

Declining Shadows



All sorts of ideas on the drafting of calendar lines on a dial, including the flat dial. The flat dial being important because this is the method used for horizontal, vertical, declining, and inclined declining dials wherein only the style height (angular) is the basis for the hours and calendar lines. Extracted from *Illustrating Time's Shadow*.

The tables, of which there are three variations can be produced with: `reference-spreadsheets.xls` which is on the web site: www.illustratingshadows.com

Also, the provided DeltaCAD macros and the JustBASIC programs provide easy ways of automating calendar data, and the DeltaCAD programs (macros) also provide highly educational animation.

Simon Wheaton-Smith
October 2009

There are indications that the seasons were of more interest to early sun dial users, and that the calendar might have been more important than the time of day.

The sun appears to oscillate annually from minus 23.5 degrees when it is over the southern tropics, to plus 23.5 degrees when it is over the northern hemisphere tropics.

This angle made with the equator is called the declination. Many of the tables show the declination for various days in the year. In particular table A2.11 shows the sun's declination for each day of the average year. This book shows two formulae, and one is:-

$$\text{DEGREES}(0.006918 - 0.399912 \cdot \cos((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.070257 \cdot \sin((2 \cdot 3.1416 \cdot (jd-1)) / 365) - 0.006758 \cdot \cos(2 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.000907 \cdot \sin(2 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) - 0.002697 \cdot \cos(3 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)) + 0.00148 \cdot \sin(3 \cdot ((2 \cdot 3.1416 \cdot (jd-1)) / 365)))$$

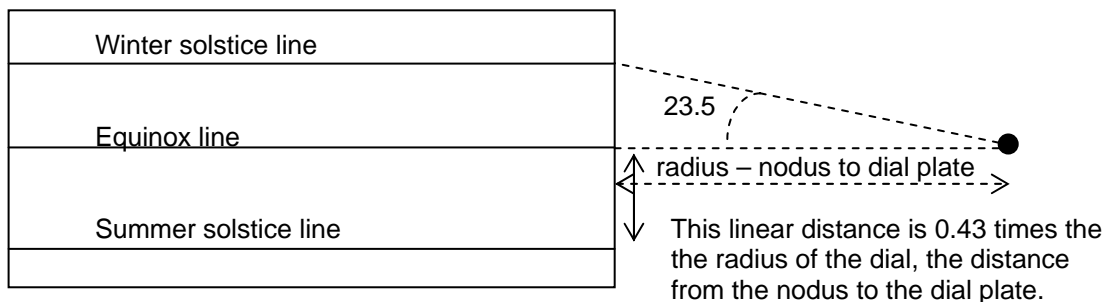
where "jd" is the day in the year 1 to 365

The declination plus the co-latitude provides the sun's altitude at noon.

There are a number of methods for drafting calendar or declination lines. Many dials have on their dial furniture just three lines or curves. They are the winter solstice (shortest day), summer solstice (longest day), and the two equinoxes, when day and night time are equal. Some dials have seven such lines, of which two are the solstices, leaving five, and those five account for ten months since all but the solstice months share a declination. However any calendar line may be drawn.

ARMILLARY DIALS

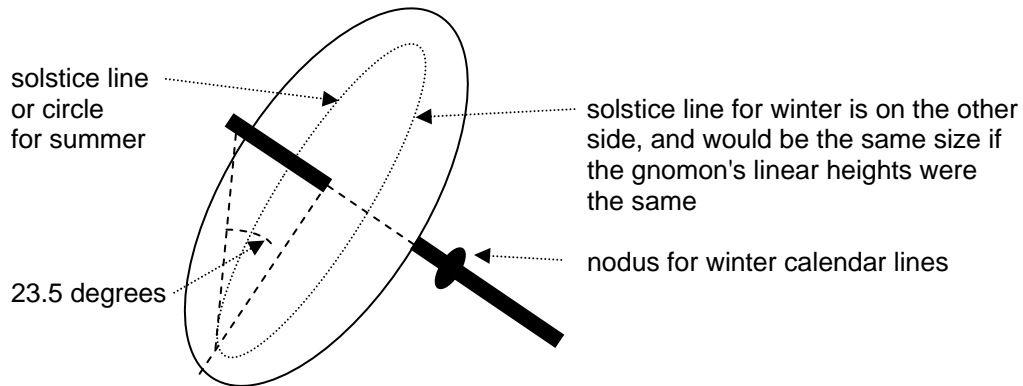
The equinox line of an armillary dial runs as a straight line in the center of the dial plate. Solstice lines parallel the equinox line. The equinox line is perpendicular to the style and dropped from the nodus. In other words a notch or blob or other nodus is needed to show the calendar information. The solstice lines are above and below the equinox line by 23.5 degrees, and simple trigonometry can be used to calculate the linear distance above and below the equinox line. Since the tangent of 23.5 degrees is 0.43, the linear distance above and below the equinox line is 0.43 times the radius of the armillary dial plate.



The above shows the circular dial plate, albeit it depicted as flat.

EQUATORIAL DIALS.

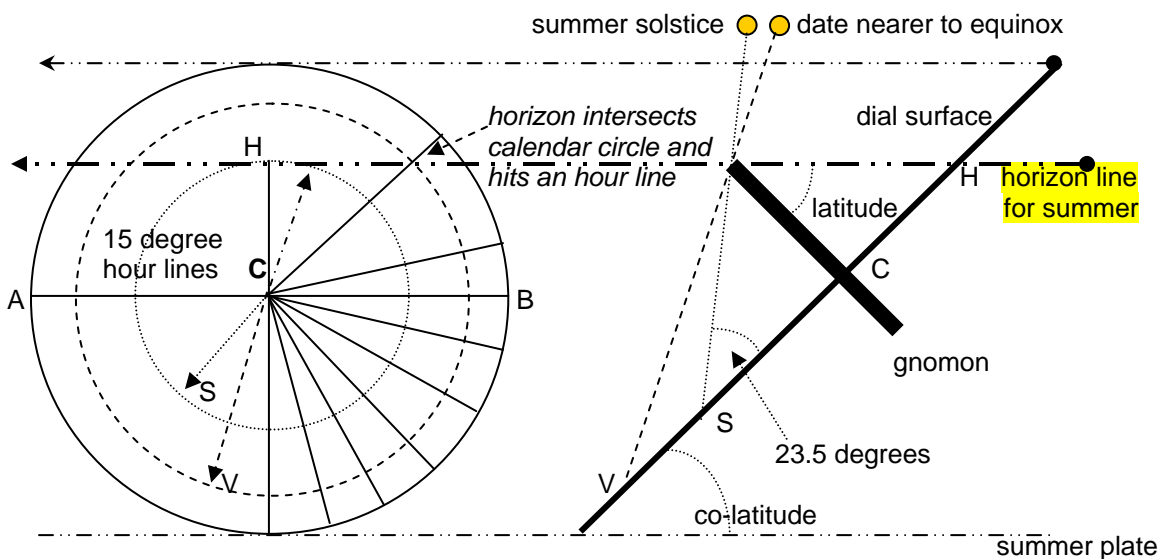
The equatorial dial has its dial plate paralleling the equator, perpendicular to its polar axis aligned style. The calendar lines are circles surrounding the gnomon. Their linear distance from the base of the gnomon is the co-tangent of the declination times the gnomon's linear height.



the radius of the calendar line, circle in this case, is the cotangent of the declination times the nodus linear height, however some spreadsheets may not support the cotan function so it is also:-

$$\text{radius of calendar circle} = \text{nodus linear height} / \tan(\text{declination})$$

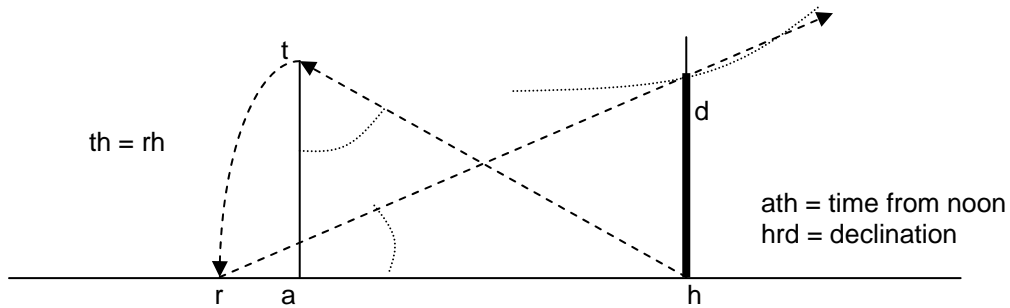
and for the solstices, the cotan of 23.5 is the reciprocal of the tan, namely $1 / 0.43$, or 2.3 times the nodus height. The winter calendar lines are on the equatorial plate's lower surface, the summer lines on the upper. There is no equinox line since at equinox, the declination is zero, placing the equinox circle at infinity. If the gnomon is used to support the dial plate, then there needs to be a clear nodus on the lower part of the gnomon in order to cast a calendar indicating shadow. An equatorial dial can show the times of sunrise and sunset, appendix 6 addresses this. Appendix table A2.12 provides all the calendar data for equatorial dials including sunrise/set data.



A horizon line is drawn from the nodus as shown above, and it crosses the 15 degree hour lines and the declination circles. For any date, the declination circle is found, and sunrise or set is the intersection of that declination circle with the hour line. Winter would use the lower nodus.

POLAR DIALS

The polar dial uses a simple geometric construction, or a simple trigonometric formula.



$$\tan (hrd) = dh / rh = dh / th, \quad \text{thus } dh = th * \tan (\text{declination}) \quad \text{and:-}$$

$$\cos (ath) = ta / th \quad \text{thus } th = ta / \cos (ath)$$

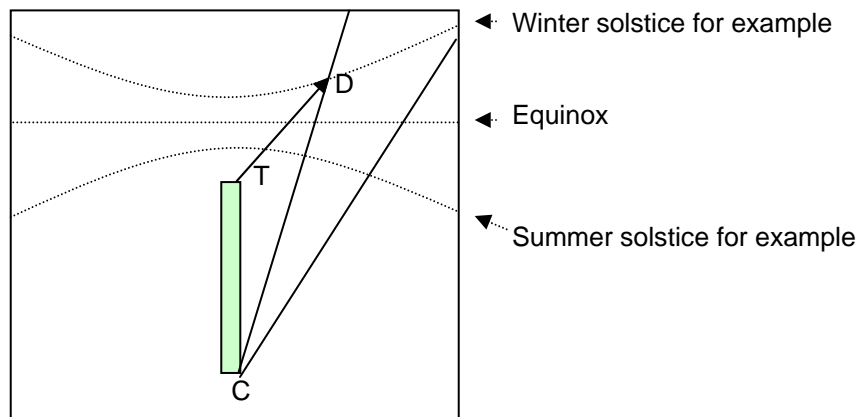
so given: $dh = th * \tan (\text{declination})$
 then:- $dh = ta * \tan (\text{declination}) / \cos (\text{time})$

The distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style linear height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines, then the points joined to form a hyperbolic curve for the solstices, or a straight line for the equinox.

MERIDIAN DIALS

The meridian dial faces true east or true west. The method is exactly the same as for the polar dial, see above, however, in this book the baseline for meridian dials is 6 o'clock (am or pm), whereas the polar dial uses 12 o'clock noon as the basis for hour lines. Some books use the 6 o'clock basis for meridian dials, some use noon, the result is formulae that look quite different due to the different reference hour, however the end results are exactly the same.

HORIZONTAL DIALS



The distance from the foot of the style, point T, and not from the dial center C, to a given hour line's declination point (TD) is determined by:-

$$\text{distance TD} = \text{style height} * \cotan(\text{sun's altitude})$$

$$\text{sun's altitude} = \arcsin(\sin(\text{dec}) * \sin(\text{lat}) + \cos(\text{dec}) * \cos(\text{lat}) * \cos(\text{time}))$$

The declination is found in several tables in the appendices. Table A2.11 has the declination tabulated by the day, the altitude tables in appendix A4.1 may be used for each hour, or their formula can be used directly, see appendix A8. Tables showing altitudes for solstice and equinox declinations for latitudes up to 59 for integer hours may be used, and are discussed later in this section. Alternatively, the azimuth may be used from T, however this does not work for noon.

VERTICAL SOUTH DIALS

The vertical dial is drawn exactly the same as a horizontal dial, but the dial is designed for the co-latitude. Thus a vertical dial for latitude 40 degrees would be the same as a horizontal dial for latitude 50 degrees. Note that a vertical dial's summer solstice is the horizontal dial's winter solstice, and the vertical dial's winter solstice is the horizontal dial's summer solstice.

VERTICAL DECLINERS

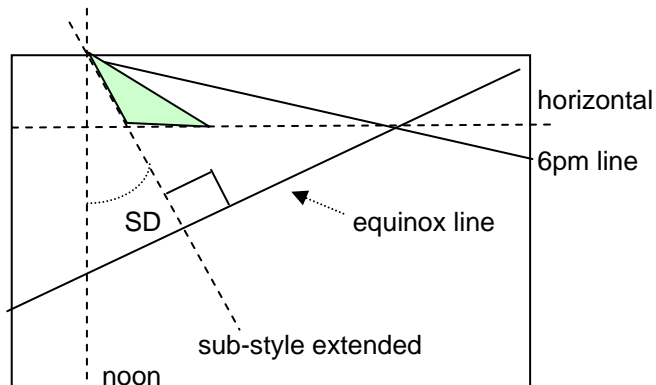
Vertical decliners have an equivalent dial that is not declining and is horizontal somewhere else on this planet. Find that location and the virtual horizontal dial with its declinations lines may be drafted, and then used by matching that horizontal dial's noon line with the vertical decliner's extended sub style line.

The terms style height (SH, an angular and not a linear distance) and difference-in-longitude (DL) have been used, so now their extra secrets need to be released.

- SH tells us the latitude where our dial would be horizontal, and
- DL gives us the longitude (by difference from our own longitude) for that location

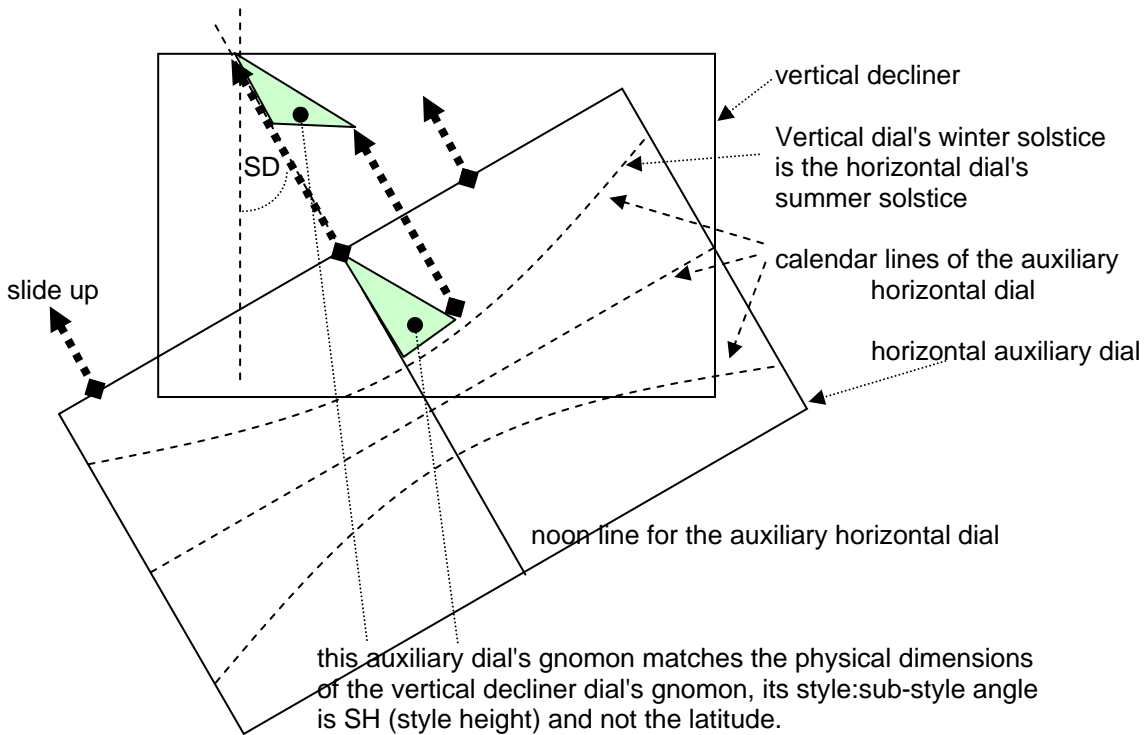
While this is an insight, some other insights exist that facilitate vertical decliner calendar lines. A key insight is that at the equinox, day time equals night time, thus 6am and 6pm solar local apparent time is when the sun rises and sets. Thus the intersection of a horizontal line from the base of the sub-style towards a reachable 6 o'clock hour line defines one point of the equinox line. From that point, moving perpendicular to the sub-style line (of a rotated gnomon), the equinox line is thus drawn.

This only works for gnomons rotated using the SD technique because the sub-style line when extended provides the angle of the equinox line because that equinox line is perpendicular to the style distance, SD.



Thus the equinox line is simply drafted as long as a 6 o'clock line is in place.

Another technique can be used for a vertical decliner's calendar lines. Again, the extended sub-style line is used, along with a horizontal auxiliary dial. This technique assumes the gnomon has been rotated using the Style Distance techniques discussed elsewhere.



A horizontal dial is built whose latitude is the vertical dial's style height and not the actual latitude, and the associated horizontal dial's gnomon dimensions equal those of the vertical dial's gnomon. The entire set of lines from the auxiliary dial is slid up such that its equinox line overlays the equinox line for the vertical decliner drawn using the methods of the previous page, or the horizontal dial is slid up so its gnomon blends with the rotated vertical dial's gnomon, same end result. The vertical dial's winter solstice is the horizontal dial's summer solstice, and the vertical dial's summer solstice is the horizontal dial's winter solstice.

Thus a vertical decliner at latitude 32, where the wall is South 10 degrees West would provide the following data:-

15.5	SD
56.6	SH

SD is 15.5 degrees, and the noon line of the auxiliary sundial will merge with the extended sub-style line.

SH is 56.6 degrees, thus the horizontal dial is designed for latitude 56.6 degrees. The physical dimensions of the vertical decliner's rotated gnomon are exactly used as-is on the auxiliary horizontal dial.

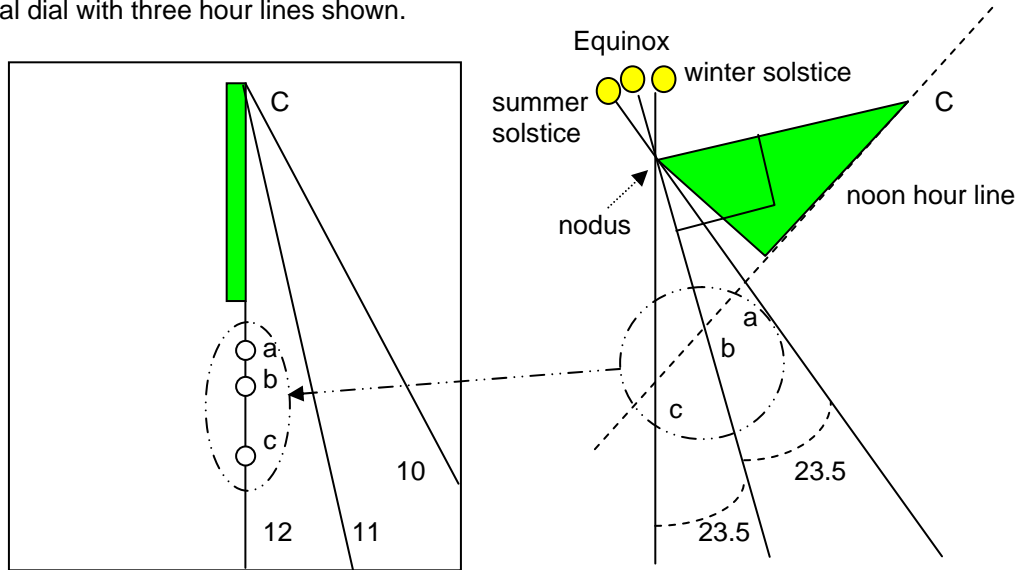
Consider double checking your design using software such as SHADOWS.

In summary, while some calendar or declination line processes may require some work, this chapter encapsulated the processes in order of complexity. The tables in the appendices facilitate calculation of the angles, and out of these lines comes the basis for exotic hours lines such as the Italian and Babylonian which need an equinox and solstice line for their construction.

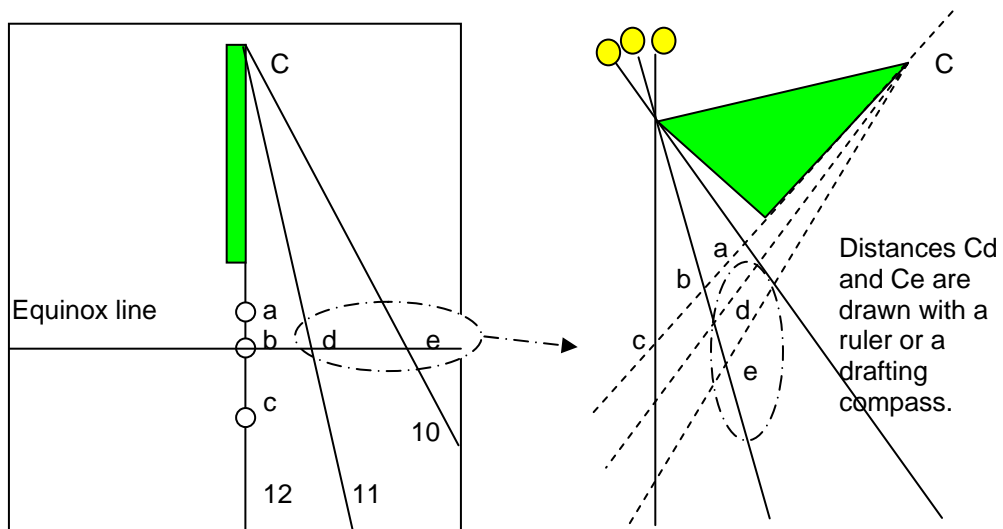
Declination lines for the horizontal (or vertical) dial using geometry.

A template is available in appendix 9 to facilitate this process.

A horizontal dial with three hour lines shown.

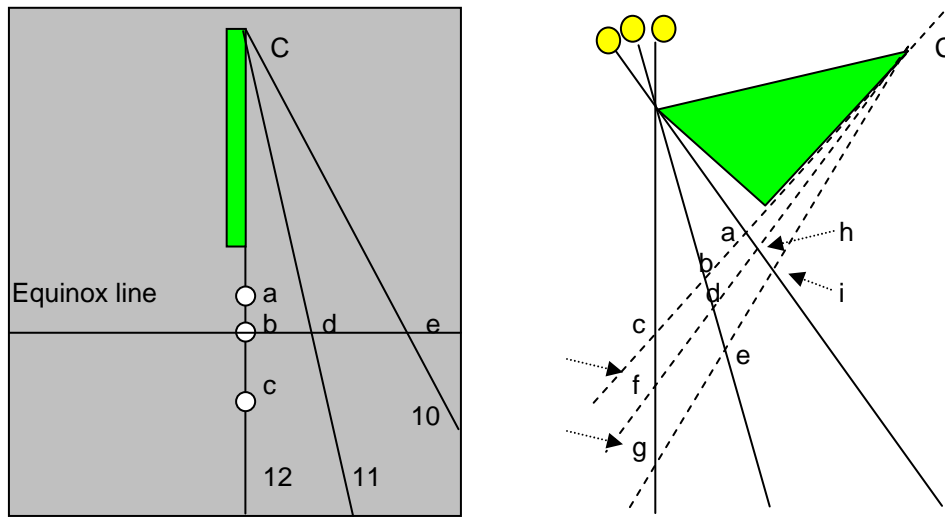


First, draw a gnomon for the dial center "C". From the nodus draw the equinox line (90 degrees to the style), and from that the solstice lines (approx 23.5 degrees on either side). The three lines (equinox and the solstices) intersect the gnomon's base line extended, or the noon line, at points a, b, and c. These three points whose distances from the dial center are Aa, Ab, and Ac are then transcribed to the dial plate (right pictorial to left pictorial).

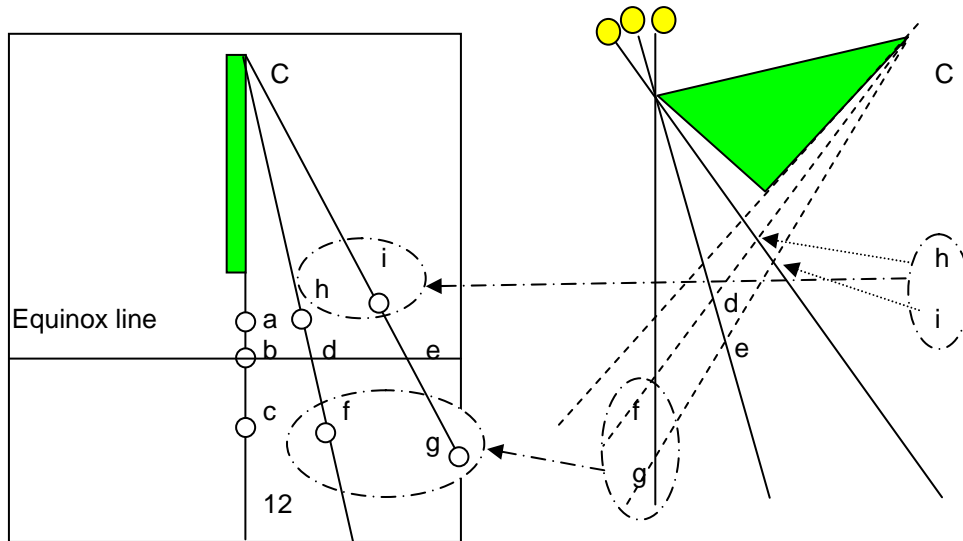


The equinox line is then drawn perpendicular to the noon line, and it produces equinox intercepts for those additional hour lines, d and e. Distances Cd and Ce are then located from the left dial plate to the right hand picture on its equinox line. This produces two more hour lines on the right hand side picture, Cd and Ce. Those hour lines on the right hand side do not have angles that match their hour lines on the dial plate, and this is because this is a projection.

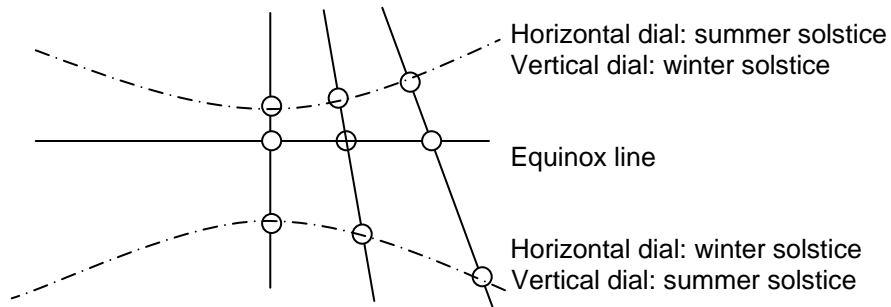
Now that there are two more hour lines, or as many as you choose, this produces intercept points for the solstice lines, namely points f, g, h and i.



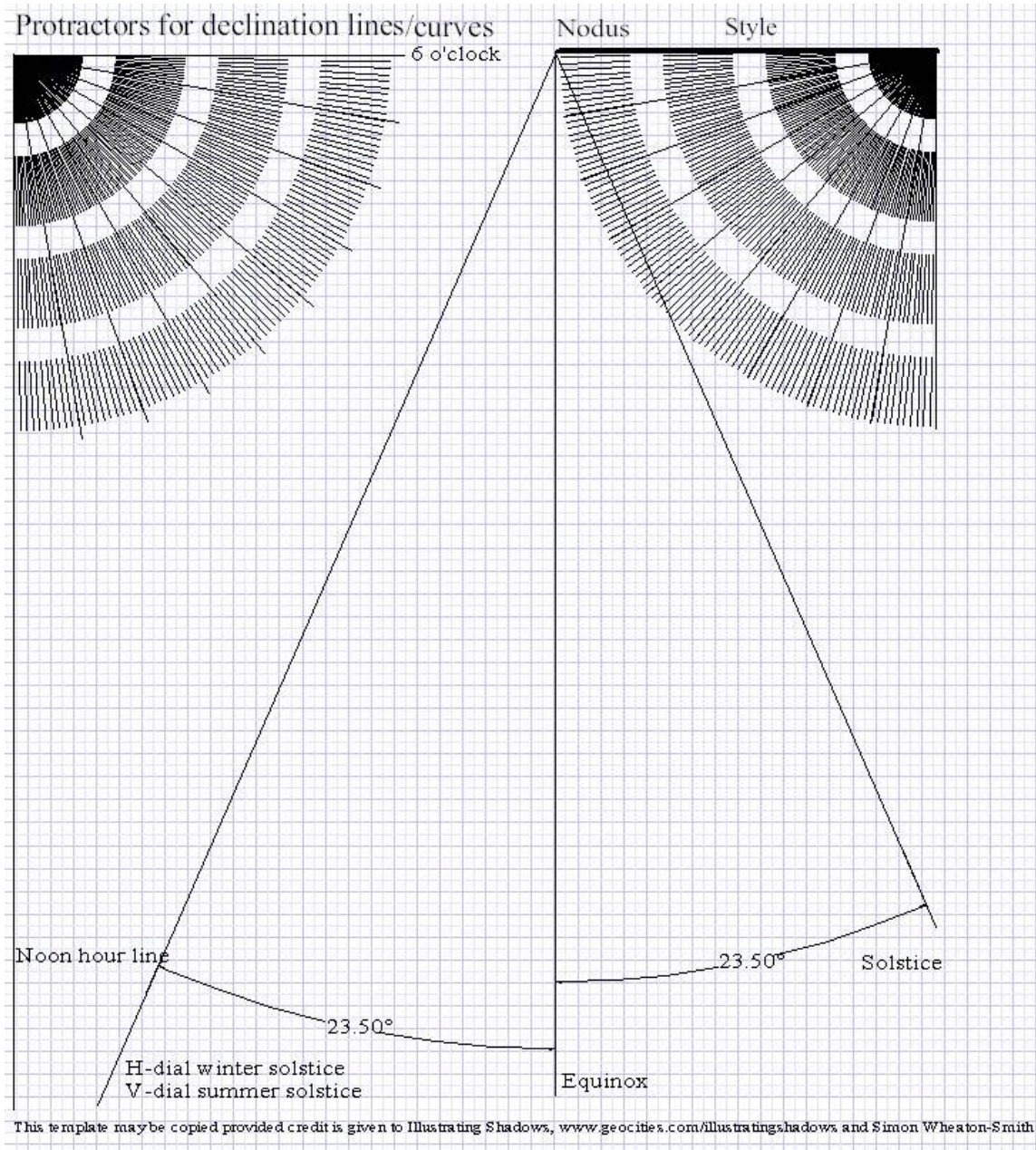
Points f, h, g, and i are now transferred back to the dial plate, from the right projection pictorial to the left picture.



When this process is completed for as many hour lines as desired, the dots are connected and then the declination lines drawn.



A DRAFTING SHEET FOR HORIZONTAL/VERTICAL DIAL DECLINATION LINES

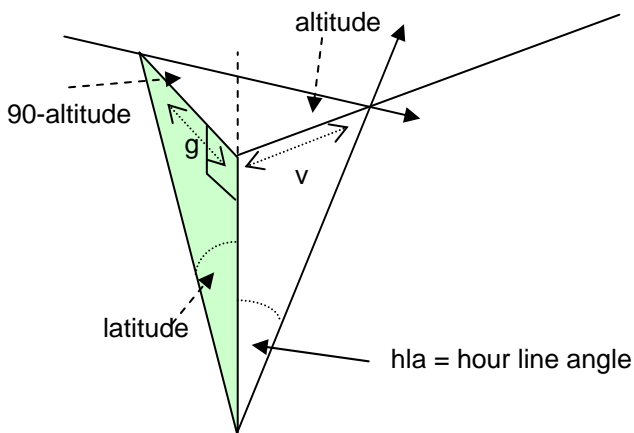


CALENDAR POINTS ON HOUR LINES

- In what follows, it is assumed that the hour lines are not longitude corrected.
- For declining dials using this method, noon in what follows is the extension of the sub-style at the style distance, and the hours are based on noon-at-the-extended-style-distance. And noon for the purposes of the calendar lines is NOT the same as the real noon on the dial plate.
- No correction for the equation of time is used.

Using the sun's altitude (using a fixed declination)

How far from the nodus base (nodus dropped perpendicular to dial plate) on a horizontal or flat dial is the declination point?



$\tan(\text{alt}) = \text{gnomon linear height} / \text{distance from gnomon base} = g / v$

thus $v = g / \tan(\text{alt})$ or $v = g * \cot(\text{alt})$

or in spreadsheet (Excel) terms which has the formula for altitude and

where the declination is a fixed number such as 23.5, 0, or -23.5, or some other value

=gnomonlinearheight /
 $(\text{TAN}(\text{ASIN}(\text{SIN}(\text{RADIANS}(\text{decl})) * \text{SIN}(\text{RADIANS}(\text{lat})) + \text{COS}(\text{RADIANS}(\text{decl})) * \text{COS}(\text{RADIANS}(\text{lat})) * \text{COS}(\text{RADIANS}(15 * (12 - \text{hh}))))))$)

This method works for all angles, and assumes a known perpendicular distance from the nodus to the dial plate. Thus this works for all style heights (latitudes including 0).

NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES
 The nodus base is a line from the nodus dropped perpendicular to the dial plate

Distance from gnomon base to the hour line for the relevant declination point

Gnomon linear height is: → 1

Latitude	Decl	1200	1100	1000	900	800	700
0.0	23.50	0.435	0.524	0.765	1.174	1.938	4.093
0.0	0.00	0.000	0.268	0.577	1.000	1.732	3.732
0.0	-23.50	0.435	0.524	0.765	1.174	1.938	4.093
5.0	23.50	0.335	0.434	0.683	1.076	1.772	3.549
5.0	0.00	0.087	0.283	0.586	1.008	1.741	3.747
5.0	-23.50	0.543	0.626	0.865	1.295	2.148	4.856
10.0	23.50	0.240	0.358	0.616	0.998	1.639	3.145
10.0	0.00	0.176	0.324	0.612	1.031	1.768	3.794
10.0	-23.50	0.662	0.742	0.984	1.444	2.417	5.996
15.0	23.50	0.149	0.296	0.566	0.937	1.534	2.837
15.0	0.00	0.268	0.386	0.655	1.069	1.813	3.873
15.0	-23.50	0.795	0.875	1.126	1.629	2.769	7.869
20.0	23.50	0.061	0.256	0.532	0.893	1.452	2.596
20.0	0.00	0.364	0.462	0.714	1.125	1.879	3.988
20.0	-23.50	0.949	1.032	1.299	1.863	3.245	11.496
25.0	23.50	0.026	0.245	0.517	0.865	1.390	2.407
25.0	0.00	0.466	0.552	0.789	1.198	1.967	4.144
25.0	-23.50	1.130	1.219	1.513	2.166	3.922	21.438
30.0	23.50	0.114	0.266	0.520	0.853	1.346	2.258
30.0	0.00	0.577	0.655	0.882	1.291	2.082	4.348
30.0	-23.50	1.351	1.450	1.787	2.573	4.958	161.839
31.0	23.50	0.132	0.273	0.523	0.852	1.339	2.232
31.0	0.00	0.601	0.677	0.903	1.312	2.108	4.395
31.0	-23.50	1.402	1.503	1.851	2.672	5.234	
32.0	23.50	0.149	0.282	0.527	0.852	1.333	2.208
32.0	0.00	0.625	0.700	0.924	1.335	2.136	4.445
32.0	-23.50	1.455	1.559	1.919	2.779	5.543	
33.0	23.50	0.167	0.291	0.531	0.852	1.327	2.184
33.0	0.00	0.649	0.724	0.946	1.358	2.165	4.497
33.0	-23.50	1.511	1.618	1.991	2.893	5.890	
34.0	23.50	0.185	0.302	0.536	0.854	1.323	2.162
34.0	0.00	0.675	0.748	0.970	1.382	2.195	4.552
34.0	-23.50	1.570	1.680	2.067	3.017	6.284	
35.0	23.50	0.203	0.313	0.542	0.855	1.318	2.141
35.0	0.00	0.700	0.773	0.994	1.407	2.227	4.609
35.0	-23.50	1.632	1.746	2.149	3.151	6.734	
Latitude	Decl	1200	100	200	300	400	500

NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from gnomon base to the hour line for the relevant declination point

Gnomon linear height is: \longrightarrow 1

Latitude	Decl	1200	1100	1000	900	800	700
35.0	23.50	0.203	0.313	0.542	0.855	1.318	2.141
35.0	0.00	0.700	0.773	0.994	1.407	2.227	4.609
35.0	-23.50	1.632	1.746	2.149	3.151	6.734	
36.0	23.50	0.222	0.325	0.548	0.858	1.315	2.121
36.0	0.00	0.727	0.798	1.018	1.434	2.261	4.670
36.0	-23.50	1.698	1.817	2.237	3.297	7.253	
37.0	23.50	0.240	0.338	0.555	0.861	1.312	2.102
37.0	0.00	0.754	0.825	1.044	1.461	2.296	4.733
37.0	-23.50	1.767	1.891	2.331	3.457	7.859	
38.0	23.50	0.259	0.351	0.563	0.865	1.310	2.085
38.0	0.00	0.781	0.852	1.071	1.490	2.333	4.800
38.0	-23.50	1.842	1.971	2.432	3.631	8.575	
39.0	23.50	0.277	0.365	0.572	0.869	1.308	2.068
39.0	0.00	0.810	0.880	1.099	1.520	2.371	4.870
39.0	-23.50	1.921	2.056	2.541	3.824	9.434	
40.0	23.50	0.296	0.380	0.581	0.874	1.307	2.052
40.0	0.00	0.839	0.909	1.128	1.552	2.412	4.944
40.0	-23.50	2.006	2.147	2.658	4.037	10.485	
41.0	23.50	0.315	0.395	0.591	0.879	1.307	2.037
41.0	0.00	0.869	0.939	1.158	1.585	2.454	5.021
41.0	-23.50	2.097	2.245	2.786	4.274	11.798	
42.0	23.50	0.335	0.410	0.601	0.885	1.307	2.023
42.0	0.00	0.900	0.970	1.189	1.619	2.499	5.102
42.0	-23.50	2.194	2.351	2.926	4.541	13.488	
43.0	23.50	0.354	0.427	0.612	0.892	1.308	2.009
43.0	0.00	0.933	1.002	1.222	1.655	2.545	5.187
43.0	-23.50	2.300	2.465	3.079	4.841	15.741	
44.0	23.50	0.374	0.443	0.624	0.900	1.310	1.997
44.0	0.00	0.966	1.035	1.256	1.693	2.594	5.277
44.0	-23.50	2.414	2.590	3.247	5.182	18.897	
45.0	23.50	0.394	0.460	0.637	0.908	1.312	1.986
45.0	0.00	1.000	1.069	1.291	1.732	2.646	5.372
45.0	-23.50	2.539	2.726	3.434	5.574	23.635	
Latitude	Decl	1200	100	200	300	400	500

Note: the spreadsheet: [reference-spreadsheets.xls](#) on the web site has the nodus base distance to an hour line, as well as the dial center to hour line distance, in interactive form.

The azimuth method is not included in these tables because it fails for an azimuth of 0, and if using azimuth to derive a distance from dial center along an hour line, that fails when the dial center is unavailable as in great declining dials. The most common method for calendar lines is the nodus base distance to the associated hour line.

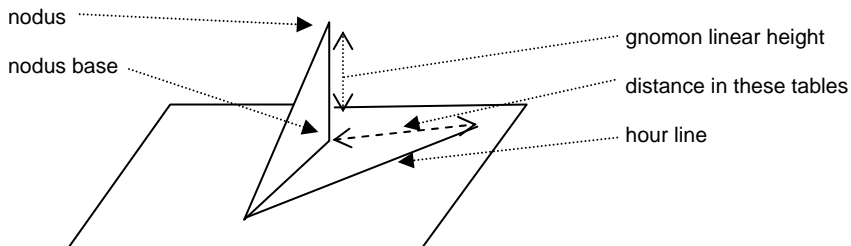
NODUS BASE TO HOUR LINE DISTANCE FOR A FLAT SUN DIAL'S DECLINATION LINES

Distance from gnomon base to the hour line for the relevant declination point

Gnomon linear height is: → 1

Latitude	Decl	1200	1100	1000	900	800	700
45.0	23.50	0.394	0.460	0.637	0.908	1.312	1.986
45.0	0.00	1.000	1.069	1.291	1.732	2.646	5.372
45.0	-23.50	2.539	2.726	3.434	5.574	23.635	
46.0	23.50	0.414	0.478	0.650	0.916	1.315	1.975
46.0	0.00	1.036	1.105	1.328	1.773	2.700	5.471
46.0	-23.50	2.675	2.875	3.641	6.029	31.544	
47.0	23.50	0.435	0.496	0.663	0.926	1.318	1.965
47.0	0.00	1.072	1.142	1.366	1.817	2.757	5.576
47.0	-23.50	2.824	3.040	3.873	6.563	47.405	
48.0	23.50	0.456	0.515	0.678	0.936	1.322	1.956
48.0	0.00	1.111	1.181	1.406	1.862	2.817	5.687
48.0	-23.50	2.989	3.222	4.134	7.200	95.340	
49.0	23.50	0.477	0.534	0.693	0.946	1.327	1.948
49.0	0.00	1.150	1.221	1.448	1.910	2.880	5.804
49.0	-23.50	3.172	3.426	4.432	7.970		
50.0	23.50	0.499	0.554	0.709	0.958	1.333	1.941
50.0	0.00	1.192	1.263	1.492	1.960	2.946	5.927
50.0	-23.50	3.376	3.655	4.773	8.924		
51.0	23.50	0.521	0.574	0.725	0.969	1.339	1.934
51.0	0.00	1.235	1.306	1.538	2.012	3.017	6.057
51.0	-23.50	3.606	3.913	5.170	10.134		
52.0	23.50	0.543	0.594	0.742	0.982	1.345	1.928
52.0	0.00	1.280	1.352	1.587	2.068	3.091	6.196
52.0	-23.50	3.867	4.209	5.635	11.720		
53.0	23.50	0.566	0.616	0.759	0.995	1.353	1.923
53.0	0.00	1.327	1.400	1.637	2.127	3.169	6.342
53.0	-23.50	4.165	4.550	6.189	13.892		
54.0	23.50	0.589	0.637	0.778	1.010	1.361	1.919
54.0	0.00	1.376	1.450	1.691	2.188	3.252	6.497
54.0	-23.50	4.511	4.949	6.861	17.047		
55.0	23.50	0.613	0.660	0.797	1.024	1.370	1.916
55.0	0.00	1.428	1.503	1.747	2.254	3.340	6.662
55.0	-23.50	4.915	5.420	7.693	22.050		

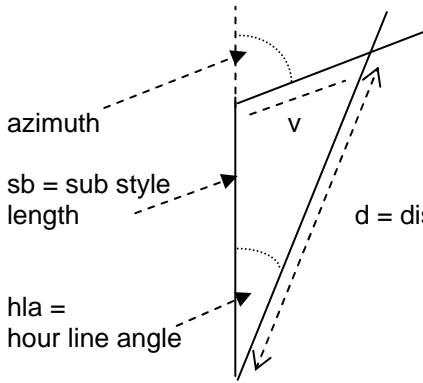
Latitude Decl 1200 100 200 300 400 500



Using the sun's azimuth:

How far along an hour line from the dial center on a horizontal dial is the declination point?

Or, what angle would the line "v" go to define a declination point.



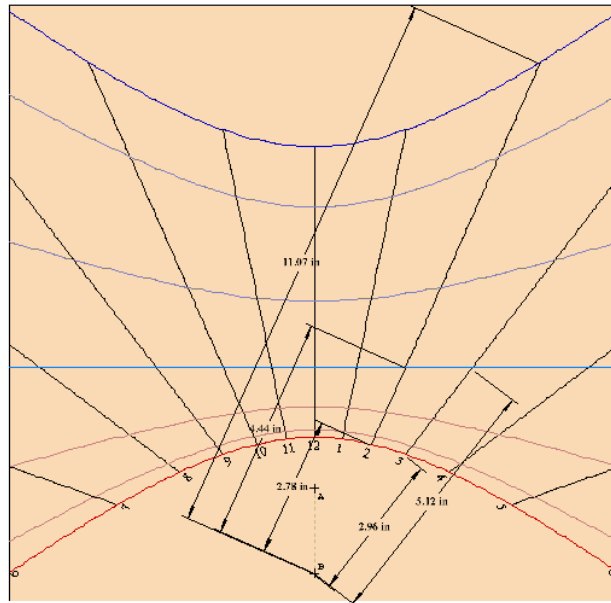
NOTE: "v" is calculated above, and in the spreadsheet for the altitude method.

d = distance along the hour line to the declination point

using the law of sines: $d / (\sin(180-\text{azimuth}) = v / (\sin(\text{hla}))$
 $d = v * \sin(180-\text{azimuth}) / (\sin(\text{hla}))$

This method fails when the azimuth is 0, and only works if the dial center is accessible. This is often not true for great decliners. For this reason, the azimuth method is not common.

Note that the tables have the noon time blocked out, this is because that at noon (LAT) the azimuth is zero.



lat 50 in SHADOWS

50.0	23.5		2.70	2.78	2.95	3.31	4.10
50.0	00		4.15	4.44	5.12	6.75	12.30
50.0	-23.5		8.95	11.05	19.15		

10 and 2 9 and 3

ADDITIONAL NOTES:

Discussed with Illustrating Time's Shadow Chapter 22 are several other techniques for deriving calendar line or curve data.

Additionally, the DeltaCAD programs (macros) provide graphical depictions of the calendar curves and even show them optionally in animation with changes to latitude.

Additionally, the JustBASIC programs offer tabular displays of calendar line data using an SH (style height) and with no longitude correction. This is because the calendar points provided are based on the SH derived local apparent time displacements and not on a resulting dial's actual location. These calendar data are transposed to the final real dial. In essence, the use of LAT hours on this surrogate dial is arbitrary. This BASIC program allows the any of the three sides of the gnomon to be specified along with the SH.

Additionally, the main spreadsheet provided with Illustrating Shadows includes these formulae.

USEFUL DECLINATIONS:

COMMON	ALTERNATIVE				
0	0		March		September
10	12	February	April	August	October
18	20	January	May	July	November
23.5	23.5		December		June