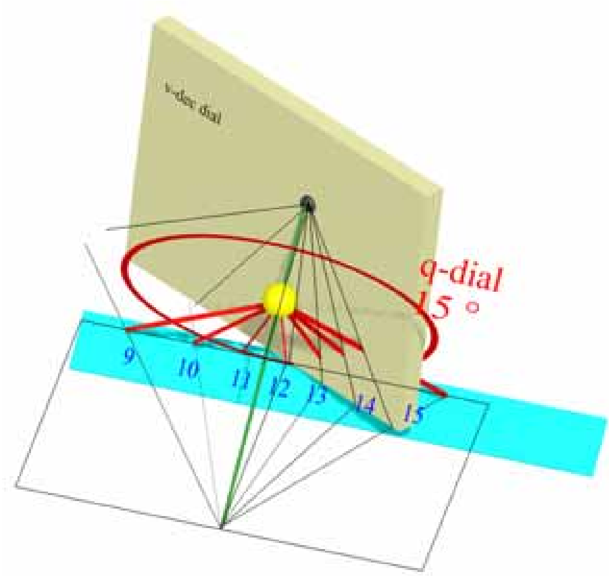
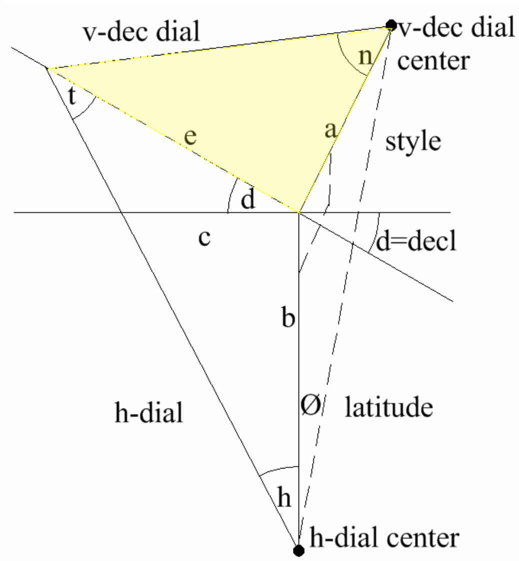
Proof of Decliner/Great Decliner Hour Line angles

In the figure to the right below, the triangle "h-dial" has among other things, sides "b", and "c" and angle "h". Side "b" is the horizontal dial's sub-style and a selected horizontal dial's hour line angle "h". Triangle "v-dec dial" has two named sides, with angle "n" being the vertical decliner's equivalent hour line angle that is associated with the horizontal dial's angle "h". Both dials share a style that connects the h-dial and v-dec dial centers, shown by a depicted dashed line. The vertical decliner's sub-style is not depicted in the figures below, and their "SD" and "SH" (style angular distance and angular height) are derived elsewhere. Declination is "d" and "Ø" is latitude.





$$a = b * \tan(Ø) \qquad [1]$$

$$\tan(n) = e / a \qquad \text{thus ...} \qquad n = \operatorname{atan}(e / a) \qquad [2] \text{ desired}$$

so $$n = \operatorname{atan}(e / b * \tan(Ø)) \qquad [3]$$

$$e / \sin(h) = b / \sin(180 - (h + (90 + d))) \qquad [4] \text{ law of sines}$$

so $$e / \sin(h) = b / \sin(90 - h - d)$$

thus $$e = b * \sin(h) / \sin(90 - h - d) \qquad [5]$$

$$n = \operatorname{atan}(e / a) = \operatorname{atan}\left[\frac{b * \sin(h) / \sin(90 - h - d)}{b * \tan(Ø)}\right] \qquad [\text{using } 2, 5, 1]$$

so $$n = \operatorname{atan}\left[\frac{b * \sin(h)}{\sin(90 - h - d) * b * \tan(Ø)}\right]$$

thus $$n = \operatorname{atan}\left[\frac{\sin(h)}{\sin(90 - h - d) * \tan(Ø)}\right] \qquad [6]$$

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and using

h = atan (sin (Ø) * tan (sun hour angle)) [for h-dial]

and since from the prior page

n = atan (sin (h) / tan (Ø) * sin(90 – h – d)) [from 6]

then using the sun's hour angle as opposed to a surrogate horizontal dial's hour line angles

then

$$n = \text{atan}\left[\frac{\sin(\ \text{atan}(\ \sin(Ø) * \tan(\text{sun hour angle})\ )\ )}{\tan(Ø) * \sin(90 - d - \text{atan}(\sin(Ø) * \tan(\text{sun hour angle})))}\right]$$

[7]

Hence, considering a spreadsheet or a procedural program implementation of a vertical dial that declines, it has hour line angles "n" equal to:-

i.e. n =

```
DEGREES(ATAN( SIN(RADIANS((DEGREES(ATAN( TAN
(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat))  ))))) /
(TAN(RADIANS(lat))*SIN(RADIANS(90-dec-
(DEGREES(ATAN( TAN(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat))
)))))) ))
```

where the hour itself, and longitude corrections are all considered.

The results match the formula usually published which is:-

n = **atan (cos(Ø)/(cos(dec) cot(ha) + sin(dec) sin(Ø))**

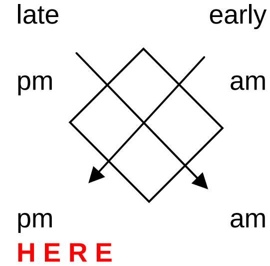
The formula [7] derived above is used in: a5.1 vdec sws formula.xls

an example of its output is shown on the next page

and the standard formula is used in:- a5.1 vertical decliner.xls vertical decliner hour line angle formula and proof

The formula in a functioning spreadsheet: a5.1 vdec sws formula.xls

Vertical Decliner dial		lat	lng	ref	dec [+w]
long diff =	3.2	32.75	108.2	105	30
				Dial faces SW	
time=				v.dec.hr.line.angle	
6				-69.23	
7				-83.92	
8				78.77	
9				59.08	
10				38.62	
11				19.59	
12				3.16	
13				-10.78	
14				-22.96	
15				-34.17	
16				-45.13	
17				-56.57	
18				-69.23	
19				-83.92	
hour		SD	37.86	SH	46.75



For NW use SE pm
for NE use SW am

Simon Wheaton-Smith vertical decliner dial formula using the clock time
=DEGREES(ATAN(SIN(RADIANS((DEGREES(ATAN(TAN(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat)))))))/(TAN(RADIANS(lat))*SIN(RADIANS(90-dec-(DEGREES(ATAN(TAN(RADIANS(15*(12-hr)+d.long))*SIN(RADIANS(lat))))))))))
This formula is nothing more than the formula using an h-dial hour line angles but the hour line angles are substituted for the formula for them.

Or if one removes the substitution of the clock time:-

=DEGREES(ATAN(SIN(RADIANS((DEGREES(ATAN(TAN(RADIANS(sun.ha))*SIN(RADIANS(lat)))))))/(TAN(RADIANS(lat))*SIN(RADIANS(90-dec-(DEGREES(ATAN(TAN(RADIANS(sun.ha))*SIN(RADIANS(lat))))))))))

Or if one removes the substitution for the sun's hor angle and deals with hour lines angles of an associated horizontal dial:-

=DEGREES(ATAN(SIN(RADIANS(hdial.hla))/(TAN(RADIANS(lat))*SIN(RADIANS(90-dec-hdial.hla)))))

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