Declining Shadows

All sorts of ideas on the drafting of calendar lines on a dial, including the flat dial. The flat dial being important because this is the method used for horizontal, vertical, declining, and inclined declining dials wherein only the style height (angular) is the basis for the hours and calendar lines. Extracted from Illustrating Time's Shadow.

The tables, of which there are three variations can be produced with: reference-spreadsheets.xls which is on the web site: www.illustratingshadows.com

Also, the provided DeltaCAD macros and the JustBASIC programs provide easy ways of automating calendar data, and the DeltaCAD programs (macros) also provide highly educational animation.

Simon Wheaton-Smith
February 2013

There are indications that the seasons were of more interest to early sun dial users, and that the calendar might have been more important than the time of day.

The sun appears to oscillate annually from minus 23.5 degrees when it is over the southern tropics, to plus 23.5 degrees when it is over the northern hemisphere tropics.

This angle made with the equator is called the declination. Many of the tables show the declination for various days in the year. In particular table A2.11 shows the sun's declination for each day of the average year. This book shows two formulae, and one is:-

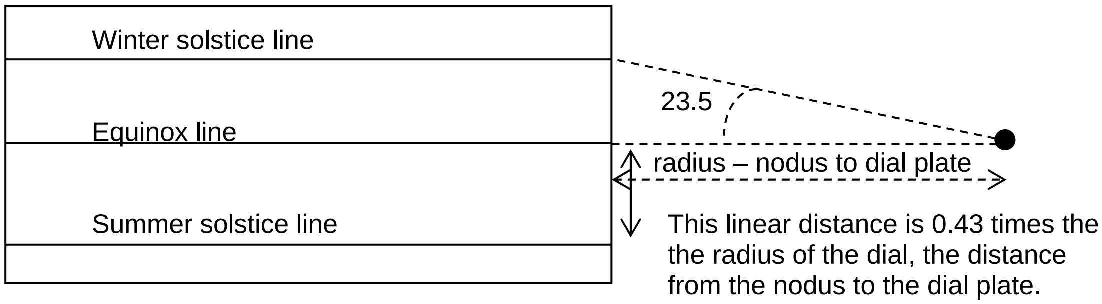
DEGREES(0.006918 - 0.399912*COS(((2*3.1416*(jd-1)) / 365)) + 0.070257*SIN(((2*3.1416*(jd-1)) / 365)) - 0.006758*COS(2*((2*3.1416*(jd-1)) / 365)) + 0.000907*SIN(2*((2*3.1416*(jd-1)) / 365)) - 0.002697*COS(3*((2*3.1416*(jd-1)) / 365)) + 0.00148*SIN(3*((2*3.1416*(jd-1)) / 365))) where "jd" is the day in the year 1 to 365

The declination plus the co-latitude provides the sun's altitude at noon.

There are a number of methods for drafting calendar or declination lines. Many dials have on their dial furniture just three lines or curves. They are the winter solstice (shortest day), summer solstice (longest day), and the two equinoxes, when day and night time are equal. Some dials have seven such lines, of which two are the solstices, leaving five, and those five account for ten months since all but the solstice months share a declination. However any calendar line may be drawn.

ARMILLARY DIALS

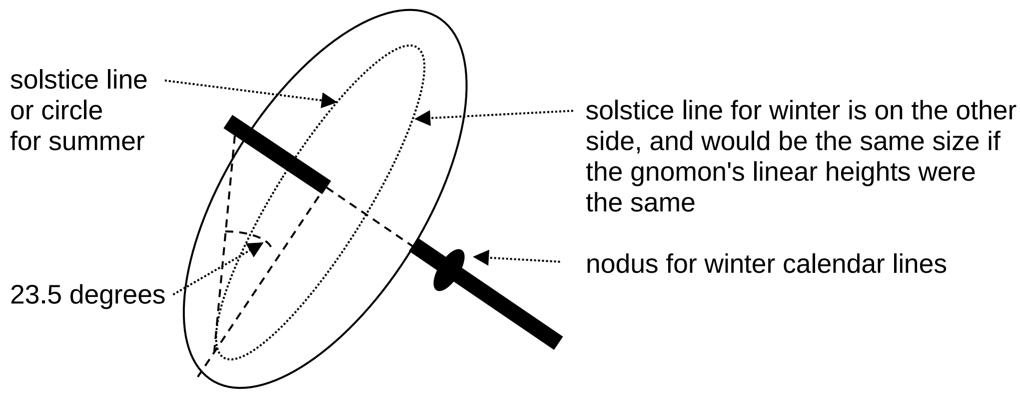
The equinox line of an armillary dial runs as a straight line in the center of the dial plate. Solstice lines parallel the equinox line. The equinox line is perpendicular to the style and dropped from the nodus. In other words a notch or blob or other nodus is needed to show the calendar information. The solstice lines are above and below the equinox line by 23.5 degrees, and simple trigonometry can be used to calculate the linear distance above and below the equinox line. Since the tangent of 23.5 degrees is 0.43, the linear distance above and below the equinox line is 0.43 times the radius of the armillary dial plate.



The above shows the circular dial plate, albeit it depicted as flat.

EQUATORIAL DIALS.

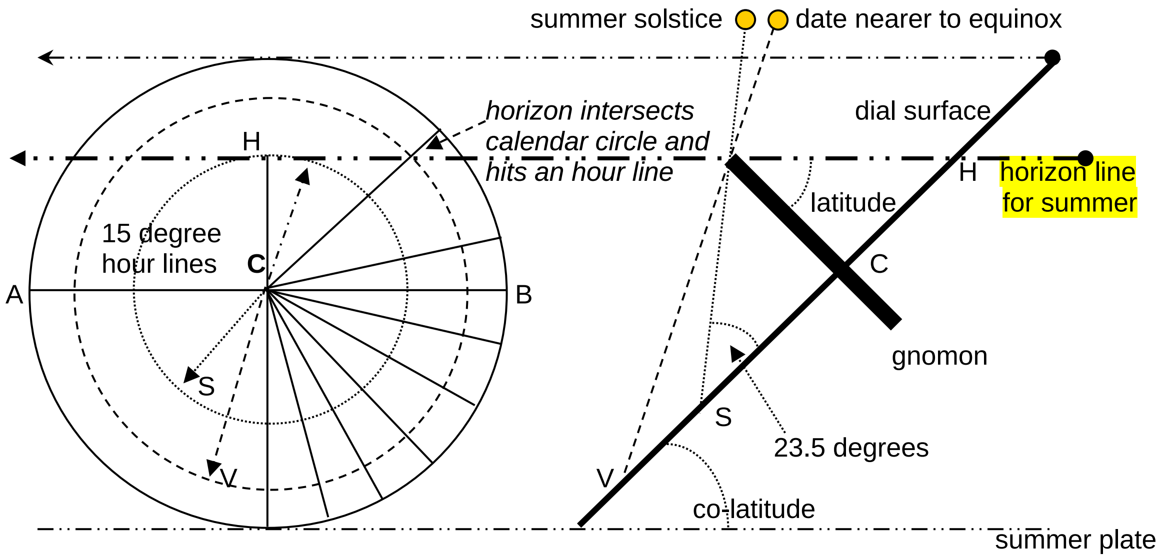
The equatorial dial has its dial plate paralleling the equator, perpendicular to its polar axis aligned style. The calendar lines are circles surrounding the gnomon. Their linear distance from the base of the gnomon is the co-tangent of the declination times the gnomon's linear height.



the radius of the calendar line, circle in this case, is the cotangent of the declination times the nodus linear height, however some spreadsheets may not support the cotan function so it is also:-

radius of calendar circle = nodus linear height / tan(declination)

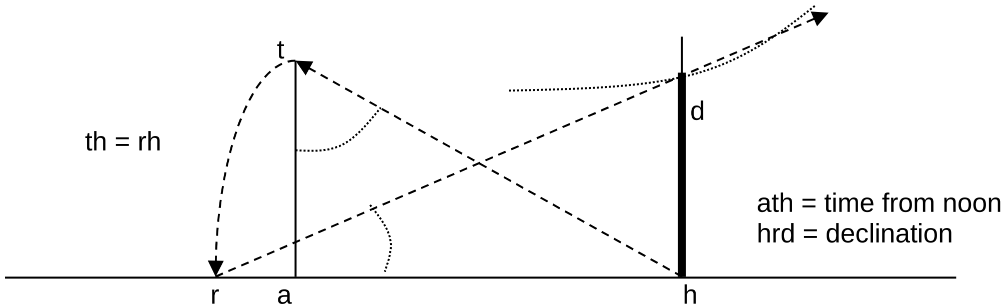
and for the solstices, the cotan of 23.5 is the reciprocal of the tan, namely 1 / 0.43, or 2.3 times the nodus height. The winter calendar lines are on the equatorial plate's lower surface, the summer lines on the upper. There is no equinox line since at equinox, the declination is zero, placing the equinox circle at infinity. If the gnomon is used to support the dial plate, then there needs to be a clear nodus on the lower part of the gnomon in order to cast a calendar indicating shadow. An equatorial dial can show the times of sunrise and sunset, appendix 6 addresses this. Appendix table A2.12 provides all the calendar data for equatorial dials including sunrise/set data.



A horizon line is drawn from the nodus as shown above, and it crosses the 15 degree hour lines and the declination circles. For any date, the declination circle is found, and sunrise or set is the intersection of that declination circle with the hour line. Winter would use the lower nodus.

POLAR DIALS

The polar dial uses a simple geometric construction, or a simple trigonometric formula.



tan (hrd) = dh / rh = dh / th,   thus dh = th * tan (declination)   and:-
cos (ath) = ta / th   thus th = ta / cos (ath)

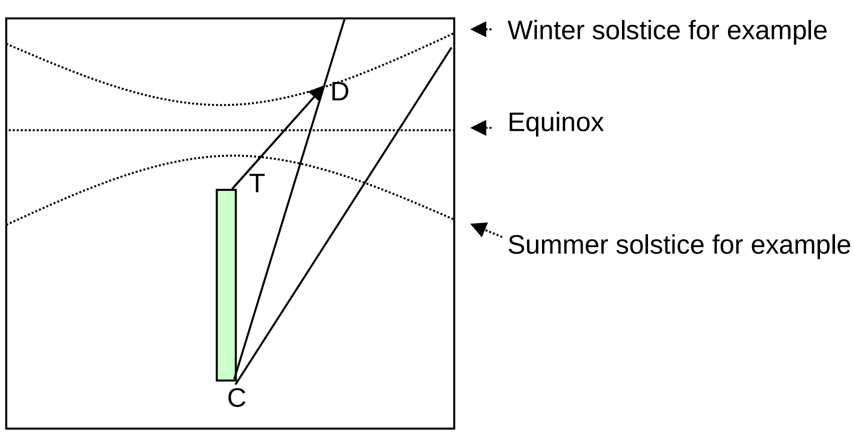
so given: dh = th * tan (declination)
then:-   dh = ta * tan (declination) / cos (time)

The distance up the hour line for the point on which the declination (calendar) line will lie is equal to the style linear height times the tangent of the declination all divided by the cosine of the time. This is repeated for several of the hour lines, then the points joined to form a hyperbolic curve for the solstices, or a straight line for the equinox.

MERIDIAN DIALS

The meridian dial faces true east or true west. The method is exactly the same as for the polar dial, see above, however, in this book the baseline for meridian dials is 6 o'clock (am or pm), whereas the polar dial uses 12 o'clock noon as the basis for hour lines. Some books use the 6 o'clock basis for meridian dials, some use noon, the result is formulae that look quite different due to the different reference hour, however the end results are exactly the same.

HORIZONTAL DIALS



The distance from the foot of the style, point T, and not from the dial center C, to a given hour line's declination point (TD) is determined by:-

distance TD = style height * cotan (sun's altitude)

sun's altitude = arcsin (sin (dec) * sin (lat) + cos (dec) * cos(lat) * cos(time))

The declination is found in several tables in the appendices. Table A2.11 has the declination tabulated by the day, the altitude tables in appendix A4.1 may be used for each hour, or their formula can be used directly, see appendix A8. Tables showing altitudes for solstice and equinox declinations for latitudes up to 59 for integer hours may be used, and are discussed later in this section. Alternatively, the azimuth may be used from T, however this does not work for noon.

VERTICAL SOUTH DIALS

The vertical dial is drawn exactly the same as a horizontal dial, but the dial is designed for the co-latitude. Thus a vertical dial for latitude 40 degrees would be the same as a horizontal dial for latitude 50 degrees. Note that a vertical dial's summer solstice is the horizontal dial's winter solstice, and the vertical dial's winter solstice is the horizontal dial's summer solstice.

VERTICAL DECLINERS ~ an overview

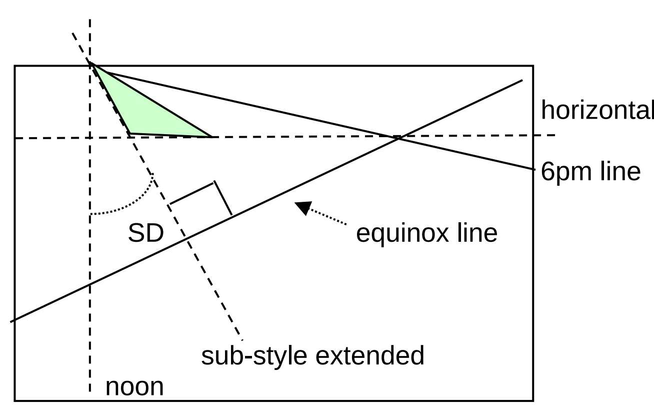
Vertical decliners have an equivalent dial that is not declining and is horizontal somewhere else on this planet. Find that location and the virtual horizontal dial with its declinations lines may be drafted, and then used by matching that horizontal dial's noon line with the vertical decliner's extended sub style line.

The terms style height (SH, an angular and not a linear distance) and difference-in-longitude (DL) have been used, so now their extra secrets need to be released.

SH tells us the latitude where our dial would be horizontal, and
DL gives us the longitude (by difference from our own longitude) for that location

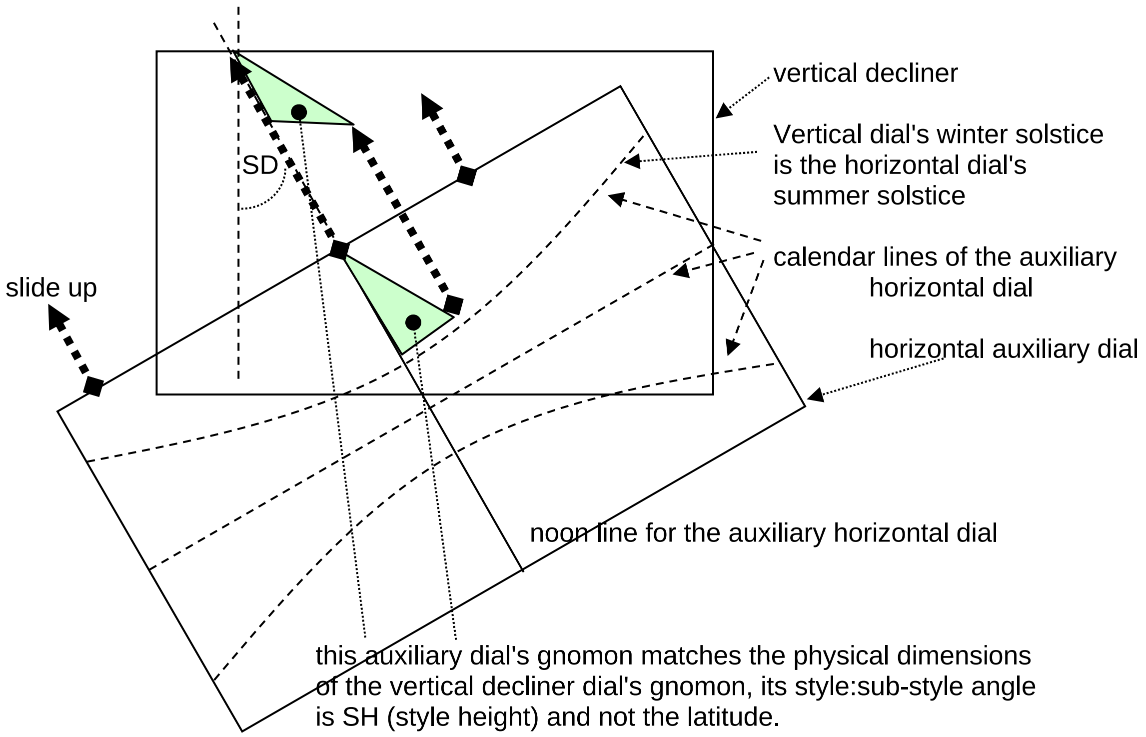
While this is an insight, some other insights exist that facilitate vertical decliner calendar lines. A key insight is that at the equinox, day time equals night time, thus 6am and 6pm solar local apparent time is when the sun rises and sets. Thus the intersection of a horizontal line from the base of the sub-style towards a reachable 6 o'clock hour line defines one point of the equinox line. From that point, moving perpendicular to the sub-style line (of a rotated gnomon), the equinox line is thus drawn.

This only works for gnomons rotated using the SD technique because the sub-style line when extended provides the angle of the equinox line because that equinox line is perpendicular to the style distance, SD.



Thus the equinox line is simply drafted as long as a 6 o'clock line is in place.

Another technique can be used for a vertical decliner's calendar lines. Again, the extended sub-style line is used, along with a horizontal auxiliary dial. This technique assumes the gnomon has been rotated using the Style Distance techniques discussed elsewhere.



A horizontal dial is built whose latitude is the vertical dial's style height and not the actual latitude, and the associated horizontal dial's gnomon dimensions equal those of the vertical dial's gnomon. The entire set of lines from the auxiliary dial is slid up such that its equinox line overlays the equinox line for the vertical decliner drawn using the methods of the previous page, or the horizontal dial is slid up so its gnomon blends with the rotated vertical dial's gnomon, same end result. The vertical dial's winter solstice is the horizontal dial's summer solstice, and the vertical dial's summer solstice is the horizontal dial's winter solstice.

Thus a vertical decliner at latitude 32, where the wall is South 10 degrees West would provide the following data:-

15.5	**SD**
56.6	**SH**

SD is 15.5 degrees, and the noon line of the auxiliary sundial will merge with the extended sub-style line.

SH is 56.6 degrees, thus the horizontal dial is designed for latitude 56.6 degrees. The physical dimensions of the vertical decliner's rotated gnomon are exactly used as-is on the auxiliary horizontal dial.

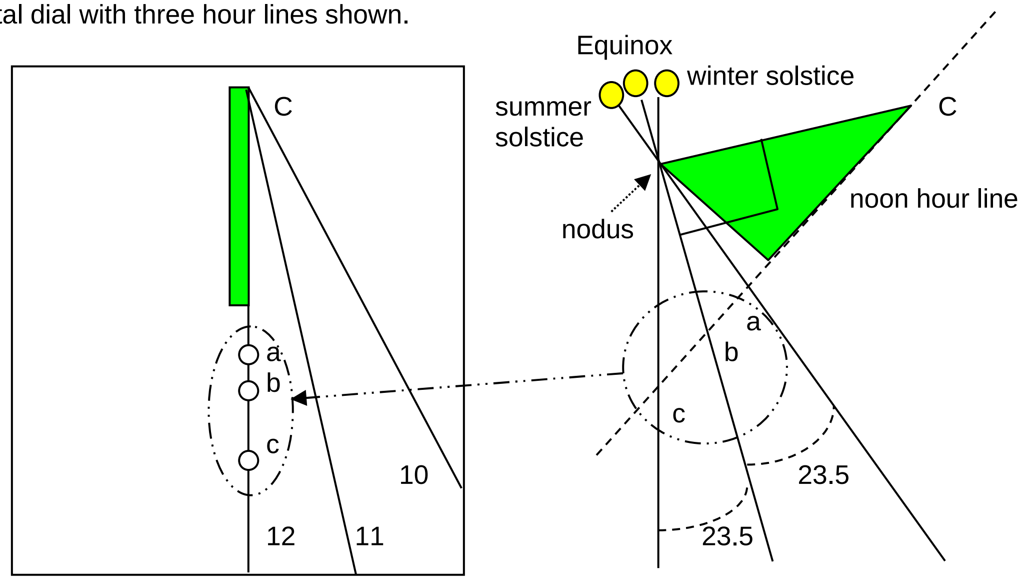
Consider double checking your design using software such as SHADOWS.

In summary, while some calendar or declination line processes may require some work, this chapter encapsulated the processes in order of complexity. The tables in the appendices facilitate calculation of the angles, and out of these lines comes the basis for exotic hour lines such as the Italian and Babylonian which need an equinox and solstice line for their construction.

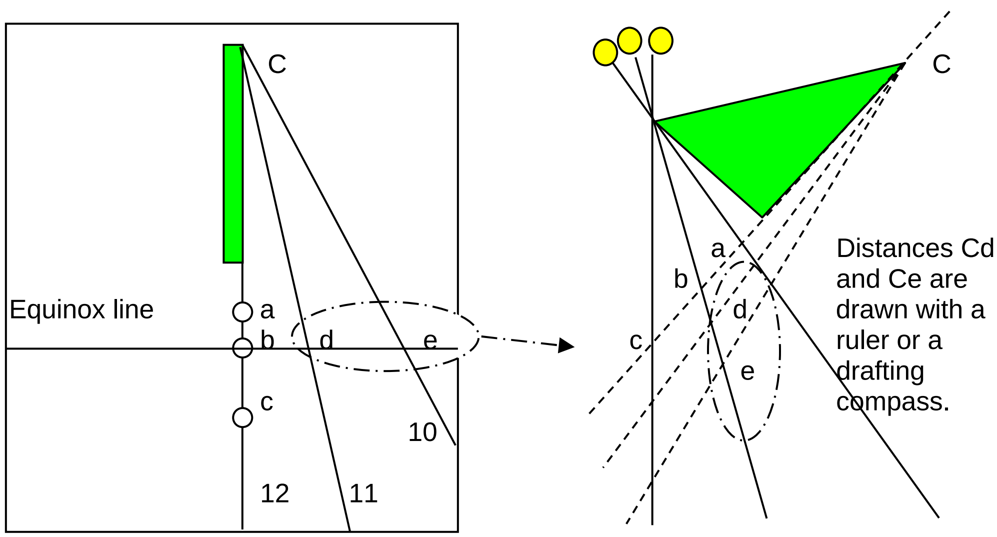
Declination lines for the horizontal (or vertical) dial using geometry.

A template is available in appendix 9 to facilitate this process.

A horizontal dial with three hour lines shown.

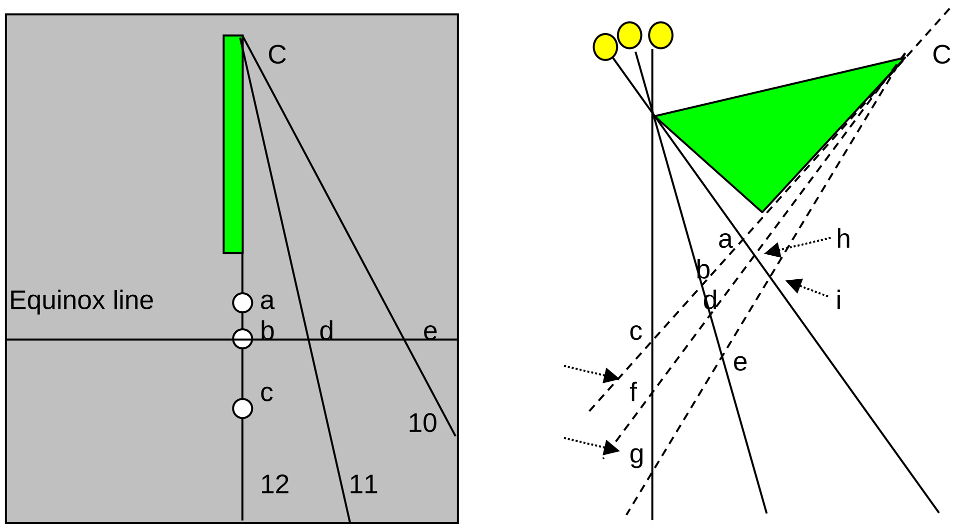


First, draw a gnomon for the dial center "C". From the nodus draw the equinox line (90 degrees to the style), and from that the solstice lines (approx 23.5 degrees on either side).The three lines (equinox and the solstices) intersect the gnomon's base line extended, or the noon line, at points a, b, and c. These three points whose distances from the dial center are Ca, Cb, and Cc are then transcribed to the dial plate (right pictorial to left pictorial).

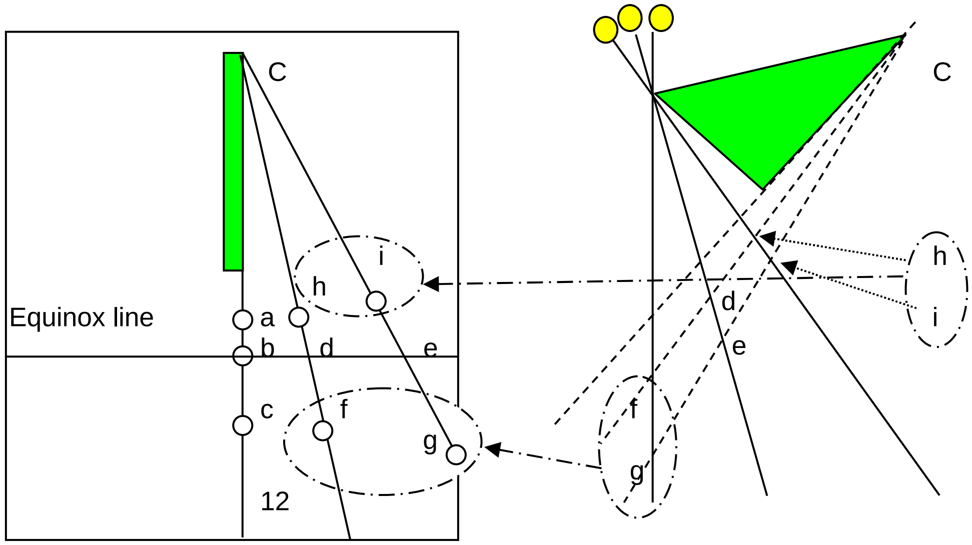


The equinox line is then drawn perpendicular to the noon line, and it produces equinox intercepts for those additional hour lines, d and e. Distances Cd and Ce are then located from the left dial plate to the right hand picture on its equinox line. This produces two more hour lines on the right hand side picture, Cd and Ce. Those hour lines on the right hand side do not have angles that match their hour lines on the dial plate, and this is because this is a projection.

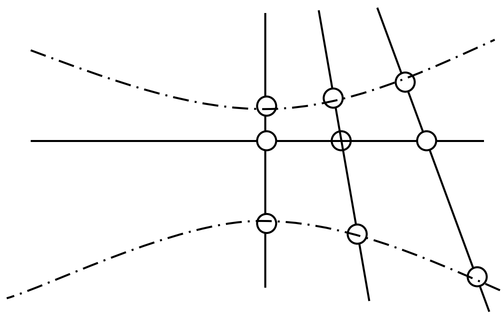
Now that there are two more hour lines, or as many as you choose, this produces intercept points for the solstice lines, namely points f, g, h and i.



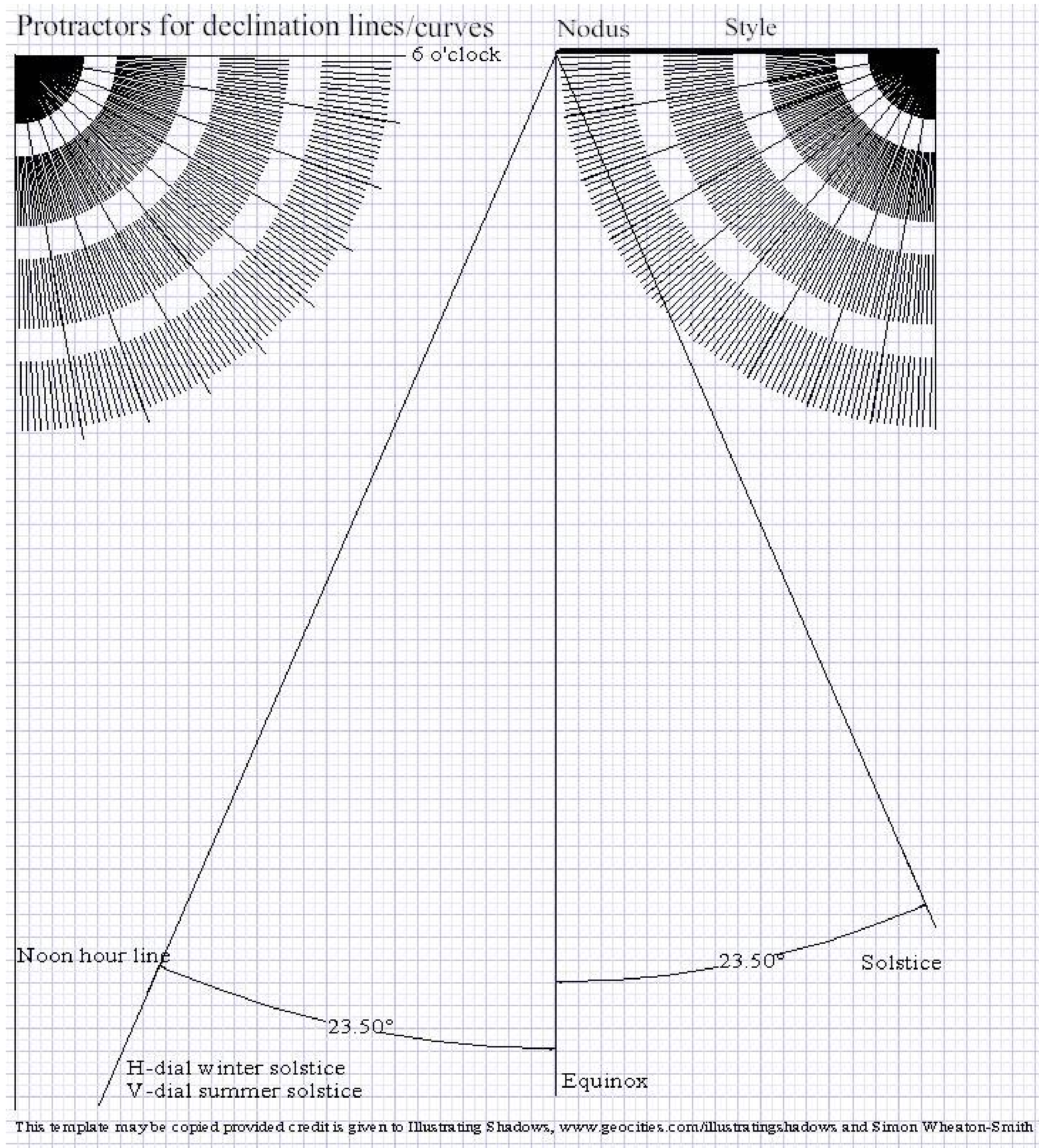
Points f, h, g, and i are now transferred back to the dial plate, from the right projection pictorial to the left picture.



When this process is completed for as many hour lines as desired, the dots are connected and then the declination lines drawn.



A DRAFTING SHEET FOR HORIZONTAL/VERTICAL DIAL DECLINATION LINES



CALENDAR POINTS ON HOUR LINES

- In what follows, it is assumed that the hour lines are not longitude corrected.
- For declining dials using this method, noon in what follows is the extension of the sub-style at the style distance, and the hours are based on noon-at-the-extended-style-distance. And noon for the purposes of the calendar lines is NOT the same as the real noon on the dial plate.
- No correction for the equation of time is used.

ADDITIONAL NOTES:

Discussed with Illustrating Time's Shadow Chapter 23 are several other techniques for deriving calendar line or curve data.

Additionally, the DeltaCAD programs (macros) provide graphical depictions of the calendar curves and even show them optionally in animation with changes to latitude.

Additionally, Programming Shadows provides some insights into methods to draft calendar or declination curves, and Supplemental Shadows has some methods that may be of interest.

Both Supplemental Shadows (over 100 pages), and Programming Shadows (over 200 pages), are provided free to download if you have acquired the big book Illustrating Time's Shadow.

Additionally, the JustBASIC programs offer tabular displays of calendar line data using an SH (style height) and with no longitude correction. This is because the calendar points provided are based on the SH derived local apparent time displacements and not on a resulting dial's actual location. These calendar data are transposed to the final real dial. In essence, the use of LAT hours on this surrogate dial is arbitrary. This BASIC program allows the any of the three sides of the gnomon to be specified along with the SH.

Additionally, the main spreadsheet provided with Illustrating Shadows includes these formulae.

USEFUL DECLINATIONS:

COMMON	ALTERNATIVE				
0	0	March		September	
10	12	February	April	August	October
18	20	January	May	July	November
23.5	23.5	December		June	