnomons
clay-dials.pdf	see also: gnomonless-armillary.pdf an 8 inch cube dial
cube-dial-design-HeyFarm.pdf	a 16 inch column off cardinal points cube dial
cubeLat479Lng1219ref120.pdf	glass 8 in cube 45 off meridian
inclined-to-decline.pdf	shallow vertical declining dial
paver-equatorial.pdf	an equatorial dial using a circular paver
pipe-armillary.pdf	armillary using an old sewer pipe
reflecting-dials.pdf	large ceiling dial
s44w-s46e-cube-dial-design.pdf	off cardinal points 16 inch column dial glass plates
s96w-dial-arrow.pdf	a vertical decliner using clay
vertical-declining-south-trigon.pdf	Italian and summer time, large home built trigon

### INDOOR DIALS

cubeLat33.pdf	small indoor cube dial meridians, H and V dials
DiptychLat33Long108.pdf	small indoor diptych vertical and horizontal dial
engraved-dials.pdf	computer engraving printer dials

### UNITED KINGDOM

ukWinshamDials.pdf	testing a dial designed for one location at a different location
ukPiltonDials.pdf	design for some Somerset dials

### MISC LOCATIONS

cubeLat479Lng1219ref120.pdf	a glass and copper 8 inch cube dial
neverBuiltChimneyDial.pdf	a beautiful chimney dial vertical decliner

## **INDEX TO THE BOOK “ILLUSTRATING TIME’S SHADOW”**

introduction	a walk around the garden and a simple horizontal dial project
chapter 1	the universe and how the stars revolve
chapter 2	the solar system and how the Earth and sun interact
chapter 3	the planet Earth and its tilted axis and the seasons, solar declination
chapter 4	the evolution of the sundial
chapter 5	the equation of time or EOT, orbits and the tilted axis
chapter 6	methods of finding north
chapter 7	geometry and math for sundials
chapter 8	general interest about sundials
chapter 9	the basic hour angle dials, equatorial, armillary, and case study
chapter 10	polar dials, and case study
chapter 11	meridian dials are like polar dials in many ways
chapter 12	the horizontal dial
chapter 13	the vertical true south/north dial
chapter 14	the general model for almost all hour angle dials
chapter 15	the vertical recliner, common on pyramid faces and roofs
chapter 16	the vertical decliner, common on walls, facing mostly the equator or pole
chapter 17	the vertical great decliner, facing mostly east or west
chapter 18	the inclined decliner part 1, introduction, and the steep inclined decliner
chapter 19	the inclined decliner part 2, the shallow inclined decliner
chapter 20	cube dials
chapter 21	altitude dials
chapter 22	azimuth dials
chapter 23	dial furniture ~ calendar or declination curve
chapter 24	dial furniture ~ Italian, Babylonian, and day of length dial furniture
chapter 25	dial furniture ~ the Analemma, and the use of “DL”
chapter 26	other interesting hour angle dials, ceiling, polarized, etc
chapter 27	night time dials
chapter 28	glass, brass, and paper dials
chapter 29	buying dials, and then making them work correctly
chapter 30	programming or use of ~ 3D-CAD, vrml, and spreadsheets
chapter 31	programming or use of ~ 2D-CAD, and other languages
chapter 32	older methods ~ DL, the Nomogram, Slide rule, and Dialing scales
chapter 33	extra thoughts and additional information
appendices	an abbreviated appendix

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[illustratingshadows@yahoo.com](mailto:illustratingshadows@yahoo.com)

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[www.illustratingshadows.com](http://www.illustratingshadows.com)  
[www.illustratingshadows.com/reference](http://www.illustratingshadows.com/reference)

regularly for articles, updates, templates, or spreadsheets. An all purpose spreadsheet covering many dialing functions is available there in Excel (.xls) format). Further, software for dialists is on the CD and website in many languages, with the source code as well as executables fully available. The programs have been tested on 32 and 64 bit Windows systems and on Windows XP, Vista, 7 and 8. In particular, many DeltaCAD programs are provided.